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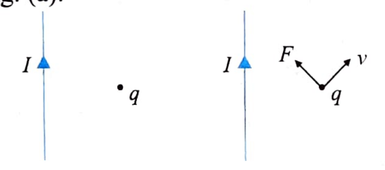
1

Magnetic Field and Magnetic Forces

INTRODUCTION

In our study of electricity, we described the interactions between charged objects in terms of electric fields. Recall that an electric field surrounds any electric charge. In addition to containing an electric field, the region of space surrounding any moving electric charge also contains a magnetic field. A magnetic field also surrounds a magnetic substance making up a permanent magnet. Historically, the symbol $\vec{B}$ has been used to represent a magnetic field, and we use this notation in this book. The direction of the magnetic field $\vec{B}$ at any location is the direction in which a compass needle points at that location. As with the electric field, we can represent the magnetic field by means of drawings with magnetic field lines.

A moving charge is a source of magnetic field. Consider this experiment: A point charge is kept near a current carrying wire as shown in Fig. (a).



(a) (b)

It is found that no force acts on charge if the charge is at rest. It means a current carrying wire does not produce electric field.

Now if the charge is given some velocity v in some direction as shown in Fig. (b), then a force is found to act on the charge as shown.

Now the question arises where from this force comes. Basically this force is due to the magnetic field produced by current carrying wire. From the above experiment we can conclude the following:

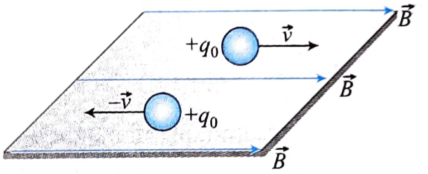
1. A current or a moving charge creates a magnetic field in the surrounding space (in addition to its electric field).
2. The magnetic field exerts a force on any other moving charge or current that is present in the field.

Like electric field, magnetic field is a vector field—that is, a vector quantity associated with each point in space. We will use the symbol $\vec{B}$ for magnetic field. At any position, the direction of $\vec{B}$ is defined as that in which the north pole of a compass needle tends to point.

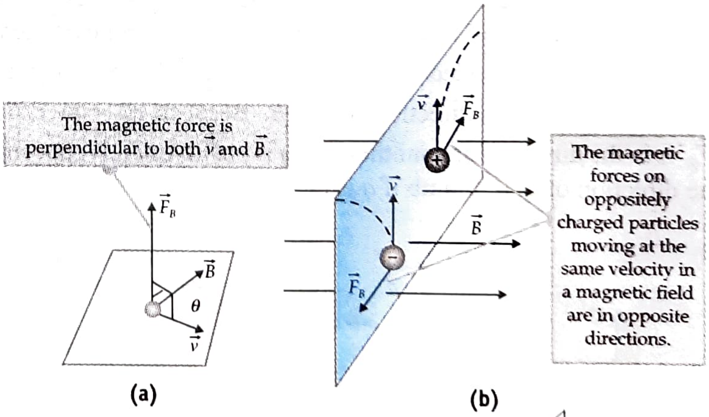
FORCE ON A MOVING CHARGE IN A MAGNETIC FIELD

We can define a magnetic field $\vec{B}$ at some point in space in terms of the magnetic force $\vec{F}_B$ the field exerts on a charged particle moving with a velocity $\vec{v}$, which we call the test object. For the time being, let's assume no electric or gravitational fields are present at the location of the test object. Experiments on various charged particles moving in a magnetic field give the following results:

- The magnitude F_B of the magnetic force exerted on the particle is proportional to the charge q and to the speed v of the particle.
- When a charged particle moves parallel to the magnetic field vector, the magnetic force acting on the particle is zero.



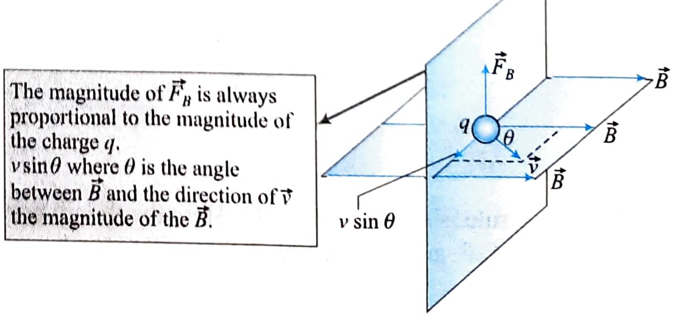
- When the particle's velocity vector makes any angle $\theta \neq 0$ with the magnetic field, the magnetic force acts in a direction perpendicular to both $\vec{v}$ and $\vec{B}$; i.e. $\vec{F}_B$ perpendicular to the plane formed by $\vec{v}$ and $\vec{B}$ [Fig. (a)]. The magnetic force exerted on a positive charge is in the direction opposite to the direction of the magnetic force exerted on a negative charge moving in the same direction [Fig. (b)].
- The magnitude of the magnetic force exerted on the moving particle is proportional to $\sin\theta$, where θ is the angle the particle's velocity vector makes with the direction of $\vec{B}$.



The magnetic force is perpendicular to both $\vec{v}$ and $\vec{B}$.

The magnetic forces on oppositely charged particles moving at the same velocity in a magnetic field are in opposite directions.

(a) (b)



The magnitude of $\vec{F}_B$ is always proportional to the magnitude of the charge q.
$v\sin\theta$ where θ is the angle between $\vec{B}$ and the direction of $\vec{v}$ the magnitude of the $\vec{B}$.

We can summarize all these results with the following equation:

$$|\vec{F}_B| = q(v\sin\theta)B$$

In vector form: $\vec{F}_B = q\vec{v}\times\vec{B}$

which by definition of the cross product is perpendicular to both $\vec{v}$ and $\vec{B}$. We can regard this equation as an operational definition of the magnetic field at some point in space. That is, the magnetic field is defined in terms of the force acting on a moving charged particle.

UNITS OF MAGNETIC FIELD STRENGTH

To discuss the motion of charges in magnetic fields, we need to know what units are used to measure the magnetic field strength. Solving above equation for the field strength and inserting the units of the other quantities gives

$$[F_B]=[q][v][B] \Rightarrow [B]=\frac{[F_B]}{[q][v]}=\frac{\text{N s}}{\text{C m}}$$

Because the ampere (A) is defined as 1 C/s, (N s)/(C m) = N/(A m). The unit of magnetic field strength has been named the **tesla** (T), in honor of Croatian-born American physicist and inventor Nikola Tesla:

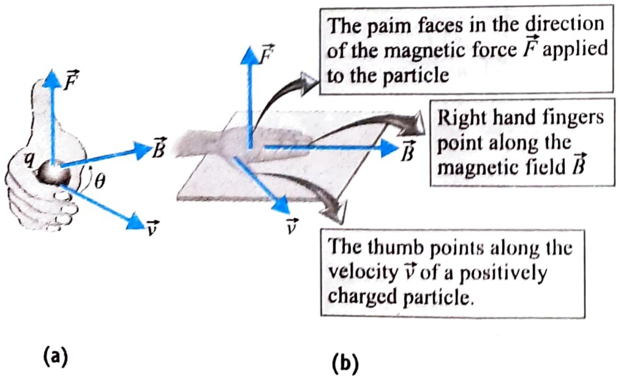
$$1\,\text{T} = 1\frac{\text{N s}}{\text{C m}} = 1\frac{\text{N}}{\text{A m}}$$

A tesla is a rather large amount of magnetic field strength. Sometimes magnetic field strength is given in gauss (G), which is not an SI unit:

$$1\,\text{G} = 10^{-4}\,\text{T}$$

RIGHT HAND RULES FOR DETERMINING THE DIRECTION OF THE MAGNETIC FORCE ACTING ON A MOVING CHARGED PARTICLE

The figure below reviews two right-hand rules for determining the direction of the cross product $\vec{v}\times\vec{B}$ and determining the direction of $\vec{F}_B$. The rule in Fig. (a) depends on our right-hand rule for the cross product in figure. Point the four fingers of your right hand along the direction of $\vec{v}$ with the palm facing $\vec{B}$ and curl them toward $\vec{B}$. Your extended thumb, which is at a right angle to your fingers, points in the direction of $\vec{v}\times\vec{B}$. Because $\vec{F}_B = q\vec{v}\times\vec{B}$, $\vec{F}_B$ is in the direction of your thumb if q is positive and is opposite the direction of your thumb if q is negative.


$\vec{F}$
q
$\vec{B}$
θ
$\vec{v}$
$\vec{F}$
$\vec{B}$
$\vec{v}$
The palm faces in the direction of the magnetic force $\vec{F}$ applied to the particle
Right hand fingers point along the magnetic field $\vec{B}$
The thumb points along the velocity $\vec{v}$ of a positively charged particle.
(a)
(b)

An alternative rule is shown in Fig. (b). Here the thumb points in the direction of $\vec{v}$ and the extended fingers in the direction of $\vec{B}$. Now, the force $\vec{F}_B$ on a positive charge extends outward from the palm. The advantage of this rule is that the force on the charge is in the direction you would push on something with your hand: outward from your palm. The force on a negative charge is in the opposite direction. You can use either of these two right-hand rules.

The magnitude of the magnetic force on a charged particle is $F_B = |q|vB\sin\theta$, where u is the smaller angle between $\vec{v}$ and $\vec{B}$. From this expression, we see that F_B is zero when $\vec{v}$ is parallel or antiparallel to $\vec{B}$ ($\theta = 0$ or 180°) and maximum when $\vec{v}$ is perpendicular to $\vec{B}$ ($\theta = 90°$).

There are important differences between electric and magnetic forces on charged particles:

- The electric force is always parallel or antiparallel to the direction of the electric field, whereas the magnetic force is perpendicular to the magnetic field.
- The electric force acts on a charged particle independent of the particle's velocity, whereas the magnetic force acts on a charged particle only when the particle is in motion and the force is proportional to the velocity.
- The electric force does work in displacing a charged particle, whereas the magnetic force associated with a constant magnetic field does no work when a charged particle is displaced.

This last statement is true because when a charge moves in a constant magnetic field, the magnetic force is always perpendicular to the displacement. Hence, the work done by the magnetic force on the particle is zero.

Proof: We observed that the force acting on a particle with charge q moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$ is

$$\vec{F} = q\vec{v}\times\vec{B}$$

The force is always perpendicular to the particle's velocity and so does no work on the particle:

$$dW = \vec{F}\cdot d\vec{s} = \vec{F}\cdot\vec{v}dt = 0, \text{ since } \vec{F} \text{ is perpendicular to } \vec{v}.$$

Magnetic of force can alter the direction of a particle's motion but cannot alter its energy. Since the force is also perpendicular to $\vec{B}$, it cannot change a particle's velocity component parallel to the field.

From the work–energy theorem, we conclude that the kinetic energy of a charged particle cannot be altered by a constant magnetic field alone. In other words, when a charge moves with a velocity, an applied magnetic field can alter the direction of the velocity vector, but it cannot change the speed of the particle.

ILLUSTRATION 1.1

A particle having mass m and charge q is released from the origin in a region in which electric field and magnetic field are given by $\vec{B} = -B_0\hat{j}$ and $\vec{E} = E_0\hat{k}$. Find the speed of the particle as a function of its z-coordinate.

Sol. Since the magnetic field does not perform any work, therefore, whatever has been the gain in kinetic energy it is only bec-ause of the work done by electric field. Applying work–energy theorem, $W_E = \Delta K$

$$qEz = \frac{1}{2}mv^2 - O \quad \text{or,} \quad v = \sqrt{\frac{2qEz}{m}}$$

LORENTZ FORCE

The combination of electric and magnetic forces is known as Lorentz forces. Let a charge particle moves in a magnetic field where both electric field ($\vec{E}$) and magnetic field ($\vec{B}$) are present. Then net force acting on the charge particle is given by

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

ILLUSTRATION 1.2

A charge $q = -4\ \mu C$ has an instantaneous velocity $\vec{v} = (2\hat{i} - 3\hat{j} + \hat{k}) \times 10^6\ ms^{-1}$ in a uniform magnetic field $\vec{B} = (2\hat{i} + 5\hat{j} - 3\hat{k}) \times 10^{-2}$ T. What is the force on the charge?

Sol. $\vec{F} = q\vec{v} \times \vec{B}$

$$= (-4 \times 10^{-6})[(2\hat{i} - 3\hat{j} + \hat{k}) \times 10^6$$

$$\times (2\hat{i} + 5\hat{j} - 3k) \times 10^{-2}]$$

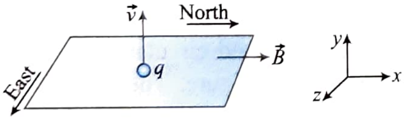
$$= -(-16\hat{i} + 32\hat{j} + 64\hat{k}) \times 10^{-2}\ \text{N}$$

$$= -16(\hat{i} + 2\hat{j} + 4\hat{k}) \times 10^{-2}\ \text{N}$$

ILLUSTRATION 1.3

A charged particle is having charge $q = 50\ \mu$C is projected vertically upward with a speed of 5.0×10^4 km/s in a region where a magnetic field of magnitude 2.0 T exits in the direction south to north. Find the magnetic force that acts on the α-particle.

Sol. Given $\vec{B} = 2.0(\hat{i})$ T, $\vec{v} = 5.0 \times 10^7 (\hat{j})$ m/s



The force acting on a charged particle moving in magnetic field is given by

$$\vec{F} = q\vec{v} \times \vec{B}$$

$$= 50 \times 10^{-6}[\{5.0 \times 10^7 (\hat{j})\} \times \{2.0(\hat{i})\}]$$

$$= 5.0 \times 10^3 (-\hat{k})\ \text{N}$$

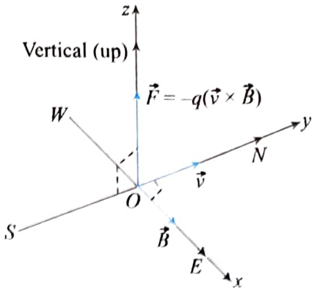
Hence the charged particle will experience a force 5 kN towards west direction.

ILLUSTRATION 1.4

An electron is moving northwards with a velocity 10^7 ms^{-1} in a magnetic field of 3 T directed eastwards. Calculate the instantaneous force on the electron. Given that charge on electron $= 1.6 \times 10^{-19}$ C.

Sol. Here, $q = 1.6 \times 10^{-19}$ C, $v = 10^7$ ms^{-1}; $B = 3$ T

As electron is moving northward and the magnetic field is eastwards, the angle between $\vec{v}$ and $\vec{B}$ is 90°.



Now, $F = qvB \sin 90°$

$$= 1.6 \times 10^{-19} \times 10^7 \times 3 \times 1 = 4.8 \times 10^{-12}\ \text{N}$$

Since the charge on electron is negative, force on electron,

$$\vec{F} = -q(\vec{v} \times \vec{B})$$

The direction of force on electron will be that of vector $-q(\vec{v} \times \vec{B})$ i.e., along vertically upwards.

Alternate: Velocity of the electron $\vec{v} = 10^7 (\hat{j})$ m/s

Magnetic field $\vec{B} = 3(\hat{i})T$

Force on electron $\vec{F} = q\vec{v} \times \vec{B}$

$$\vec{F} = (-1.6 \times 10^{-19})[10^7 \hat{j} \times 3\hat{i}]$$

$$= -4.8 \times 10^{-12} [\hat{j} \times \hat{i}]$$

$$= -4.8 \times 10^{-12} [-\hat{k}] = 4.8 \times 10^{-12} (\hat{k})\ \text{N}$$

ILLUSTRATION 1.5

A magnetic field of $(5.0 \times 10^{-3} \hat{k})$ T exerts a force of $(2.0\hat{i} + 1.5\hat{j}) \times 10^{-10}$ N on a particle having a charge of 1.0×10^{-9} C and going in the x-y plane. Find the velocity of the particle.

Sol. Let velocity of the charged particle be $(v_x\hat{i} + v_y\hat{j} + v_z\hat{k})$

Force on the charged particle $F = q(\vec{v} \times \vec{B})$

Yaha se banay links

$$(2.0\hat{i} + 1.5\hat{j}) \times 10^{-10} = 1.0 \times 10^{-9}[(v_x\hat{i} + v_y\hat{j} + v_z\hat{k}) \times 5.0 \times 10^{-3} (\hat{k})]$$

$$(2.0\hat{i} + 1.5\hat{j}) \times 10^{-10} = 5.0 \times 10^{-9}[v_x(\hat{i} \times \hat{k}) + v_y(\hat{j} \times \hat{k})]$$

$$(200\hat{i} + 150\hat{j}) = 5(-v_x\hat{j} + v_y\hat{i}) \qquad \text{...(i)}$$

From Eq. (i), we get $v_x = -\dfrac{200}{5}\hat{i}$ m/s $= -40\hat{i}$ m/s

and $v_y = \dfrac{150}{5}\hat{j}$ m/s $= 30\hat{j}$ m/s

Hence velocity of the charged particle $\vec{v} = (-40\hat{i} + 30\hat{j})$ m/s

ILLUSTRATION 1.6

A charged particle is projected in a magnetic field of $(5.0\hat{i} - 2.0\hat{j}) \times 10^{-3}$ T. If the acceleration of the particle is found to be $(x\hat{i} + 5.0\hat{j}) \times 10^{-6}$ m/s^2. What should be the value of x?

Sol. Force acting on a charged particle in magnetic field is given by $\vec{F} = q\,\vec{v} \times \vec{B}$

It means the force vector (or acceleration vector) and magnetic field vector should be perpendicular to each other.

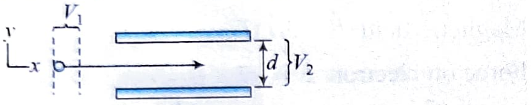
Hence, $\vec{a}.\vec{B} = 0 \Rightarrow ((x\hat{i} + 5.0\,\hat{j}) \times 10^{-6}).((5.0\hat{i} - 2.0\,\hat{j}) \times 10^{-3}) = 0$

$\Rightarrow \quad [5.0x + 5.0 \times (-2.0)] \times 10^{-9} = 0$

$\Rightarrow \quad 5.0x + 5.0 \times (-2.0) = 0 \text{ or } x = 2$

ILLUSTRATION 1.7

In figure given, an electron gets accelerated from rest through potential difference $V_1 = 500\,\text{V}$. It enters the gap between two parallel plates having separation $d = 20.0$ mm and potential difference $V_2 = 100\,\text{V}$. The upper plate is at the higher potential. Neglecting fringing and assuming that the electron's velocity vector is perpendicular to the electric field vector between the plates, find the magnitude and direction of uniform magnetic field which allows the electron to travel in a straight line in the gap?(Mass of electron $m_e = 9.0 \times 10^{-31}$ kg)



Sol. Straight-line motion will result from zero net force acting on the system; we ignore gravity. Due to electric field, the electron experiences force in upward direction, hence due to magnetic field it will experience force in down wards direction. Thus,

$$\overline{F} = q(\overline{E} + \vec{v} \times \overline{B}) = 0. \text{ Note that } \vec{v} \perp \overline{B} \text{ so } |\vec{v} \times \overline{B}| = vB.$$

Hence, $\overline{E} = -\vec{v} \times \overline{B} \Rightarrow |E| = |vB|$

The electric field between the plates,

$$E = \frac{V_2}{d} = \frac{100}{20 \times 10^{-3}} = 5.0 \times 10^4 \text{ V/m}$$

The electron gets accelerated through potential difference $V_1 = 500\,\text{V}$

Hence its kinetic energy, $K = eV_1 = \frac{1}{2}m_e v^2$

The speed of the electron,

$$v = \sqrt{\frac{2eV_1}{m_e}} = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 500}{9.0 \times 10^{-31}}} = \frac{4}{3} \times 10^7 \text{ m/s}$$

Hence magnetic field, $B = \frac{E}{v} = \frac{5.0 \times 10^4}{4/3 \times 10^7} = 3.75 \times 10^{-3}$ T

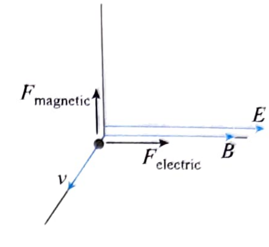
Since the velocity of the electron is in the +*x* direction, and the force due to magnetic field acts in downwards direction then using the right-hand rule we conclude that the magnetic field must point in the -*z* direction. In unit-vector notation, we have

$$\vec{B} = -(3.75 \times 10^{-3} \text{ T})\hat{k}$$

ILLUSTRATION 1.8

A magnetic field has a magnitude of 1.5×10^{-3} T, and an electric field has a magnitude of 4.0×10^3 N/C. Both fields point in the same direction. A positive 1.8 μC charge moves at a speed of 2.0×10^6 m/s in a direction that is perpendicular to both fields. Determine the magnitude of the net force that acts on the charge.

Sol. The magnetic field applies the maximum magnetic force to the moving charge, because the motion is perpendicular to the field. This force is perpendicular to both the field and the velocity. The electric field applies an electric force to the charge that is in the same direction as the field, since the charge is positive. These two forces are shown in the drawing, and they are perpendicular to one another. Therefore, the magnitude of the net force acting on the charge, $F = \sqrt{F^2_{\text{magnetic}} + F^2_{\text{electric}}}$



The magnetic force $\vec{F}_{\text{mag}} = q\,\vec{v} \times \vec{B}$ has a magnitude of $F_{\text{magnetic}} = |q|vB \sin\theta$, where $|q|$ is the magnitude of the charge, B is the magnitude of the magnetic field, v is the speed, and $\theta = 90°$ is the angle of the velocity with respect to the field. Thus,

$$F_{\text{magnetic}} = |q|vB$$

Andthe electric force has a magnitude of $F_{\text{electric}} = |q|E$

Hence,

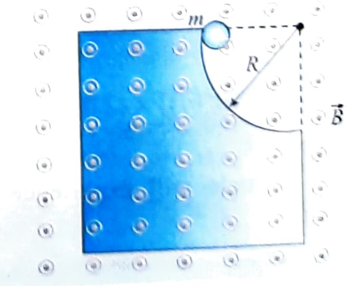
$$F = \sqrt{F^2_{\text{magnetic}} + F^2_{\text{electric}}} = \sqrt{(|q|vB)^2 + (|q|E)^2} = |q|\sqrt{(vB)^2 + E^2}$$

$$= (1.8 \times 10^{-6})\sqrt{[(2.0 \times 10^6)(1.5 \times 10^{-3})]^2 + (4.0 \times 10^3)^2}$$

$$= 9.0 \times 10^{-3} \text{ N}$$

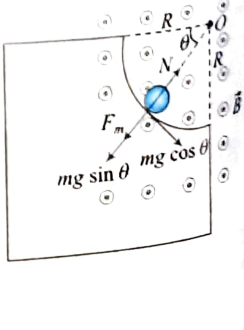
ILLUSTRATION 1.9

A charged sphere of mass m and charge q starts sliding from rest on a vertical fixed circular track of radius R from the position as shown in figure. There exists a uniform and constant horizontal magnetic field of induction B. Find the maximum force exerted by the track on the sphere.



Sol. Magnetic force on sphere

$F_m = qvB$ (directed radially outward)



$\because \quad N - mg\sin\theta - qvB = \frac{mv^2}{R}$

$\Rightarrow \quad N = \frac{mv^2}{R} + mg\sin\theta + qvB$

CONCEPT APPLICATION EXERCISE 1.1

1. A charged particle of mass 5 mg and charge $q = +2\ \mu\text{C}$ has velocity $\vec{v} = 2\hat{i} - 3\hat{j} + 4\hat{k}$. Find out the magnetic force on the charged particle and its acceleration at this instant due to magnetic field $\vec{B} = 3\hat{j} - 2\hat{k}$. $\vec{v}$ and $\vec{B}$ are in m s^{-1} and Wb m^{-2}, respectively.

2. A charged particle has acceleration $\vec{a} = 2\hat{i} + x\hat{j}$ in a magnetic field $\vec{B} = -3\hat{i} + 2\hat{j} - 4\hat{k}$. Find the value of x.

3. A particle with charge −5.60 nC is moving in a uniform magnetic field $\vec{B} = -(1.25\,\text{T})\hat{k}$. The magnetic force on the particle is measured to be
$$\vec{F} = -(3.36 \times 10^{-7}\ \text{N})\hat{i} + (7.42 \times 10^{-7}\ \text{N})\hat{j}.$$
   (a) Calculate all components of velocity of the particle from this information.
   (b) Calculate the scalar product $\vec{v}\cdot\vec{F}$. What is the angle between v and F?

4. A particle with charge 7.00 μC is moving with velocity $\vec{v} = -(4 \times 10^3\ \text{m s}^{-1})\hat{j}$. The magnetic force on the particle is measured to be $\vec{F} = +(8.4 \times 10^{-2}\ \text{N})\hat{i} - (5.60 \times 10^{-2}\ \text{N})\hat{k}$.
   (a) Calculate all the components of the magnetic field you can from this information.
   (b) Calculate the scalar product $\vec{B}\cdot\vec{F}$. What is the angle between $\vec{B}$ and $\vec{F}$?

5. When a proton has a velocity $\vec{v} = (2\hat{i} + 3\hat{j}) \times 10^6\ \text{ms}^{-1}$, it experiences a force $\vec{F} = -(1.28 \times 10^{-13}\ \hat{k})$ N. When its velocity is along the z-axis, it experiences a force along the x-axis. What is the magnetic field?

6. The force on a charged particle moving in a magnetic field can be computed as the vector sum of the force due to each separate component of the magnetic field. As an example, a particle with charge q is moving with speed v in the $-y$ direction. It is moving in a uniform magnetic field $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$.
   (a) What is the force $\vec{F}$ exerted on the particle by the magnetic field?
   (b) If $q > 0$, what must the signs of the components of $\vec{B}$ be if the components of $\vec{F}$ are all non-negative?
   (c) If $q < 0$ and $B_x = B_y = B_z > 0$, find $\vec{F}$ in terms of $|q|$, v and B_x.

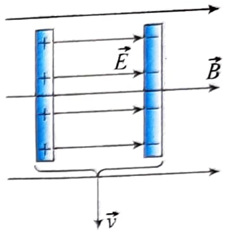
7. A particle of charge $q > 0$ is moving at speed v in the $+z$ direction through a region of uniform magnetic field. The magnetic force on the particle $\vec{F} = F_0(3\hat{i} + 4\hat{j})$, where F_0 is a positive constant.
   (a) Determine the components B_x, B_y and B_z or at least as many of the three components as is possible from the information given.
   (b) If it is given in addition that the magnetic field has magnitude $6F_0/qv$, determine the magnitude of B_z.

8. A charged particle of mass 10 g and charge 50 μT moves through auniform magnetic field, in a region where the free-fall acceleration is $-10\hat{j}$ m/s^2. The velocity of the particle is a constant $20\hat{i}$ km/s, which is perpendicular to the magnetic field. What, then, is the magnetic field?

9. An electron is moving through a uniform magnetic field given by $\vec{B} = B_x\hat{i} + (3.0B_x)\hat{j}$. At a particular instant, the electron has velocity $\vec{v} = (2.0\hat{i} + 4.0\hat{j})$ m/s and the magnetic force acting on it is $(6.4 \times 10^{-19}\ \text{N})\hat{k}$. Find B_x.

10. Two charged particles move in the same direction with respect to the same magnetic field. Particle 1 travels three times faster than particle 2. However, each particle experiences a magnetic force of the same magnitude. Find the ratio $|q_1|/|q_2|$ of the magnitudes of the charges.

11. The drawing shows a parallel plate capacitor that is moving with a speed of 20 m/s through a 5.0-T magnetic field. The velocity v is perpendicular to the magnetic field. The electric field within the capacitor has a value of 200 N/C, and each plate has an area of 5.0×10^{-4} m^2. What is the magnetic force (magnitude and direction) exerted on the positive plate of the capacitor?



12. The electrons in the beam of a television tube have a kinetic energy of 4.5×10^{-15} J. Initially, the electrons move horizontally from west to east. The vertical component of the earth's magnetic field points down, toward the surface of the earth, and has a magnitude of 2.25×10^{-5} T.
   (a) In what direction are the electrons deflected by this field component?
   (b) What is the acceleration of an electron in part (a)? (Mass of electron $m_e = 9.0 \times 10^{-31}$ kg)

ANSWERS

1. $2 \times 10^{-6}[-6\hat{i} + 4\hat{j} + 6\hat{k}]$ N; $0.8[-3\hat{i} + 2\hat{j} + 3\hat{k}]$ ms^{-2}

2. 3

3. (a) $v_x = -106$ ms^{-1}; $v_y = -48$ ms^{-1} (b) $\vec{v}\cdot\vec{F} = 0$; 90°

4. (a) $B_x = -2$ T, $B_z = -3$ T (b) $\vec{B}\cdot\vec{F} = 0$; 90° **5.** $-(0.4\hat{j})$ T

6. (a) $qvB_x\hat{k} - qvB_z\hat{i}$
(b) $B_x > 0$, $B_z < 0$, sign of B_y does not matter
(c) $|q|vB_x\hat{i} - |q|vB_x\hat{k}$

7. (a) $B_x = \dfrac{4F_0}{qv}$, $B_y = \dfrac{-3F_0}{qv}$, B_z is arbitrary (b) $\pm\dfrac{11F_0}{qv}$

8. $(-0.10\ \text{T})\hat{k}$ **9.** −2.0 T **10.** $\dfrac{1}{3}$

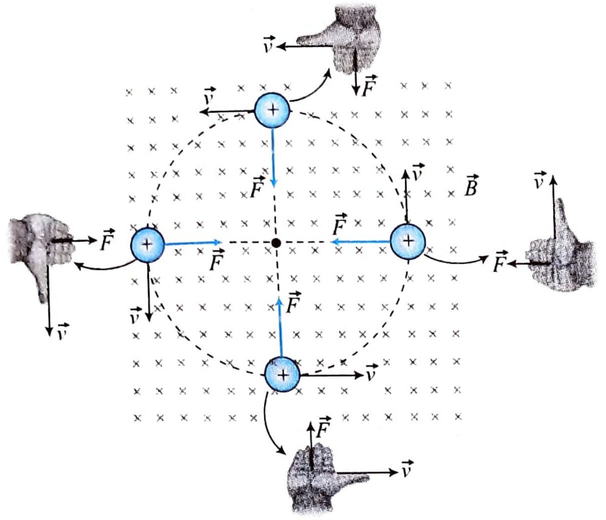
11. 8.85×10^{13} N, directed out of the page

12. (a) Due south (b) 4.0×10^{14} m/s^2

MOTION OF A CHARGED PARTICLE IN A MAGNETIC FIELD

Consider a charged particle of mass m projected in a uniform magnetic field $\vec{B}$ with an initial velocity vector $\vec{v}$ perpendicular to the field (figure below). B points away from the reader.

A force will act on the particle as shown which is perpendicular to the velocity at any instant. And also magnitude of the velocity will remain same, only direction will change. Finally the particle will move in a circular path.



In this situation, the particle moves in a circle with constant speed v and the magnetic force of magnitude $F = |q|vB$ supplies the centripetal force that keeps the particle moving in a circle. So we can write

$$\frac{mv^2}{r} = qvB \text{ or } r = \frac{mv}{qB}$$

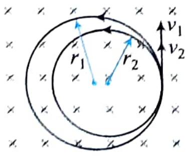
The above equation gives the radius of the circular path.

The angular speed ω of the particle is given by $\omega = \frac{v}{r} = \frac{qB}{m}$.

- The time period T of the motion is given by $T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$
- The time taken by charged particle to rotate an angle θ in magnetic field, $t = \left(\frac{\theta}{2\pi}\right)T = \frac{m\theta}{qB}$
- Frequency [or the number of revolutions completed in unit time] is given by $f = \frac{1}{T} = \frac{qB}{2\pi m}$.

Important Point:

We see that time period is independent of velocity given to the charge particle. If two identical charge particles are given different speeds from a point in the same direction, then they will return to the initial point simultaneously.



CHARGED PARTICLE ENTERING INTO MAGNETIC FIELD REGION FROM OUTSIDE

If a charged particle enters into an uniform magnetic field region, the particle starts moving in circular arc, but it can not complete a full circle.

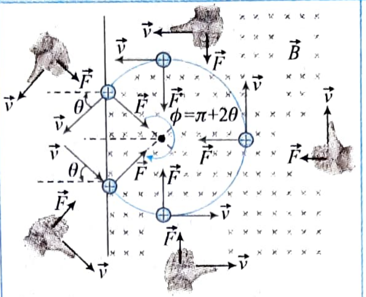
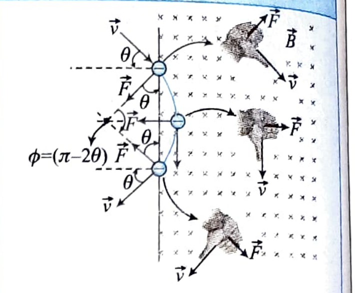
Motion of a positively charged particle	Motion of a negatively charged particle
 $\phi=\pi+2\theta$ θ $\vec{v}$ $\vec{F}$ $\vec{B}$	 $\phi=(\pi-2\theta)$ θ $\vec{v}$ $\vec{F}$ $\vec{B}$
Time taken by charged particle in magnetic field $t = \left(\frac{\phi}{2\pi}\right)T = \left(\frac{\phi}{2\pi}\right)\frac{2\pi m}{qB} = \frac{m\phi}{qB}$ $\Rightarrow t = \frac{m(\pi+2\theta)}{qB}$	Time taken by charged particle in magnetic field $t = \left(\frac{\phi}{2\pi}\right)T = \left(\frac{\phi}{2\pi}\right)\frac{2\pi m}{qB} = \frac{m\phi}{qB}$ $\Rightarrow t = \frac{m(\pi-2\theta)}{qB}$

ILLUSTRATION 1.10

Doubly-ionized helium ions are projected with a speed of 10 km s^{-1} in a direction perpendicular to a uniform magnetic field of magnitude 1.0 T. Find **(a)** the force acting on an ion, **(b)** the radius of the circle in which it circulates and **(c)** the time taken by an ion to complete the circle.

Sol.

(a) $F = qvB\sin\theta$

$$= (2\times1.6\times10^{-19})\times(10\times10^3)\times1.0\times\sin 90^\circ$$

$$= 3.2\times10^{-15}\text{ N}$$

(b) $r = \frac{mv}{qB} = \frac{(4\times1.67\times10^{-27})\times10\times10^3}{(2\times1.6\times10^{-19})\times1.0} = 2.1\times10^{-4}\text{ m}$

(c) $T = \frac{2\pi m}{qB} = \frac{2\pi\times(4\times1.67\times10^{-27})}{(2\times1.6\times10^{-19})\times1.0} = 1.32\times10^{-7}\text{ s}$

ILLUSTRATION 1.11

A stream of protons and deuterons in a vacuum chamber enters a uniform magnetic field. Both protons and deuterons have been subjected to same accelerating potential, hence the kinetic energies of the particles are the same. If the ion-stream is perpendicular to the magnetic field and the protons move in a circular path of radius 15 cm, find the radius of the path traversed by the deuterons. Given that mass of deuteron is twice that of a proton.

Sol. The radius r of the path is given by

$$\frac{mv^2}{r} = qvB \quad\text{or}\quad r = \frac{mv}{qB}$$

For proton, $r_p = \frac{m_p v_p}{q_p B}$

For deuteron, $r_d = \frac{m_d v_d}{q_d B}$

$\therefore \quad \frac{r_d}{r_p} = \frac{m_d v_d}{m_p v_p} \times \frac{q_p}{q_d}$ ...(i)

Given that $\frac{1}{2} m_p v_p^2 = \frac{1}{2} m_d v_d^2$ also $q_p = q_d$

Now $\frac{v_d}{v_p} = \left(\frac{m_p}{m_d}\right)^{1/2}$ ...(ii)

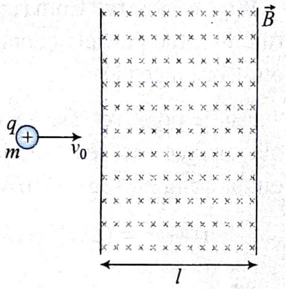
Subsituting the value of v_d/v_p from Eq. (ii) in Eq. (i) we get

$$\frac{r_d}{r_p} = \frac{m_d}{m_p}\left(\frac{m_p}{m_d}\right)^{1/2} = \left(\frac{m_d}{m_p}\right)^{1/2} = \sqrt{2} \qquad (\because \; m_d = 2m_p)$$

or $r_d = \sqrt{2}.r_p = 1.414 \times 0.15 = 0.212$ m

ILLUSTRATION 1.12

A positive charged particle having charge q and mass m is projected into a region having perpendicular uniform magnetic field B. The particle enter into a magnetic field region with speed v_0 normal to the boundary of magnetic field as shown in the figure. Find the angle of deviation of the particle from its path when it comes out of the magnetic field region, if the width of the region



(i) l is slightly greater than $\frac{mv_0}{qB}$

(ii) $l = \frac{mv_0}{qB}$

(iii) $l = \frac{mv_0}{2qB}$

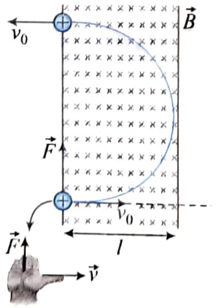
Sol. When the particle enters into uniform magnetic field region, its path becomes circular. The radius of the circular path,

$$R = \frac{mv_0}{qB}$$

Case I: l is slightly greater than $\frac{mv_0}{qB}$

In this case the radius of the circular path is less than the width of the magnetic field region. Hence the particle will return to the same side of the magnetic field region.

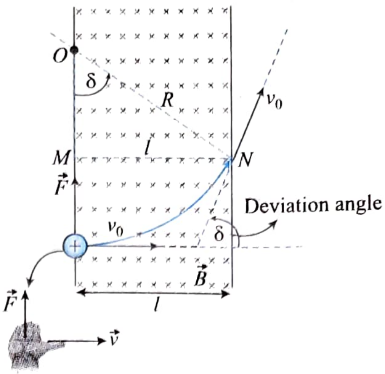
Hence, the angle of deviation should be 180°.



If $l < \frac{mv_0}{qB}$, the particle will came out from other side of magnetic field region as shown in figure.

In ΔOMN

$$\sin\delta = \frac{l}{R} = \frac{qBl}{mv_0} \qquad \text{...(i)}$$



Here δ is the angle of deviation.

Case II: If $l = \frac{mv_0}{qB}$

$$\sin\delta = 1 \quad \text{or} \quad \delta = \frac{\pi}{2}$$

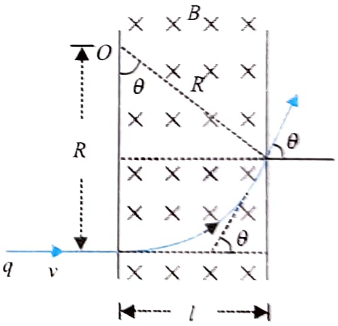
Case III: If $l = \frac{mv_0}{2qB}$

From (i), $\sin\delta = \frac{1}{2}$ or $\delta = \frac{\pi}{6}$

ILLUSTRATION 1.13

An α-particle is accelerated by a potential difference of 10^4 V. Find the change in its direction of motion, if it enters normally in a region of thickness 0.1 m having transverse magnetic induction of 0.1 tesla. (Given: mass of α-particle 6.4×10^{-27} kg).

Sol. The situation is shown in figure.



When a charged particle with charge q is accelerated through a potential difference V volt, then

$$\frac{1}{2}mv^2 = qV$$

or $\quad v = \sqrt{\left(\frac{2qV}{m}\right)}$ ...(i)

α-particle in magnetic field moves in a circle of radius R which is given by

$$R = \frac{mv}{qB} \quad \text{or} \quad R = \frac{1}{B}\sqrt{\left(\frac{2mV}{q}\right)} \qquad \text{...(ii)}$$

The change in direction of α-particle (θ) from the figure is given by

$$\sin\theta = \frac{l}{R} = lB\sqrt{\left(\frac{q}{2mV}\right)}$$

Here $l = 0.1$ m, $B = 0.1$ tesla, $V = 10^4$ volt

$q = 2e = 2 \times 1.6 \times 10^{-19} = 3.2 \times 10^{-19}$ C

and $m = 6.4 \times 10^{-27}$ kg

$$\therefore \quad \sin\theta = 0.1 \times 0.1 \times \sqrt{\left(\frac{3.2 \times 10^{-19}}{2 \times 6.4 \times 10^{-27} \times 10^4}\right)} = \frac{1}{2}$$

or $\quad \theta = 30°$

Note: In the above illustration,

1. If d were slightly less than (mv/qB), then the particle leaves the magnetic field just before it is able to turn back. Hence the angle of deviation is 90°.
2. If d were slightly greater than (mv/qB), then the particle turns around and completes a semicircle. When it leaves the magnetic field its angle of deviation is 180°.

ILLUSTRATION 1.14

A charged particle of mass m and charge q is accelerated through a potential difference of V volts. It enters a region of uniform magnetic field which is directed perpendicular to the direction of motion of the particle. Find the radius of circular path moved by the particle in magnetic field.

Sol. Since the particle is accelerated through V volts, therefore its kinetic energy will be equal to qV.

or $\frac{1}{2}mv^2 = qV$. Therefore, $v = \sqrt{\frac{2qV}{m}}$.

Radius of circular path is given by $R = \frac{mv}{qB} = \sqrt{\frac{2mV}{qB^2}}$.

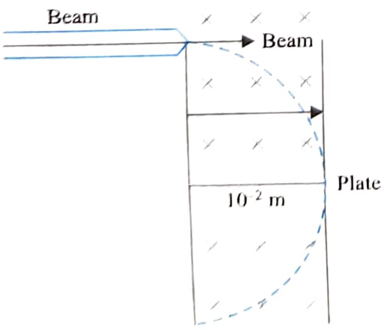
ILLUSTRATION 1.15

A beam of charged particle , having kinetic energy 10^3 eV, contains masses 8×10^{-27} kg and 1.6×10^{-26} kg emerge from the end of an accelerator tube. There is a plate at distance 10^{-2} m from the end of the tube and placed perpendicular to the beam. Calculate the magnitude of the smallest magnetic field which can prevent the beam from striking the plate.

Sol. Let $\vec{B}$ be required magnetic field and E_k the kinetic energy. Maximum radius of circular path for the beam not to strike the plane

$$r = \frac{mv}{qB} = \frac{\sqrt{2mE_k}}{qB} \Rightarrow B = \frac{\sqrt{2mE_K}}{qr}$$

Here $r = 10^{-2}$ m is same for both types of particles. For the particle of larger mass, B required will be more. This B will be minimum value required to prevent the beam from striking the plate.

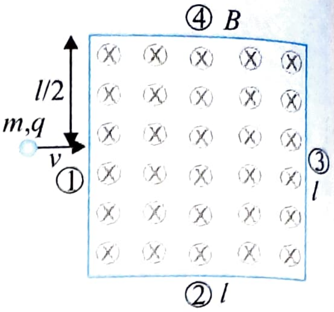


$$B_{\min} = \frac{\sqrt{2 \times 1.6 \times 10^{-26} \times (10^3 \times 1.6 \times 10^{-19})}}{1.6 \times 10^{-19} \times 10^{-2}}$$

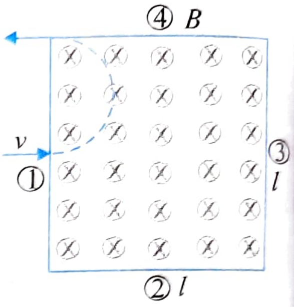
$$= \frac{1.6\sqrt{2} \times 10^{-21}}{1.6 \times 10^{-21}} = \sqrt{2}\text{ T} = 1.414\text{ T}$$

ILLUSTRATION 1.16

The magnetic field is confined in a square region. A positive charged particle of charge q and mass m is projected as shown in figure. Find the limiting velocities of the particle so that it may come out of face (1), (2), (3), and (4).



Sol. For the positive charge coming out from face (1), the radius of the path in magnetic field should be less than or equal to $l/4$. For limiting case



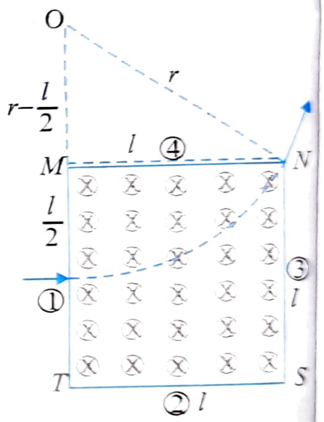
$$r_{\max} = \frac{l}{4} = \frac{mv}{qB}$$

$$\Rightarrow \quad v_{\max} = \frac{qBl}{4m}$$

Hence, if the velocity is $< \frac{qBl}{4m}$, the charge particle comes out of face (1).

We can observe from right palm rule that the particle cannot come out from face (1).

For a positive charge coming out of face (4), let particle come out at point N from ΔOMN.



$$(OM)^2 = (ON)^2 + (MN)^2$$

$$r^2 = \left(r - \frac{l}{2}\right)^2 + l^2$$

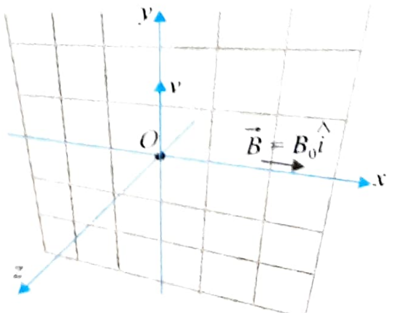
$$\Rightarrow \quad r = \frac{5}{4}l$$

If the particle comes out from face (4), $r < \frac{5}{4}l \Rightarrow \frac{mv}{qB} < \frac{5}{4}l$

or $v < \frac{5}{4}\frac{qBl}{m}$. If velocity $v > \frac{5}{4}\frac{qBl}{m}$, the particle will come out from face (3).

ILLUSTRATION 1.17

A charge particle of mass m and charge q is projected with velocity v along y-axis at $t = 0$.



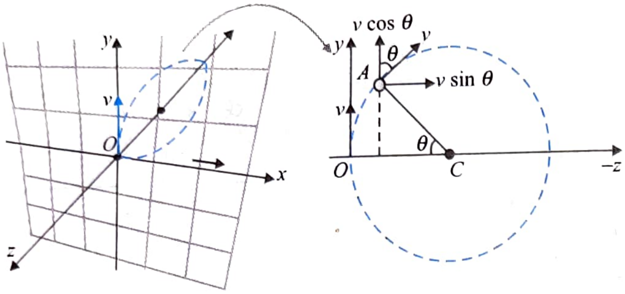
Find the velocity vector and position vector of the particle $\vec{v}(t)$ and $\vec{r}(t)$ in relation with time.

Sol. Radius of the circular path $R = \dfrac{mv}{qB}$

Angular velocity of the particle $\omega = \dfrac{v}{R} = \dfrac{qB}{m}$

The speed of the particle will remain unchanged. The center of circular path will be located on $-z$-axis at point C. Let us take time interval t. during this time interval the particle will rotate an angle $\theta = \omega t = \dfrac{qBt}{m}$.

Let us draw the front view of the circular path and locate the instantaneous position of the particle.



Writing velocity vector at time t.

$$\vec{v} = v\cos\theta\,\hat{j} - v\sin\theta\,\hat{k}$$

$$\vec{v}(t) = v\cos\left(\frac{qBt}{m}\right)\hat{j} - v\sin\left(\frac{qBt}{m}\right)\hat{k}$$

Now writing y and z coordinates of the particle.

y-coordinate of $P = R\sin\theta$

z-coordinate of $P = -(R - R\cos\theta)$

$$\vec{r}(t) = R\sin\frac{qBt}{m}\,\hat{j} - R\left(1-\cos\frac{qBt}{m}\right)\hat{k}$$

$$\left[\text{where} \quad R = \frac{mv}{qB}\right]$$

IF PARTICLE IS PROJECTED AT SOME ANGLE WITH MAGNETIC FIELD (HELICAL PATHS)

If a charged particle is projected into uniform magnetic field in such a way that the velocity of a charged particle has a component parallel to the magnetic field, the particle will move in a helical path about the direction of the field vector. Figure, for example, shows the velocity vector $\vec{v}$ of such a particle resolved into two components, one parallel to $\vec{B}$ and one perpendicular to it:

$$v_{\parallel} = v\cos\alpha \quad \text{and} \quad v_{\perp} = v\sin\alpha \qquad \text{...(i)}$$

With perpendicular component of velocity, it moves in a circular path of radius $r = \dfrac{mv_{\perp}}{qB} = \dfrac{mv\sin\alpha}{qB}$ and, with parallel component of velocity it moves along the field lines.

The linear distance travelled (along the field line) in one revolution (time period) is called pitch (p) (figure). The parallel component determines the pitch p of the helix—that is, the distance between adjacent turns.

Pitch: $p = v_{\parallel}T = \dfrac{2\pi m v_{\parallel}}{qB} = \dfrac{2\pi m v\cos\alpha}{qB}$

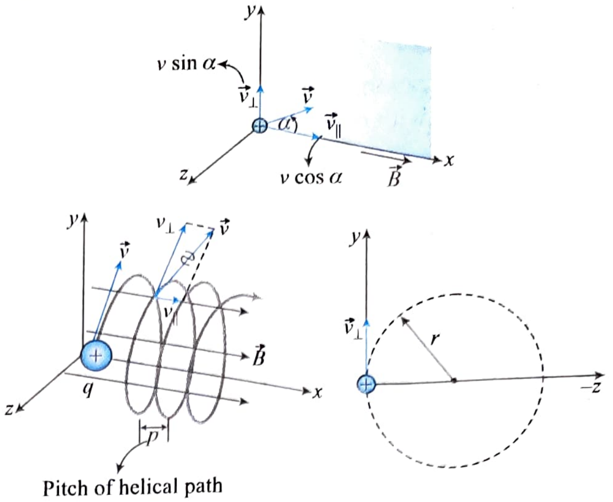
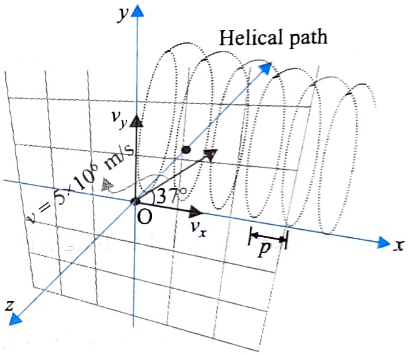


ILLUSTRATION 1.18

A proton (charge 1.6×10^{-19} C, mass = 1.60×10^{-27} kg) is shot with a speed 5×10^6 m s^{-1} at an angle of 37° with the X-axis. A uniform magnetic field $B = 0.30$ T exists along the X-axis. Show that path of the proton is a helix. Find the radius and pitch of the helix.

Sol. The situation is shown in figure.



$$v_x = v\cos 37° = (5\times 10^6)\left(\frac{4}{5}\right) = 4.0\times 10^6\ \text{ms}^{-1}$$

$$v_y = v\sin 37° = (5\times 10^6)\left(\frac{3}{5}\right) = 3.0\times 10^6\ \text{ms}^{-1}$$

Since the velocity has both components, parallel and transverse to magnetic field, so the resulting path will be helix. Due to combined action of v_x, v_y and B, the proton moves in a helical path.

The radius of helix is

$$r = \frac{mv_y}{qB} = \frac{(1.60\times 10^{-27})(3\times 10^6)}{(1.6\times 10^{-19})\times 0.3} = 0.10\ \text{m}$$

Time taken to complete one circle:

$$T = \frac{2\pi m}{qB} = \frac{2\pi \times 1.6\times 10^{-27}}{1.6\times 10^{-19}\times 0.3} = \frac{2\pi}{3}\times 10^{-7}\ \text{s}$$

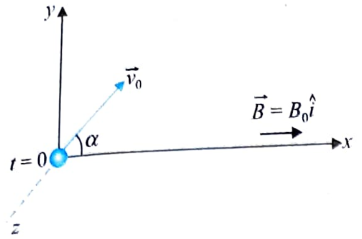
The pitch of the helix

= Distance travelled by the proton along x-axis in time T.

$$p = v_x \times T = (4.0\times 10^6)\times\left(\frac{2\pi}{3}\times 10^{-7}\right) = \frac{8\pi}{30}\ \text{m}$$

ILLUSTRATION 1.19

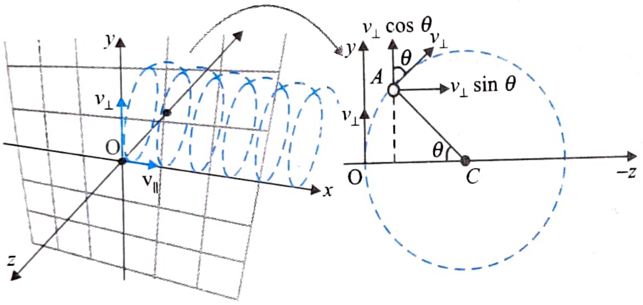
If in previous illustration, The charge particle is projected at angle α with x-axis with magnitude of velocity. Find



(a) velocity vector in function of time $\vec{v}(t)$.

(b) position vector in function of time $\vec{r}(t)$.

Sol. We can make components of the velocity of the particle along x-axis and y-axis. The component which is along the magnetic field will remain unchanged. But the component perpendicular to the magnetic field will provide circular motion. In a combined way, the path of the particle will be helical.



The component of velocity parallel to magnetic field

$$v_{\|} = v\cos\alpha$$

The component of velocity perpendicular to magnetic field

$$v_\perp = v\sin\alpha$$

The y and z components of velocity and position vector can be calculated in the same way as in previous illustration. The only difference is in place of v we substitute $v_\perp$. Hence, velocity vector $[\vec{v}_\perp(t)]$ in function of time can be written as

$$\vec{v}_\perp(t) = v_\perp \cos\frac{qB}{m}t\,\hat{j} - v_\perp \sin\frac{qB}{m}t\,\hat{k}$$

Hence net velocity

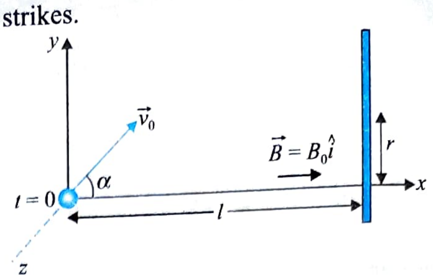
$$\vec{v} = v_{\|}\hat{i} + v_\perp \cos\frac{qB}{m}t\,\hat{j} - v_\perp \sin\frac{qB}{m}t\,\hat{k}$$

And position vector in function of time

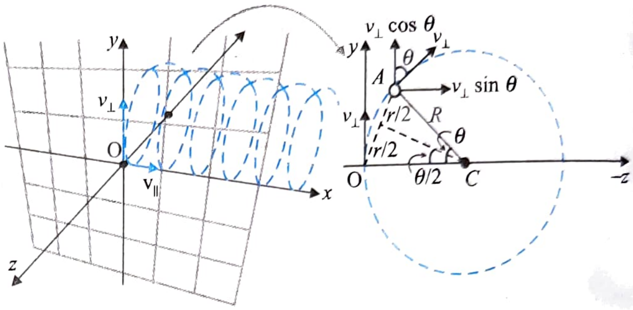
$$\vec{r}(t) = v_{\|}t\,\hat{i} + R\sin\frac{qBt}{m}\hat{j} - R\left(1-\cos\frac{qBt}{m}\right)\hat{k}$$

ILLUSTRATION 1.20

A non-relativistic charge q of mass m originates at a point A lying on x-axis and moves with velocity v at an angle α to the x-axis. A screen is located at a distance l from A. Find the distance r from the x-axis to the point on the screen into which the charge strikes.



Sol. Time taken by particle to reach the screen: $t = \dfrac{l}{v\cos\alpha}$



Radius of helical path: $R = \dfrac{mv\sin\alpha}{qB}$

angle rotated during this time $\theta = \omega t = \dfrac{qB}{m}t$

Required distance:

$$r = 2R\sin\left(\frac{\theta}{2}\right) = \frac{2mv\sin\alpha}{qB}\sin\left(\frac{qB}{2m}\frac{l}{v\cos\alpha}\right)$$

ILLUSTRATION 1.21

An electron accelerated by a potential difference $V = 1.0$ kV moves in a uniform magnetic field at an angle $\alpha = 30°$ to the vector B whose modulus is $B = 29$ mT. Find the pitch of the helical trajectory of the electron.

Sol. Time period of the electron moving in helical path

$$T = \frac{2\pi m}{eB} = \frac{2\times 3.14\times(9.1\times10^{-31})}{(1.6\times10^{-19})(29\times10^{-3})} = 1.232\times10^{-9}\text{ s}$$

The KE acquired by the electron is given by

$$\frac{1}{2}mv^2 = eV \quad \text{or} \quad v = \sqrt{\frac{2eV}{m}}$$

$$\therefore \quad v = \sqrt{\left[\frac{2\times(1.6\times10^{-19})\times10^3}{9.1\times10^{-31}}\right]} = 1.875\times10^7\text{ m s}^{-1}$$

Now, if $v_{\|}$ be the velocity of electron parallel to magnetic field, then

$$v_{\|} = v\cos\alpha = 1.875\times10^7\cos30 = 1.624\times10^7\text{ m s}^{-1}$$

$$\therefore \quad \text{Pitch} = v_{\|}\times T = (1.624\times10^7)(1.232\times10^{-9})$$

$$= 2\times10^{-2}\text{ m} = 2\text{ cm}$$

PATH OF A CHARGED PARTICLE IN BOTH ELECTRIC AND MAGNETIC FIELDS

Consider a particle of charge q and mass m released from the origin with velocity $\vec{v}=v_0\hat{i}$ into a region of uniform electric and magnetic fields parallel to y-axis, *i.e.*, $\vec{E}=E_0\hat{j}$ and $\vec{B}=B_0\hat{j}$. The electric field accelerates the particle in y-direction, i.e., y component of velocity goes on increasing with acceleration,

$$a_y=\frac{F_y}{m}=\frac{F_e}{m}=\frac{qE_0}{m}.$$

The magnetic field rotates the particle in a circle in x–z plane (perpendicular to magnetic field). The resultant path of the particle is a helix with increasing pitch. The axis of the helix is parallel to y-axis. Velocity of the particle at time t would be,

$$\vec{v}(t)=v_x\hat{i}+v_y\hat{j}+v_z\hat{k}$$

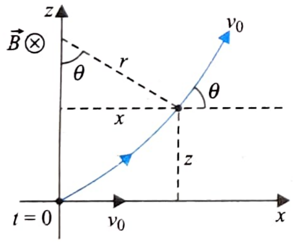
Here $v_y=a_y t=\dfrac{qE_0}{m}t$

and $v_x^2+v_z^2=\text{constant}=v_0^2;\quad \theta=\omega t=\dfrac{Bq}{m}t$

$$v_x=v_0\cos\theta=v_0\cos\left(\frac{Bqt}{m}\right)$$

and $v_z=v_0\sin\theta=v_0\sin\left(\dfrac{Bqt}{m}\right)$

$$\vec{v}(t)=v_0\cos\left(\frac{Bqt}{m}\right)\hat{i}+\left(\frac{qE_0}{m}t\right)\hat{j}+v_0\sin\left(\frac{Bqt}{m}\right)\hat{k}$$



Similarly, position vector of particle at time t can be given by,

$$\vec{r}(t)=x\hat{i}+y\hat{j}+z\hat{k}$$

Here, $y=\dfrac{1}{2}a_y t^2=\dfrac{1}{2}\left(\dfrac{qE_0}{m}\right)t^2$

$$x=r\sin\theta=\left(\frac{mv_0}{Bq}\right)\sin\left(\frac{Bqt}{m}\right)$$

and $z=r(1-\cos\theta)=\left(\dfrac{mv_0}{Bq}\right)\left[\left\{1-\cos\left(\dfrac{Bqt}{m}\right)\right\}\right]$

$$\vec{r}(t)=\left(\frac{mv_0}{Bq}\right)\sin\left(\frac{Bqt}{m}\right)\hat{i}+\frac{1}{2}\left(\frac{qE_0}{m}\right)t^2\hat{j}$$
$$+\left(\frac{mv_0}{Bq}\right)\left[\left\{1-\cos\left(\frac{Bqt}{m}\right)\right\}\right]\hat{k}$$

ILLUSTRATION 1.22

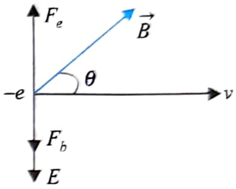
An electron accelerated through a potential difference of 2.5 kV, moves horizontally into a region of space in which there is a downward directed uniform electric field of magnitude 10 kVm^{-1}.

(a) In what direction must a magnetic field be applied so that the electron moves undeflected? Ignore the gravitational force. What is the magnitude of the smallest magnetic field possible in this case?

(b) What happens if the charge is a proton that passes through the same combination of fields?

Sol. Velocity of electron: $\dfrac{1}{2}mv^2=2.5\times10^{-3}\times1.6\times10^{-19}$

$$\Rightarrow\quad v=\sqrt{\frac{2\times2.5\times10^3\times1.6\times10^{-19}}{9.1\times10^{-31}}}=2.96\times10^7\text{m s}^{-1}$$



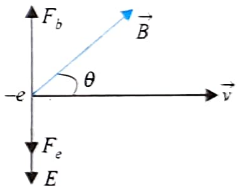
(a) For the electron to move undeflected: (magnetic field should be perpendicular to E, but it can be at any angle with $\vec{v}$)

$$F_b=F_e\Rightarrow evB\sin\theta=eE$$

$$\Rightarrow\quad B=\frac{E}{v\sin\theta}$$

for smallest B, $\sin\theta$ is maximum, i.e. $\theta=90°$

$$\Rightarrow\quad B_{\min}=\frac{E}{v}=\frac{10\times10^3}{2.96\times10^7}=3.38\times10^{-4}\text{T}$$



(b) B or $B_{\min}$ is independent of magnitude and sign of charge if the particle moves undeflected. So in this case also

$$B_{\min}=3.38\times10^{-4}\text{T}$$

But we see that direction of both F_e and F_b is opposite to that in part (a).

$$B_{\min}=\frac{E}{v}=\frac{10\times10^3}{2.96\times10^7}=3.38\times10^{-4}\text{T}$$

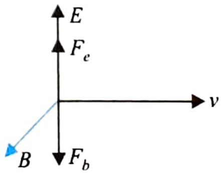
ILLUSTRATION 1.23

A non-relativistic proton beam passes without deviation through a region of space where there are uniform transverse mutually perpendicular electric and magnetic fields with E = 120 kV m^{-1} and B = 50 mT. Then the beam strikes a grounded target. Find the force which the beam acts on the target if the beam current is equal to i = 0.8 mA.
Mass of protons = 1.67×10^{-27} kg

Sol. Here the proton beam passes without deviation, hence

$$F_b=F_e\quad\Rightarrow\quad Bev=eE$$

$$\therefore\quad v=\frac{E}{B}=\frac{120\times10^3}{50\times10^{-3}}=2.4\times10^6\text{ m s}^{-1}$$



Number of protons reaching per second

$$\frac{nq}{t} = 0.8 \times 10^{-3}$$

or $$\frac{n \times 1.6 \times 10^{-19}}{1} = 0.8 \times 10^{-3} \quad \text{or} \quad n = 5 \times 10^{15}$$

Total mass of proton reaching per second

$$= (5 \times 10^{15}) \times (1.67 \times 10^{-27}) = 8.35 \times 10^{-12} \text{ kg}$$

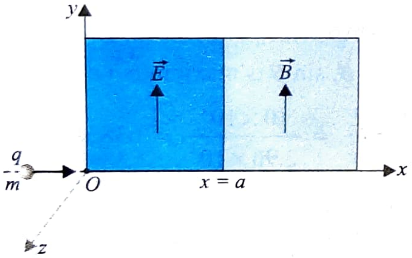
$$\text{Force} = \frac{\text{Change in momentum}}{\text{time}}$$

$$= [(8.35 \times 10^{-12})(2.4 \times 10^{6})]/1$$

$$= 20.04 \times 10^{-6} \text{ N} = 20.04 \text{ μN}$$

ILLUSTRATION 1.24

A positively charged particle, having charge q, is accelerated by a potential difference V. This particle moving along the x-axis enters a region where an electric field E exists. The direction of the electric field is along positive y-axis. The electric field exists in the region bounded by the lines $x = 0$ and $x = a$. Beyond the line $x = a$ (i.e., in the region $x > a$), there exists a magnetic field of strength B, directed along the positive y-axis. Find



(a) at which point does the particle meet the line $x = a$?

(b) the pitch of the helix formed after the particle enters the region $x \geq a$.

(Mass of the particle is m.)

Sol. The work done by the potential difference gets stored as its kinetic energy.

$$\therefore \quad \frac{1}{2}mv^2 = qV \Rightarrow v = \left(\frac{2qV}{m}\right)^{1/2} \qquad \text{...(i)}$$

(a) When it enters the region $x \in [0, a)$, it experiences an electric field $\vec{E} = E\hat{j}$

Time taken to cross the region:

$$\Rightarrow \quad t = \frac{a}{v} = a\left(\frac{m}{2qV}\right)^{1/2} \qquad \text{...(ii)}$$

The distance travelled in y-direction during this time is

$$y = \frac{1}{2}\frac{qE}{m}t^2 = \frac{1}{2} \times \frac{qE}{m} \times a^2 \times \frac{m}{2qV}$$

$$\Rightarrow \quad y = \frac{1}{4}\frac{Ea^2}{V}$$

Hence, the particle meets the line $x = a$ at point

$$(x, y) = \left(a, \frac{1}{4}\frac{Ea^2}{V}\right)$$

(b) Now, velocity of the particle as it crosses the line $x = a$

$$\vec{v} = \left(\frac{2qV}{m}\right)^{1/2}\hat{i} + \frac{qE}{m}\left(\frac{2qV}{m}\right)^{1/2}\hat{j}$$

Magnetic field in this region, $\vec{B} = B\hat{j}$

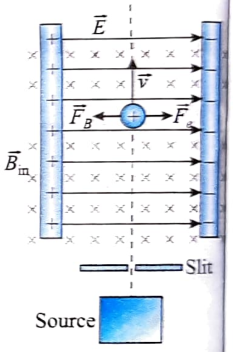
Hence, the time period of revolution, $t = \dfrac{2\pi m}{qB}$

$$\text{Pitch } p = v_{\|}t = \frac{2\pi m}{mqB} \times aE\left(\frac{mq}{2V}\right)^{1/2}$$

$$p = \frac{\pi a E}{B}\left(\frac{2m}{Vq}\right)^{1/2}$$

APPLICATIONS INVOLVING CHARGED PARTICLES MOVING IN A MAGNETIC FIELD

VELOCITY SELECTOR



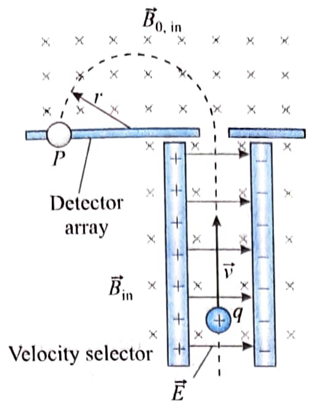
In many experiments involving moving charged particles, it is important that all particles move with essentially the same velocity, which can be achieved by applying a combination of an electric field and a magnetic field oriented as shown in figure. A uniform electric field is directed to the right (in the plane of the page as shown in the figure), and a uniform magnetic field is applied in the direction perpendicular to the electric field (into the page in the figure). If q is positive and the velocity $\vec{v}$ is upward, the magnetic force $q\vec{v} \times \vec{B}$ is to the left and the electric force $q\vec{E}$ is to the right. When the magnitudes of the two fields are chosen so tha $qE = qvB$, the charged particle is modeled as a particle i equilibrium and moves in a straight vertical line through the region of the fields. From the expression $qE = qvB$, we find that

$$v = \frac{E}{B} \qquad \text{...(i}$$

Only those particles having this speed pass undeflected through the mutually perpendicular electric and magnetic fields. The magnetic force exerted on particles moving at speeds greater than that is stronger than the electric force, and the particles are deflected to the left. Those moving at slower speeds are deflected to the right.

MASS SPECTROMETER

A mass spectrometer separates ions according to their mass-to-charge ratio. In one version of this device, known as the Bainbridge mass spectrometer, a beam of ions first passes through a velocity selector and then enters a second uniform magnetic field $\vec{B}_0$ that has the same direction as the magnetic field in the selector. Upon entering the second magnetic field, the ions move in a semicircle of radius r before striking a detector array at P. If the ions are positively charged, the beam deflects to the left as shown in figure. If the ions are negatively charged, the beam deflects to the right. From equation, we can express the ratio m/q as



$$\frac{m}{q}=\frac{rB_0}{v}$$

Using Eq. (i) gives

$$\frac{m}{q}=\frac{rB_0B}{E}$$

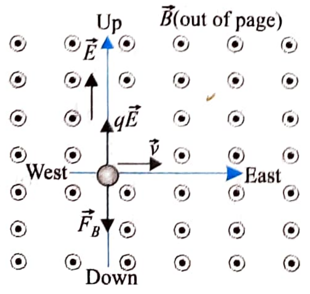
Therefore, we can determine m/q by measuring the radius of curvature and knowing the field magnitudes B, B_0, and E. In practice, one usually measures the masses of various isotopes of a given ion, with the ions all carrying the same charge q. In this way, the mass ratios can be determined even if q is unknown.

ILLUSTRATION 1.25

A velocity selector has an electric field of magnitude 2500 N/C, directed vertically upward, and a horizontal magnetic field that is directed south. Charged particles, traveling east at a speed of 6.0×10^3 m/s, enter the velocity selector and are able to pass completely through without being deflected. When a different particle with an electric charge of $+4.00 \times 10^{-12}$ C enters the velocity selector traveling east, the net force (due to the electric and magnetic fields) acting on it is 2.0×10^{-9} N, pointing directly upward. What is the speed of this particle?

Sol. The magnitude F_B of the magnetic force acting on the particle is related to its speed v by $F_B = |q_0|vB\sin\theta$, where B is the magnitude of the magnetic field, q_0 is the particle's charge, and θ is the angle between the magnetic field B and the particle's velocity v. As the drawing shows, the vector v (east, to the right) is perpendicular to the vector B (south, out of the page). Therefore, $\theta = 90°$, then

$$F_B=|q_0|vB\sin 90°=|q_0|vB \qquad \text{...(i)}$$



In addition to the magnetic force, there is also an electric force of magnitude F_E acting on the particle. This force magnitude does not depend upon the speed v of the particle, as we see from $F_E = |q_0|E$. The particle is positively charged, so the electric force acting on it points upward in the same direction as the electric field. By right-hand, the magnetic force acting on the positively charged particle points down, and is therefore opposite to the electric force. The net force on the particle points upward, so we conclude that the electric force is greater than the magnetic force. Thus, the magnitude F of the net force acting on the particle is equal to the magnitude of the electric force minus the magnitude of the magnetic force:

$$F=F_E-F_B \qquad \text{...(ii)}$$

We also note that particles traveling at a speed $v_0 = 6.0 \times 10^3$ m/s experience no net force. Therefore, $F_E = F_B$ for particles moving at the speed v_0.

Substituting Eq. (i) and $F_E = |q_0|E$ into Eq. (ii) yields

$$F=|q_0|E-|q_0|vB \qquad \text{...(iii)}$$

The magnetic field magnitude B is not given, but, as noted above, for particles with speed $v_0 = 6.0 \times 10^3$ m/s, the magnetic force of Eq. (i), $F_B = |q_0|v_0B$, is equal to the electric force $F_E = |q_0|E$. Therefore, we have that

$$\cancel{|q_0|}v_0B=\cancel{|q_0|}E \quad \text{or} \quad B=\frac{E}{v_0} \qquad \text{...(iv)}$$

Substituting Eq. (iv) into Eq. (iii) yields

$$F=|q_0|E-|q_0|vB=|q_0|E-\frac{|q_0|vE}{v_0} \qquad \text{...(v)}$$

Solving Eq. (v) for v, we obtain

$$\frac{|q_0|vE}{v_0}=|q_0|E-F \quad \text{or} \quad \frac{v}{v_0}=1-\frac{F}{|q_0|E}$$

or

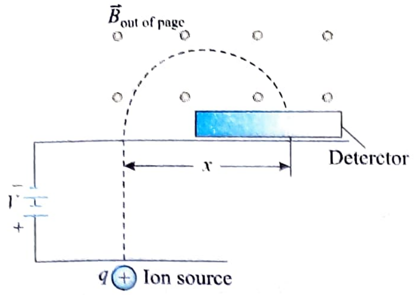
$$v=v_0\left(1-\frac{F}{|q_0|E}\right) \qquad \text{...(vi)}$$

Substituting the given values into Eq. (vi), we find that

$$v=(6.0\times10^3)\left[1-\frac{2.0\times10^{-9}}{(4.00\times10^{-12})(2500)}\right]=4.8\times10^3 \text{ m/s}$$

ILLUSTRATION 1.26

A mass spectrometer separates ions according to their ratio of charge to mass. Such devices are widely used in silence and engineering to analyze unknown mixtures and to separate isotopes of chemical elements. Figure shows ions of charge q and mass m first being accelerated from rest through a potential difference V and then entering a region of uniform magnetic field B pointing out of the page. Only the magnetic force acts on the ions in this region, so they undergo circular motion and, after half an orbit, land on a detector. Find an expression for the horizontal distance x from the entrance slit to the point where an ion lands on the detector.



Sol. This problem is about charged particle undergoing circular motion in a uniform magnetic field. The distance we are asked for is the diameter of the particle's circular path.

Equation, $r = mv/qB$, shows that the path radius depends on the field and on the particle's mass, charge, and speed. We know everything but the speed, so this becomes a two-step problem in which we will first find the speed. We are given the potential difference—energy per unit charge—so we can use energy conservation to find the kinetic energy and hence the ion's speed in the magnetic field region. Then we will use equation to find the radius of the ion's circular path.

A charge q gains kinetic energy qV in "falling" through a potential difference V, so an ion's kinetic energy once it enters the magnetic field is

$$\frac{1}{2}mv^2 = qV$$

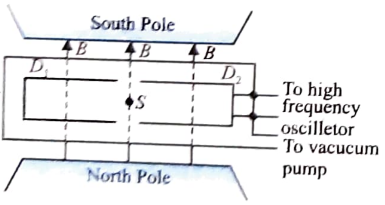
Solving for v gives $v = \sqrt{2qV/m}$.

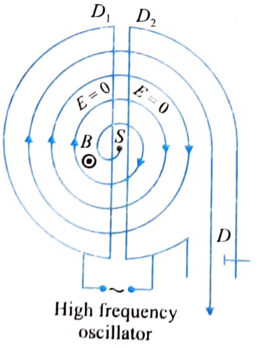
Our answer, the path diameter x, is then twice the radius given in equation.

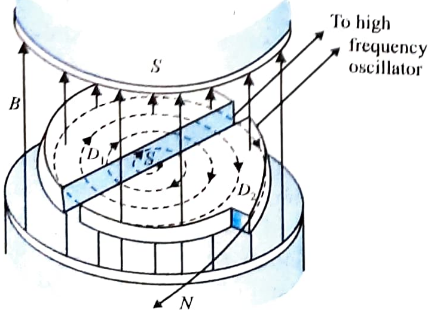
$$x = 2r = \frac{2mv}{qB} = \frac{2m\sqrt{2qV/m}}{qB} = \frac{2}{B}\sqrt{\frac{2mV}{q}}$$

CYCLOTRON

It is a device which is used to accelerate charged particles.







Construction

It consists of two hollow flat semicircular metal boxes D_1 and D_2 called the "dees" on account of their shape like the letter D. The two dees are separated by narrow parallel gap. A high frequency oscillator, which provides an alternating potential of the order of 10^4 to 10^5 volt at a frequency of 10^7 cycles is connected between the two dees. The oscillator establishes an alternating electric field in the air gap, i.e., the electric field is once directed toward D_1 and then toward D_2. Thus D_1 and D_2 become alternately positive and negative at the same rate as the frequency of the oscillator. A source S is placed at the center of the dees which supplies the positive ions to be accelerated. These dees are mounted inside a vacuum chamber. The chamber is mounted horizontally between the pole pieces NS of a huge electromagnet capable of producing a vertical field of about 1.6 T.

Working

The positive ions emitted from the source will be acceleraed in the gap towards the dees which is negative at that time. Let it be D_2.Since there is no electric field inside the dees, the positive ions move with constant speed along circles of constant radius under the influence of magnetic field which is **perpendicular** to the dees. If by the time the ions emerge from D_2, the polarity of the applied potential is reversed (i.e., the dee D_1 now becomes negative), the positive ions will again face the negative dee and thus will be again accelerated by the field in the gap. Since their velocity is increased, they will now move through D_1 along circular arc of greater radius as shown in figure. Here the time of passage to complete the semi circle in the dee remains the same as in D_2. If the time of travel in D_1 is equal to half the time period of the oscillator voltage, the positive ions after coming from D will find the reversed field and hence they are accelerated again in the gap D_1D_2. In this way, the positive ions move faster and faster in ever-expanding circles until they reach the outer edge of the dees where they are deflected by deflector plate and strike the target. Here it should be remembered that the time required for the positive ions to make one complete turn within dees is the same for all speeds and is equal to the time period of the oscillator.

Theory

When a particle of mass m and charge q moves with a velocity v in the magnetic field of flux density B, then the radius r of the circular path is given by

$$r = \frac{mv}{qB} \qquad \text{...(i}$$

As the speed v increases every time the particle passes through the gap, so radius r also increases.

For resonance between the applied ac voltage and the moving particle, the time taken by the particle to travel the circular arc within any of the dees must be equal to half the time period of the applied oscillator voltage. Or the frequency of oscillator voltage should be same as the frequency of revolution of charge particle. So the frequency of oscillator is given by

$$f = \frac{1}{T} = \frac{qB}{2\pi m}$$

The value of q/m being fixed for an ion hence the value of f is adjusted corresponding to B or vice-versa.

Energy of a Particle Accelerated by a Cyclotron

The ion will have maximum energy when it will travel at the boundary of dee. If the outside radius of dee is R then according to Eq. (i), the maximum velocity v_m of the ion may be written as

$$v_m = \frac{RqB}{m} \qquad \text{...(ii)}$$

and so the maximum kinetic energy of the ion will be given by

$$E_m = \frac{1}{2}mv_m^2 = \frac{R^2q^2B^2}{2m} \qquad \text{...(iii)}$$

Limitations of Cyclotron

The maximum available particle energy is limited due to the following factors:

1. due to the limited power and frequency of the oscillator.
2. due to the maximum strength of the magnetic field which can be produced, and
3. the energy of charged particle emerging from cyclotron is limited due to variation of mass with velocity, i.e.,

$$m = \frac{m_0}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

where m_0 is the rest mass, m, the mass in motion when velocity is v and c, the velocity of light.

The frequency of rotation of charged particle becomes

$$f = \frac{qB}{2\pi m} = \frac{qB\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}{2\pi m_0}$$

Thus, the frequency of rotation of charged particle decreases with increase in velocity. Consequently, the charged particle takes a longer time to complete semicircular path. Now the particle continuously goes on lagging behind the alternating potential difference till a stage is reached when it is no longer accelerated further.

The frequency of the charged particle can be kept constant by making $B\sqrt{\left(\frac{1-v^2}{c^2}\right)}$ constant. To achieve this, the magnetic field B is increased as the velocity of particle increases. Alternatively, the frequency of applied alternating potential difference may be varied so that it is always equal to the frequency of rotation of charged particle.

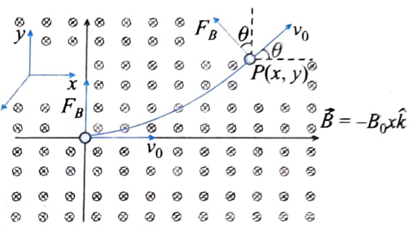
MOTION OF A CHARGED PARTICLE IN NON-UNIFORM MAGNETIC FIELD

It is known that a charged particle in a uniform magnetic field exhibits circular motion with specified frequency of revolution and specified radius which depends on the charge and mass of particle and magnitude magnetic field. But when a charged particle moves in non-uniform magnetic field, the path of the charged particle no longer remains circular. The magnetic field always applies force on the charged particle perpendicular to its motion, and the magnitude of this force is not constant. Hence, the particle moves in a curved path. In illustration given below, we will learn to analyze this type of motion.

ILLUSTRATION 1.27

A particle of charge q and mass m is projected from the origin with velocity $\vec{v} = v_0\hat{i}$ in a non-uniform magnetic field $\vec{B} = -B_0x\hat{k}$. Here v_0 and B_0 are positive constants of proper dimensions. Find the maximum positive x-coordinate of the particle during its motion.

Sol. As the magnetic field is not constant, the particle will move in a curve path. Let at point $P(x, y)$ its velocity vector make an angle θ with positive x-axis.



Then the magnetic force acting on the particle will be at an angle θ with the positive y-direction.

So, $$a_y = \left(\frac{F_B}{m}\right)\cos\theta$$

$$a_y = \left(\frac{F_B}{m}\right)\cos\theta \text{ where } F_B = Bqv_0\sin 90° = (B_0x)qv_0$$

$$\frac{dv_y}{dt} = \frac{(B_0x)qv_0}{m}\cos\theta$$

or $$\left(\frac{dv_y}{dt}\right)\left(\frac{dx}{dx}\right) = \left(\frac{B_0qx}{m}\right)(v_0\cos\theta)$$

or $$\left(\frac{dv_y}{dx}\right)\left(\frac{dx}{dt}\right) = \left(\frac{B_0qx}{m}\right)(v_x)$$

Thus $$\frac{dv_y}{dx} = \left(\frac{B_0q}{m}\right)x \qquad \left(\text{As } v_x = \frac{dx}{dt}\right)$$

$$\frac{dv_y}{dx} = \left(\frac{B_0q}{m}\right)x \Rightarrow dv_y = \left(\frac{B_0q}{m}\right)x\,dx \qquad \text{...(i)}$$

At maximum x-displacement, velocity is along $+y$-direction. As magnetic field do no work on moving charge hence its speed remains unchanged. Now integrating equation (i) both sides

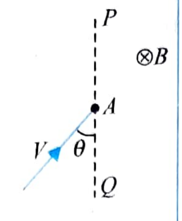
$$\int_0^{v_0} dv_y = \left(\frac{B_0q}{m}\right)\int_0^{x_{max}} x\,dx$$

or $$v_0 = \left(\frac{B_0q}{m}\right)\left(\frac{x_{max}^2}{2}\right)$$

or $$x_{max} = \sqrt{\frac{2mv_0}{B_0q}}$$

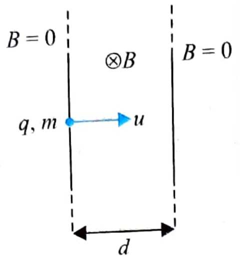
CONCEPT APPLICATION EXERCISE 1.2

1. A positive charge particle of charge q and mass m enters into a uniform magnetic field with velocity v as shown in figure. There is no magnetic field to the left of PQ. Find **(a)** time spent, **(b)** distance travelled in the magnetic field, **(c)** impulse of magnetic force.



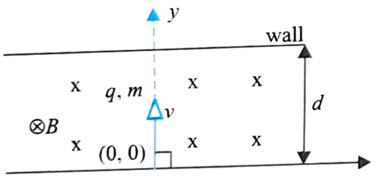
2. Repeat above Question 1, if the charge is negative and the angle made by the boundary with the velocity is $\theta = \pi/6$.

3. A uniform magnetic field of strength B exists in a region of width d. A particle of charge q and mass m is shot perpendicularly (as shown in figure) into the magnetic field. Find the time spent by the particle in the magnetic field if



(a) $d > \dfrac{mu}{qB}$ **(b)** $d < \dfrac{mu}{qB}$

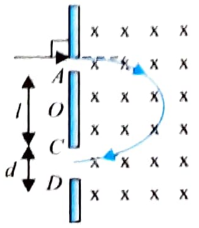
4. In figure, what should be the speed of the charged particle so that it cannot collide with the upper wall? Also, find the coordinates of the point where the particle strikes the lower plate in the limiting case of velocity.



5. A particle with charge 6.0×10^{-19} C travels in a circular orbit with radius 5.0 mm due to the force exerted on it by a magnetic field of magnitude 2.0 T and perpendicular to the orbit.
   (a) What is the magnitude of the linear momentum $\vec{p}$ of the particle?
   (b) What is the magnitude of the angular momentum $\vec{L}$ of the particle?

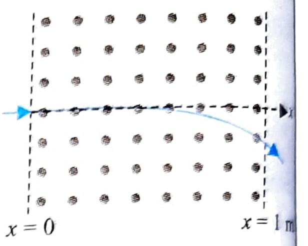
6. A particle of mass m and charge q is accelerated by a potential difference V volt and made to enter a magnetic field region at an angle θ with the field. At the same moment, another particle of same mass and charge is projected in the direction of the field from the same point. Magnetic field induction is B. What would be the speed of second particle so that both particles meet again and again after regular interval of time. Also, find the time interval after which they meet and the distance travelled by the second particle during that interval.

7. A beam of equally charged particles after being accelerated through a voltage V enters into a magnetic field B as shown in figure. It is found that all the particles hit the plate between C and D. Find the ratio between the masses of the heaviest and lightest particles of the beam.



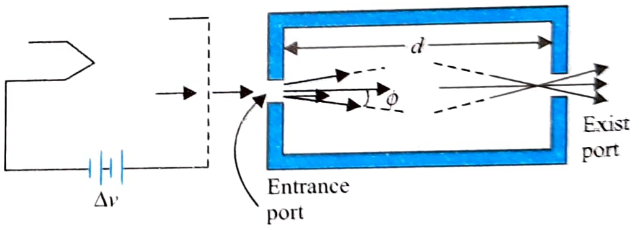
8. A proton and an alpha particle are projected in a magnetic field which exists in the width of region d. Compare the angles of deviation suffered by the proton and the alpha particle if before entering the magnetic field both the particles.
   (a) have the same momentum,
   (b) have the same kinetic energy, and
   (c) are accelerated through the same potential difference.
   Take $m_\alpha = 4m_p$, $q_\alpha = 2q_p$.

9. Protons having a kinetic energy of 50 MeV are moving in the positive x-direction and enter a magnetic field $B = 0.5\ \text{mT}(\hat{k})$ directed out of the plane of the page and extending from $x = 0$ to $x = 1$ m as shown in figure.

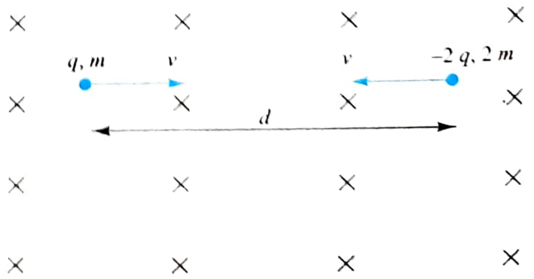


   (a) Calculate the y-component of the protons' momentum as they leave the magnetic field.
   (b) Find the angle ϕ between the initial velocity vector of the proton beam and the velocity vector after the beam emerges from the field. Ignore relativistic effects and note that 1 eV = 1.60×10^{-19} J and mass of proton $m_p = 1.6 \times 10^{-27}$ kg.

10. Electrons in a beam are accelerated from rest through a potential difference ΔV. The beam enters an experimental chamber through a small hole. As shown in figure, the electron velocity vector lie within a narrow cone of half angle ϕ oriented along the beam axis. We wish to use a uniform magnetic field directed parallel to the axis to focus the beam, so that all of the electrons can pass through a small exit port on the opposite side of the chamber after they travel the length d of the chamber. What is the required magnitude of the magnetic field?



11. A charged particle $+q$ of mass m is placed at a distance d from another charged particle $-2q$ of mass $2m$ in a uniform magnetic field B as shown in figure. If the particles are projected towards each other with same speed v,



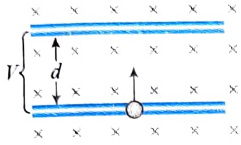
   (a) find the maximum value of projection speed v_m so that the two particles do not collide.

(b) find the time after which collision occurs between the particles if projection speed equals $2v_m$.

(c) Assuming the collision to be perfectly inelastic, find the radius of the particle in subsequent motion.

(Neglect the electric force between the charges.)

12. In the given figure, a charged particle of mass m, charge $-q$, and having low (negligible) speed enters the region between two plates of potential difference V and plate separation d, initially headed directly toward the top plate. A uniform magnetic field of magnitude B is normal to the plane of the paper. Find the minimum value of magnetic field (B) such that the electron will not strike the top plate.



ANSWERS

1. (a) $\dfrac{m2\theta}{qB}$ (b) $\dfrac{mv}{qB}\times 2\theta$ (c) $-2mv\sin\theta\hat{i}$

2. (a) $\dfrac{5\pi m}{3qB}$ (b) $\dfrac{5\pi r}{3}$ (c) $-mv\hat{j}$

3. (a) $\dfrac{\pi m}{qB}$ (b) $\dfrac{m}{qB}\sin^{-1}\left(\dfrac{d}{R}\right)$

4. $\dfrac{qBd}{m}$; $(-2d, 0, 0)$

5. (a) 6.0×10^{-23} kgms^{-1} (b) 3.0×10^{-23} kgm^{2}s^{-1}

6. $\sqrt{\dfrac{2qV}{m}}\cos\theta$; $\dfrac{2\pi m}{qB}$; $\pi\sqrt{\dfrac{8Vm}{qB^2}}\cos\theta$

7. $\left(\dfrac{l+d}{l}\right)^2$

8. (a) $\dfrac{1}{2}$ (b) 1 (c) $\sqrt{2}$

9. (a) 8.0×10^{-23} kg m/s (b) 30°

10. $\dfrac{2\pi m}{qd}\left(\sqrt{\dfrac{2\Delta Ve}{m}}\right)\cos\phi$

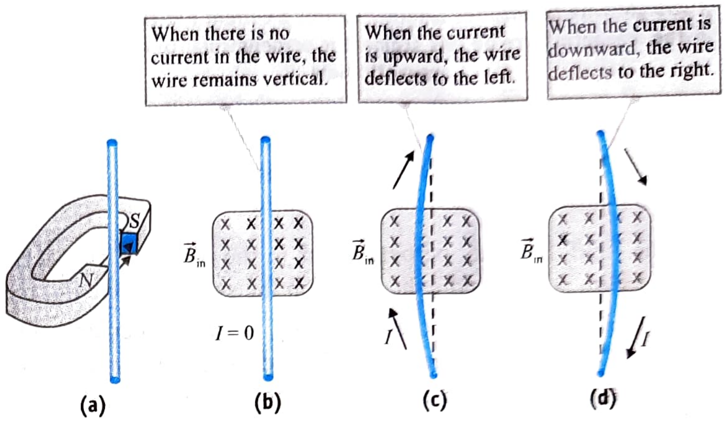
11. (a) $\dfrac{Bqd}{2m}$ (b) $\dfrac{\pi m}{6qB}$ (c) $\cdot\sqrt{3}d$

12. $\sqrt{\dfrac{mV}{2qd^2}}$

FORCE ON A CURRENT-CARRYING WIRE

MAGNETIC FORCE ACTING ON A CURRENT CARRYING CONDUCTOR

If a magnetic force is exerted on a single charged particle when the particle moves through a magnetic field, it should not surprise you that a current-carrying wire also experiences a force when placed in a magnetic field. The current is a collection of many charged particles in motion; hence, the resultant force exerted by the field on the wire is the vector sum of the individual forces exerted on all the charged particles making up the current. The force exerted on the particles is transmitted to the wire when the particles collide with the atoms making up the wire. One can demonstrate the magnetic force acting on a current-carrying conductor by hanging a wire between the poles of a magnet as shown in figure. For ease in visualization, part of the horseshoe magnet in part (a) is removed to show the end face of the south pole in parts (b) through (d) of figure. The magnetic field is directed into the page and covers the region within the shaded squares. When the current in the wire is zero, the wire remains vertical as in Fig. (b). When the wire carries a current directed upward as in Fig. (c), however, the wire deflects to the left. If the current is reversed as in Fig. (d), the wire deflects to the right.

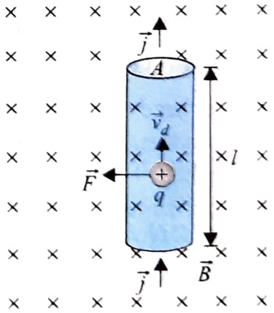

When there is no current in the wire, the wire remains vertical.

When the current is upward, the wire deflects to the left.

When the current is downward, the wire deflects to the right.

Let us quantify this discussion by considering a straight segment of wire of length l and cross-sectional area A carrying a current I is placed uniform magnetic field $\vec{B}$.

The figure below shows a straight segment of a conducting wire, with length l and cross-sectional area A; the current is from bottom to top. The wire is in a uniform magnetic field $\vec{B}$, perpendicular to the plane of the diagram and directed into the plane. Let us assume first that the moving charges are positive.

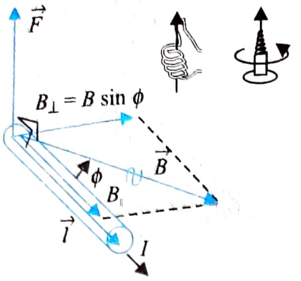


Forces on a moving positive charge in a current carrying conductor.

The drift velocity $\vec{v}_d$ is upward, perpendicular to $\vec{B}$. The average force on each charge is $\vec{F} = q\vec{v}_d \times \vec{B}$, directed to the left as shown in the figure; since $\vec{v}_d$ and $\vec{B}$ are perpendicular, the magnitude of the force is $F = qv_d B$.

We can derive an expression for the total force on all the moving charges in a length l of a conductor with cross-sectional area A. The number of charges per unit volume is n; a segment of conductor with length l has volume Al and contains a number of charges equal to nAl. The total force $\vec{F}$ on all the moving charges in this segment has magnitude

$$F = (nAl)(qv_dB) = (nqv_dA)(lB) \quad \text{...(i)}$$



A straight wire segment of length $\vec{l}$ carries a current I in the direction of $\vec{l}$. The magnetic force on this segment is perpendicular to both $\vec{l}$ and the magnetic field $\vec{B}$.

The current density is $J = nqv_d$. The product JA is the total current I, so we can rewrite Eq. (i) as,

$$F = IlB \qquad \text{...(ii)}$$

If the field $\vec{B}$ is not perpendicular to the wire but makes an angle ϕ with it, we handle the situation the same way we did for a single charge. Only the component of $\vec{B}$ perpendicular to the wire (and to the drift velocities of the charges) exerts a force; this component is $B_\perp = B \sin \phi$. The magnetic force on the wire segment is then

$$F = IlB_\perp = IlB \sin \phi \qquad \text{...(iii)}$$

The force is always perpendicular to both the conductor and the field, with the direction determined by the same right hand rule we used for a moving positive charge (as shown in figure). Hence, this force can be expressed as a vector product, just like the force on a single moving charge. We represent the segment of wire with a vector $\vec{l}$ along the wire in the direction of the current; then the force $\vec{F}$ on this segment is

$$\vec{F} = I\vec{l} \times \vec{B} \text{ (magnetic force on a straight wire segment)} \quad \text{...(iv)}$$

If the conductor is not straight, we can divide it into infinitesimal segments $d\vec{l}$. The force $d\vec{F}$ on each segment is

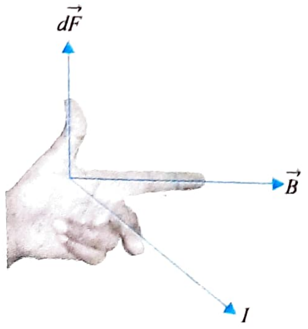
$$d\vec{F} = I\vec{l} \times \vec{B} \text{ (magnetic force on an infinitesimal wire section)}$$

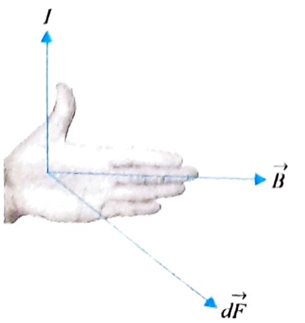
Integrating this, we get total force $\vec{F} = \int I d\vec{l} \times \vec{B}$ ...(v)

DIRECTION OF FORCE ON A CURRENT-CARRYING WIRE IN MAGNETIC FIELD

Left Hand Rule

If the thumb and first two fingers of the left hand are held each at right angles to the other, with the first finger pointing in the direction of the field and the second finger in the direction of the current, then the thumb predicts the direction of the thrust or force (figure).





Right Hand Palm Rule

Stretch the fingers and thumb of right hand at right angles to each other. If the fingers point in the direction of field $\vec{B}$ and thumb in the direction of current I, the normal to palm will point in the direction of force (or motion).

Regarding the force on a current carrying conductor in a magnetic field, it is worth mentioning that:

As the force $BI\ dL \sin \theta$ is not a function of position r, the magnetic force on a current element is non-central [a central force is of the form $F = Kf(r)\vec{n}_r$]

The force $d\vec{F}$ is always perpendicular to both $\vec{B}$ and $I\, d\vec{L}$ though $\vec{B}$ and $I\, d\vec{L}$ may or may not be perpendicular to each other.

ILLUSTRATION 1.28

A wire of length l carries a current i along the X-axis. A magnetic field exists which is given as

$$\vec{B} = B_0(\hat{i} + \hat{j} + \hat{k})\ \text{T}$$

Find the magnitude of the magnetic force acting on the wire.

Sol. Here current is passing along X-axis

So $\vec{F} = i\vec{l} \times \vec{B} = i\{(l\hat{i}) \times (B_0\hat{i} + B_0\hat{j} + B_0\hat{k})\}$

$$= i\{lB_0\hat{k} - lB_0\hat{j}\} = ilB_0\{-\hat{j} + \hat{k}\}$$

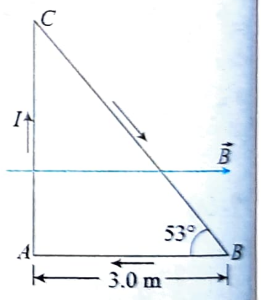
$$|\vec{F}| = ilB_0\sqrt{1+1} = \sqrt{2}\, ilB_0$$

ILLUSTRATION 1.29

A loop of wire has the shape of a right triangle (see the drawing) and carries a current of I = 5.0 A. A uniform magnetic field is directed parallel to side AB and has a magnitude of 2.0 T.

(a) Find the magnitude and direction of the magnetic force exerted on each side of the triangle;

(b) Determine the magnitude of the net force exerted on the triangle.



Sol. The magnitude of the magnetic force exerted on a long straight wire is given by equation, $F = ILB \sin \theta$. The direction of the magnetic force is predicted by right-hand rule. The net force on the triangular loop is the vector sum of the forces on the three sides.

(a) The direction of the current inside AB is opposite to the direction of the magnetic field, so the angle θ between them is $\theta = 180°$.

The magnitude of the magnetic force,

$$F_{AB} = ILB\sin\theta = ILB\sin 180° = 0\ \text{N}$$

For the side BC, the angle is $\theta = 53°$, and the length of the side,

$$L = \frac{3.0\ \text{m}}{\cos 53°} = 5.0\,\text{m}$$

The magnetic force is

$$F_{BC} = ILB\sin\theta = (5.0)(5.0)(2.0)\sin 53° = 40.0\,\text{N}$$

An application of right-hand rule shows that the magnetic force on side BC is directed perpendicularly out of the paper, toward $+z$ direction.

For the side AC, the angle is $\theta = 90°$. We see that the length of the side is $L = (3.0)\tan 53° = 4.0$ m

The magnetic force is

$$F_{AC} = ILB\sin\theta$$
$$= (5.0)(4)(2.0)\sin 90° = 40.0\text{ N}$$

An application of right-hand no. 1 shows that the magnetic force on side AC is directed perpendicularly into the paper, toward $-z$ direction.

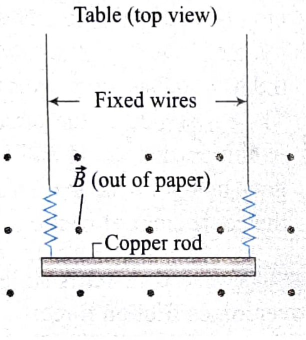
(b) The net force is the vector sum of the forces on the three sides. Taking the positive direction as being out of the paper,

The net force is

$$\Sigma F = 0\text{ N} + 40.0\text{ N} + (-40.0\text{ N}) = 0\text{ N}$$

ILLUSTRATION 1.30

A copper rod of length 1.0 m is lying on a frictionless table (see the drawing). Each end of the rod is attached to a fixed wire by an unstretched spring that has a spring constant of $k = 75$ N/m. A magnetic field with a strength of 0.20 T is oriented perpendicular to the surface of the table. **(a)** What must be the direction of the current in the copper rod that causes the springs to stretch? **(b)** If the current is 10 A, by how much does each spring stretch?



Sol.

(a) From right-hand rule, if we extend the right hand so that the fingers point in the direction of the magnetic field, and the thumb points in the direction of the current, the palm of the hand faces the direction of the magnetic force on the current.

The springs will stretch when the magnetic force exerted on the copper rod is downward, toward the bottom of the page. Therefore, if you extend your right hand with your fingers pointing out of the page and the palm of your hand facing the bottom of the page, your thumb points left-to-right along the copper rod. Thus, the current flows left-to-right in the copper rod.

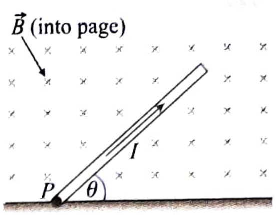
(b) The downward magnetic force exerted on the copper rod is,

$$F = ilB\sin\theta$$
$$= (10)(1.0)(0.20)\sin 90° = 2.0\text{ N}$$

The force required to stretch each spring is $F = kx$, where k is the spring constant. Since there are two springs, the magnetic force F exerted on the rod must equal $F = 2kx$. Solving for x, we find that

$$x = \frac{F}{2k} = \frac{2.0\text{ N}}{2(50\text{ N/m})} = 2.0\times10^{-2}\text{ m}$$

ILLUSTRATION 1.31



The drawing shows a thin, uniform rod that has a length of 0.50 m and a mass of 0.20 kg. This rod lies in the plane of the paper and is attached to the floor by a hinge at point P. A uniform magnetic field of 0.50 T is directed perpendicularly into the plane of the paper. There is a current $I = 4.0$ A in the rod, which does not rotate clockwise or counter clockwise. Find the angle.

Sol. Since the rod does not rotate about the axis at P, the net torque relative to that axis must be zero; $\Sigma\tau = 0$. There are two torques that must be considered, one due to the magnetic force and another due to the weight of the rod. We consider both of these to act at the rod's centre of gravity, which is at the geometrical center of the rod (length = L), because the rod is uniform. According to right-hand rule, the magnetic force acts perpendicular to the rod and is directed up and to the left in the drawing. Therefore, the magnetic torque is a counter clockwise(positive) torque. The magnitude F of the magnetic force as $F = ILB\sin 90.0°$, since the current is perpendicular to the magnetic field. The weight is mg and acts downward, producing a clockwise (negative) torque.

Setting the sum of these torques equal to zero will enable us to find the angle θ that the rod makes with the ground.

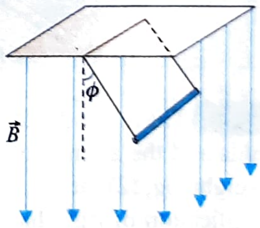
$$\Sigma\tau = \tau_{\text{magnetic}} + \tau_{\text{weight}} = 0$$

$$(ILB)(L/2) - (mg)[(L/2)\cos\theta] = 0 \text{ or } \cos\theta = \frac{ILB}{mg}$$

$$\cos\theta = \frac{(4.0)(0.50)(0.50)}{(0.20)(10)} = \frac{1}{2} \Rightarrow \theta = 60°.$$

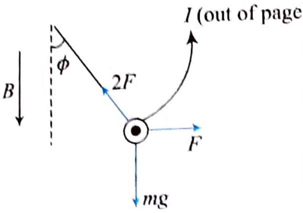
ILLUSTRATION 1.32

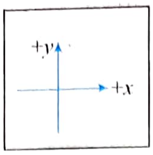
A horizontal wire is hung from the ceiling of a room by two massless strings. The wire has a length of 0.20 m and a mass of 0.080 kg. A uniform magnetic field of magnitude 0.075 T is directed from the ceiling to the floor. When a current of $I = 40$ A exists in the wire, the wire swings upward and, at equilibrium, makes an angle ϕ with respect to the vertical, as the drawing shows. Find **(a)** the angle ϕ and **(b)** the tension in each of the two strings.



Sol. There are four forces that act on the wire: the magnetic force (magnitude F), the weight mg of the wire, and the tension in each of the two strings (magnitude T in each string). Since there are two strings, the following drawing shows the total tension as $2T$. The magnitude F of the magnetic force is given by $F = ILB\sin\theta$, where I is the current, L is the length of the wire, B is the magnitude of the magnetic field, and θ is the angle between the wire and the magnetic field. In this problem $\theta = 90°$.

(a) The direction of the magnetic force is given by right-hand rule. The drawing shows an end view of the wire, where it can be seen that the magnetic force (magnitude = F) points to the right, in the $+x$ direction.





In order for the wire to be in equilibrium, the net force ΣF_x and ΣF_y must be zero.

$$\Sigma F_x = 0 \Rightarrow 2T\sin\phi = F_B = ILB\sin 90^\circ \qquad \text{...(i)}$$

$$\Sigma F_y = 0 \Rightarrow 2T\cos\phi = mg \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{\sin\phi}{\cos\phi} = \tan\phi = \frac{ILB}{mg} = \frac{(40)(0.20)(0.075)}{(0.080)(10)} = \frac{3}{4}$$

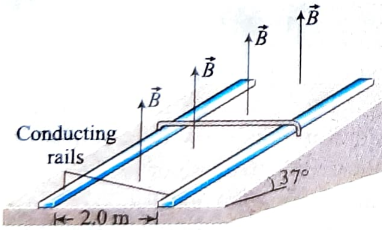
Hence, $\phi = 37^\circ$

(b) The tension in each wire can be found directly from Eq. (ii):

$$T = \frac{mg}{2\cos\phi} = \frac{(0.080\text{ kg})(10\text{ m/s}^2)}{2\cos 37^\circ} = 0.50\text{ N}$$

ILLUSTRATION 1.33

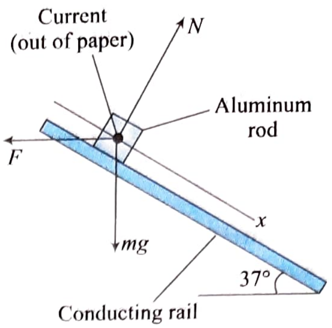
Two conducting rails in the drawing are tilted upward so they each make an angle of 37° with respect to the ground. The vertical magnetic field has a magnitude of 0.050 T. The 0.20-kg aluminium rod (length = 2.0 m) slides without friction down the rails at a constant velocity. How much current flows through the rod?



Sol. The following drawing shows a side view of the conducting rails and the aluminum rod. Three forces act on the rod: (1) its weight mg, (2) the magnetic force F, and the normal force N. An application of right-hand rule shows that the magnetic force is directed to the left, as shown in the drawing. Since the rod slides down the rails at a constant velocity, its acceleration is zero. If we choose the x-axis to be along the rails, the net force along the x-direction is zero: $\Sigma F_x = ma_x = 0$. Using the components of F and mg that are along the x-axis,

$$F\cos 37^\circ = mg\sin 37^\circ \qquad \text{...(i)}$$

The magnetic force is given by Eq. (i) as $F = ILB\sin\theta$, where $\theta = 90^\circ$ is the angle between the magnetic field and the current. We can use these two relations to find the current in the rod.

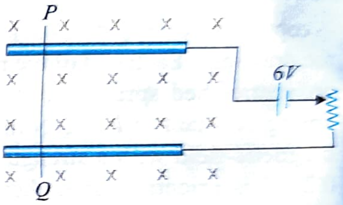


Substituting the expression $F = ILB\sin 90^\circ$ into Newton's second law and solving for the current I, we obtain

$$I = \frac{mg\sin 37^\circ}{(LB\sin 90^\circ)\cos 37^\circ}$$

$$= \frac{(0.20\text{ kg})(10\text{ m/s}^2)\sin 37^\circ}{(2.0\text{ m})(0.050\text{ T})\sin 90^\circ\cos 37^\circ} = 14\text{ A}$$

ILLUSTRATION 1.34



A metal wire PQ of mass 10 g lies at rest on two horizontal metal rails separated by 5 cm. A vertically downward magnetic field of magnitude 0.800 T exists in the space. The resistance of the circuit is slowly decreased and it is found that when the resistance goes below 20 Ω, the wire PQ starts sliding on the rails. Find the coefficient of friction.

Sol. The wire starts sliding when magnetic force on the wire overcomes friction force.

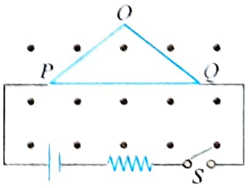
$$\mu N = F \quad \Rightarrow \quad \mu mg = ilB$$

$$\mu \times .10 \times 10^{-3} \times 10 = \frac{6}{20} \times 5 \times 10^{-2} \times 0.8$$

$$\mu = \frac{6 \times 5 \times 10^{-2} \times 0.8}{20 \times 0.10 \times 10^{-3} \times 10} = 0.12$$

ILLUSTRATION 1.35

Figure (a) shows a rod PQ of length 20 cm and mass 200 g suspended through a fixed point O by two threads of lengths 20 cm each. A magnetic field of strength 0.500 T exists in the vicinity of the wire PQ as shown in the figure.



The wire connecting PQ with the battery are loose and exert no force on PQ.

(a) Find the tension in the threads when the switch S is open.

(b) A current of 2 A is established when the switch S is closed. Find the tension in the threads now.

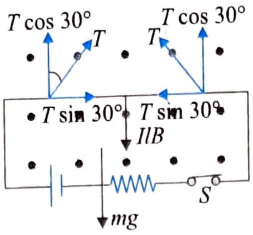
Sol.

(a) When switch S is open

$$2T\cos 30° = mg$$

$$\Rightarrow T = mg/2\cos 30°$$

$$= \frac{2}{\sqrt{3}} = 1.13 \text{ N}$$



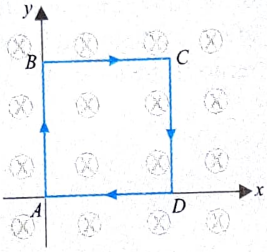
(b) When the switch is closed and a current passed through the circuit $= 2$ A

Then, $2T'\cos 30° = mg + ilB$

$$2T'\left(\frac{\sqrt{3}}{2}\right) = 2.2 \Rightarrow T' = \frac{2.2}{\sqrt{3}} \text{ N}$$

ILLUSTRATION 1.36

A square loop of edge l and carrying a current i, is placed with its edges parallel to the XY axes. There is present a non-uniform magnetic field in the region $\vec{B} = B_o\left(1+\frac{x}{l}\right)(-\hat{k})$. Find the magnitude of the net magnetic force experienced by the loop.



Sol. The magnetic field in the region

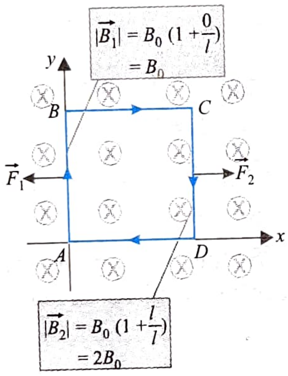
$$B = B_0\left(1+\frac{x}{l}\right)\vec{k}$$

Force on AB,

$$F_1 = iB_0\left(1+\frac{0}{l}\right)l = iB_0 l$$

Force on CD,

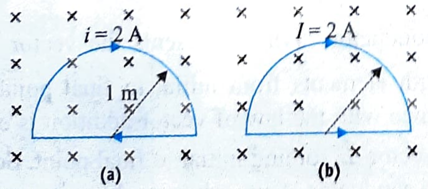
$$F_2 = iB_0\left(1+\frac{l}{l}\right)l = iB_0 \cdot 2l$$


$|\vec{B}_1| = B_0\left(1+\frac{0}{l}\right) = B_0$
y
B
C
$\vec{F}_1$
$\vec{F}_2$
A
D
x
$|\vec{B}_2| = B_0\left(1+\frac{l}{l}\right) = 2B_0$

The forces due to wires BC and AD will be equal and opposite. Hence, the force on BC and AD will cancel. Net force, $F_2 - F_1 = iB_0 l$

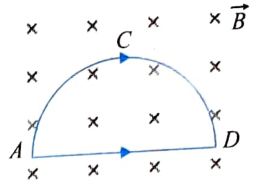
ILLUSTRATION 1.37

In figure, a semicircular wire loop is placed in uniform magnetic field $B = 1.0$ T. The plane of the loop is perpendicular to the magnetic field. Current $i = 2$ A flows in the loop in the direction shown. Find the magnitude of the magnetic force in both the cases (a) and (b). The radius of the loop is 1 m.



Sol. Refer Fig. (a): It forms a closed loop and the current completes the loop. Therefore, net force on the loop in uniform field should be zero.

Refer Fig. (b): In this case although it forms a closed loop, but current does not complete the loop. Hence, net force is not zero.



$$\vec{F}_{ACD} = \vec{F}_{AD}$$

$$\therefore \vec{F}_{\text{loop}} = \vec{F}_{ACD} + \vec{F}_{AD} = 2\vec{F}_{AD}$$

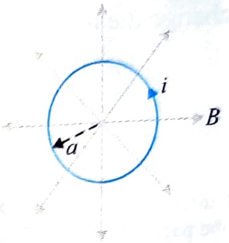
$$\therefore |\vec{F}_{\text{loop}}| = 2|\vec{F}_{AD}|$$

$$= 2ilB\sin\theta\ (l = 2r = 2.0 \text{ m})$$

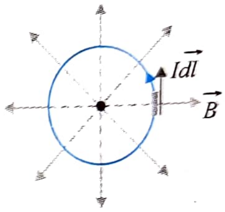
$$= (2)(2)(2)(1)\sin 90° = 8 \text{ N}$$

ILLUSTRATION 1.38

A circular loop of radius a, carrying a current i, is placed in a two-dimensional magnetic field. The center of the loop coincides with the center of the field. The strength of the magnetic field at the periphery of the loop is B. Find the magnetic force on the wire.



Sol. Let us select a current element on circular loop $Id\vec{l}$. The magnetic field in radial direction will be perpendicular to current element vector. From left hand rule or right palm rule, we can find the direction of force $d\vec{F}$ on the element which comes to be perpendicular into the plane of the paper.



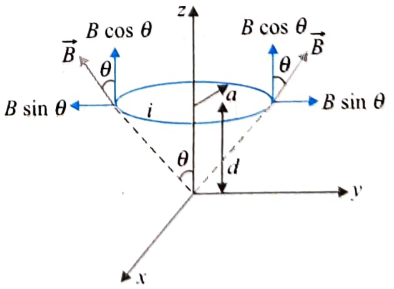
$$\text{Required force } F = \int dF = \int idlB\sin 90° = \int IdlB$$

$$= 2\pi aiB \text{ perpendicular to the plane.}$$

ILLUSTRATION 1.39

A hypothetical magnetic field existing in a region is given by $\vec{B} = B_0\vec{e}_r$ where $\vec{e}_r$ denotes the unit vector along the radial direction. A circular loop of radius a, carrying a current i, is placed with its plane parallel to the X-Y plane and the center at $(0, 0, d)$. Find the magnitude of the magnetic force acting on the loop.

Sol. Magnetic force due to $B\cos\theta$ will be cancelled out. There will be no force due to magnetic field due to $B\cos\theta$ in downward direction.



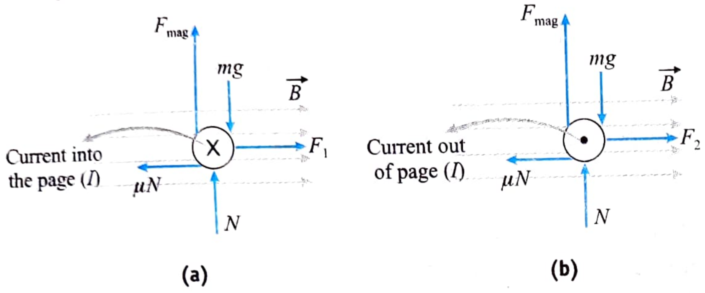
Required magnetic force

$$F = \int dF = \int_0^{2\pi a} idlB\sin\theta = \frac{2\pi i a^2 B}{\sqrt{a^2+d^2}}$$

ILLUSTRATION 1.40

A conducting wire of length l is placed on a rough horizontal surface, where a uniform horizontal magnetic field B perpendicular to the length of the wire exists. Least values of the forces required to move the rod when a current I is established in the rod are observed to be F_1 and F_2 ($< F_1$) for the two possible directions of the current through the rod, respectively. Find the weight of the rod and the coefficient of friction between the rod and the surface.

Sol. Changing the direction of current in the wire, we can change the normal reaction on the wire by the surface.



(a) (b)

In one case, magnetic force on the wire will be in upward direction while in the other case it will be in the downward direction. Hence, normal reaction, $N = mg \pm Bil$.

$$f_{(\text{friction limiting})} = \mu(mg \pm Bil) \text{ as } F_1 > F_2$$

$$F_1 = \mu(mg + Bil) \quad \text{...(i)}$$

and $F_2 = \mu(mg - Bil)$ ...(ii)

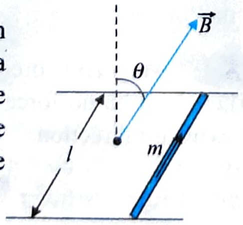
From Eqs (i) and (ii), $\dfrac{F_1}{F_2} = \dfrac{mg + Bil}{mg - Bil}$

$$mg = Bil\left[\frac{F_1 + F_2}{F_1 - F_2}\right] \quad \text{...(iii)}$$

From Eqs. (i) and (iii), we get $\mu = \dfrac{F_1 - F_2}{2Bil}$

ILLUSTRATION 1.41

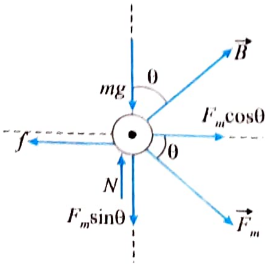
A conductor (rod) of mass m, length l carrying a current i is subjected to a magnetic field of induction B. If the coefficients of friction between the conducting rod and rail is μ, find the value of i if the rod starts sliding.



Sol. The magnetic force is $F_m = ilB \sin \varphi$, where φ_B = Angle between current and magnetic field = 90°.

or $F_m = ilB$

but it acts at an angle θ with horizontal as shown in figure according to Fleming's left hand rule or right palm rule.



Then the net force along x- and y-axes are

$$F_y = N - (mg + F_m \sin \theta) = 0$$

or $N = mg + F_m \sin \theta$ ...(i

$$F_x = f - F_m \cos \theta = 0$$ ...(ii

or $f = F_m \cos \theta$

Law of friction, $f = \mu N$ ...(ii

By using the above equations

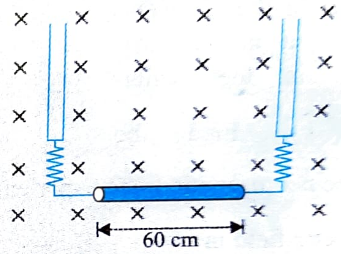
$$\mu(mg + F_m \sin \theta) = F_m \cos \theta$$

or $F_m = \dfrac{\mu mg}{\cos\theta - \mu \sin\theta}$ or $ilB = \dfrac{\mu mg}{\cos\theta - \mu \sin\theta}$

or $i = \dfrac{\mu mg}{lB(\cos\theta - \mu \sin\theta)}$

ILLUSTRATION 1.42

A wire of 60 cm length and mas 16 gm is suspended by a pair of flexible leads in a magnetic field of induction 0.40 T. What are the magnitude and direction of the current required to remove the tension in the supporting leads?



Sol. We know that the magnetic force on a wire is given by

$$F = ilB \sin \theta$$

Here B is at right angle to i and hence $\theta = 90°$.

So $F = ilB$

Setting $F = mg$, the weight of the wire, the magnitude of the current required to remove the tension in the supporting lead is

$$i = \frac{mg}{lB} = \frac{(1.0 \times 10^{-2}\,\text{kg})(10\,\text{m s}^{-2})}{(0.6\,\text{m})(0.4\,\text{Wb m}^{-2})} = 0.41 \text{ A.}$$

Since the magnetic force should act in upward direction so the direction of current should be from left to right.

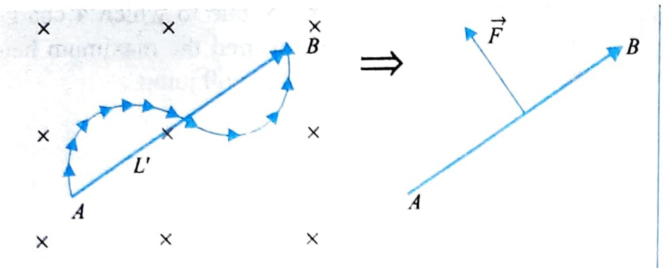
Important Points:

- In case of current carrying conductor in a magnetic field, if the field is uniform, i.e., $\vec{B}$ = constant,

$$\vec{F} = \int I\, d\vec{L} \times \vec{B} = I\left[\int d\vec{L}\right] \times \vec{B}.$$

For a conductor, $\int d\vec{L}$ represents the vector sum of all the length elements from initial to final point, which in accordance with the law of vector addition is equal to the length vector $\vec{L}$ joining initial to final point. So, a current carrying conductor of any arbitrary shape in a uniform field experiences a force

$$\vec{F} = i\left[\int d\vec{L}\right] \times \vec{B} = I\vec{L} \times \vec{B}$$



where $\vec{L}$ is the length vector joining initial and final points of the conductor as shown in figure.

- If the current carrying conductor in the form of a closed loop of any arbitrary shape is placed in a uniform field,

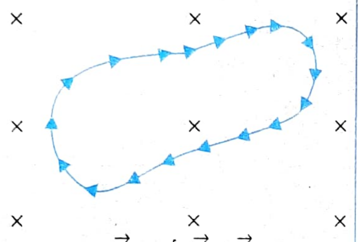
$$\vec{F} = \oint I d\vec{L} \times \vec{B} = I\left[\oint d\vec{L}\right] \times \vec{B}.$$

For a closed loop, the vector sum of $d\vec{L}$ is always zero.

So, $\vec{F} = 0$

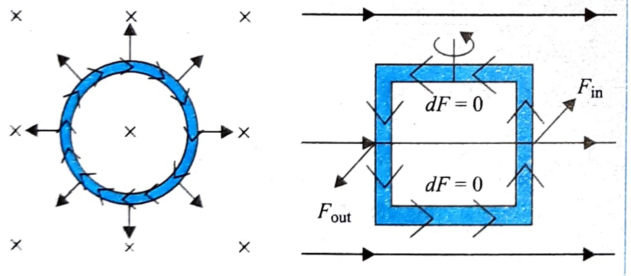
$$\left[\text{as } \oint d\vec{L} = 0\right]$$

i.e., the net magnetic force on a current loop in a uniform magnetic field is always zero as shown in figure.



$\vec{F} = I(\oint d\vec{L}) \times \vec{B} = 0$

- A current carrying loop in a uniform magnetic field Here, it must be kept in mind that in this situation different parts of the loop may experience elemental force due to which the loop may be under tension or may experience a torque as shown in figure.



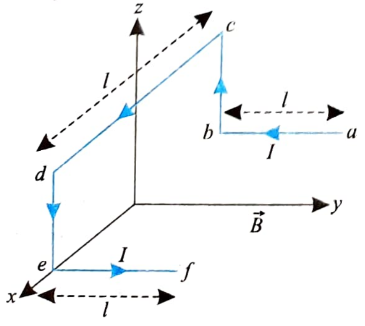
$\vec{F} = 0$
$\vec{\tau} = 0$

Current loop in a uniform field

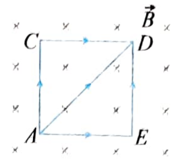
$\vec{F} = 0$
$\vec{\tau} \neq 0$

CONCEPT APPLICATION EXERCISE 1.3

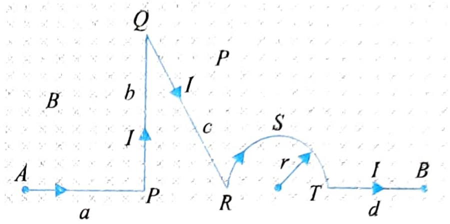
1. A wire *abcdef* with each side of length *l* bent as shown in figure and carrying a current *I* is placed in a uniform magnetic field *B* parallel to +*y* direction What is the force experienced by the wire?



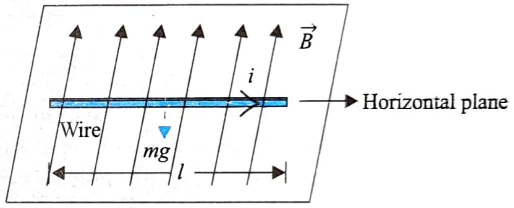
2. A square of side 2.0 m is placed in a uniform magnetic field $\vec{B} = 2.0\ T$ in a direction perpendicular to the plane of the square inwards. Equal current $i = 3.0$. A is flowing in the directions shown in figure. Find the magnitude of magnetic force on the loop.



3. Calculate the force on a current carrying wire in a uniform magnetic field as shown in figure.

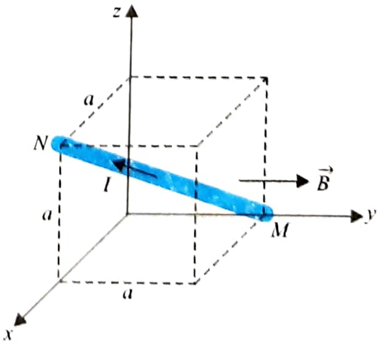


4. A straight wire of mass 200 g and length 1.0 m carries a current of 2 A. It is suspended in mid-air by a uniform horizontal magnetic field *B*. What is the magnitude of the magnetic field? Take $g = 10\ \text{m/s}^2$



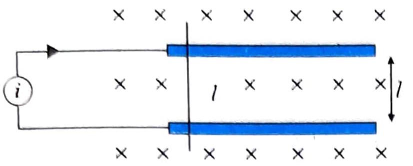
5. The horizontal component of the earth's magnetic field at a certain place is 3×10^{-5} T and the direction of the field is from the geographic south to the geographic north. A very long straight conductor is carrying a steady current of 1 A. What is the force per unit length on it when it is placed on a horizontal table and the direction of the current is
   (a) east to west;
   (b) south to north?

6. A straight wire (*MN*) lies along a body diagonal of an imaginary cube of side $a = \sqrt{2}$ m, and carries a current of 5 A (as shown in figure). Find the force on it due to a uniform field $\vec{B} = 1.0\,\hat{j}$ T.



7. The figure below shows two long metal rails placed horizontally and parallel to each other at a separation

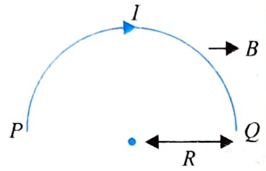
7. A uniform magnetic field B exists in the vertically downward direction. A wire of mass m can slide on the rails. The rails are connected to a constant current source which drives a current i in the circuit. The friction coefficient between the rails and the wire is μ.



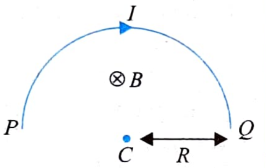
(a) What should be the minimum value of μ which can prevent the wire from sliding on the rails?

(b) Describe the motion of the wire if the value of μ is half the value found in the previous part.

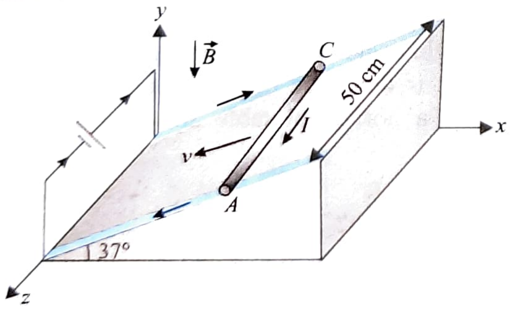
8. In the figure, a semicircular wire is placed in a uniform field $\vec{B}$ directed towards right. Find the resultant magnetic force and torque on it.



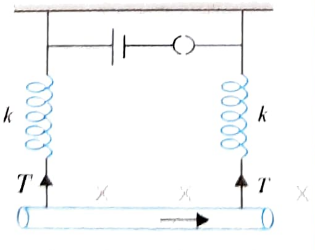
9. In the figure, find the resultant magnetic force and torque about C and P.



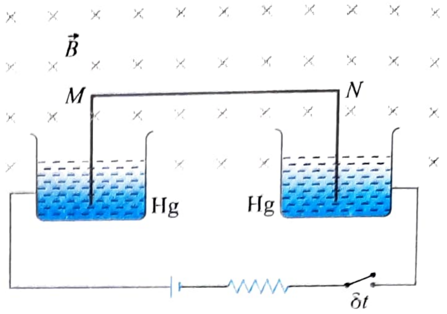
10. In figure, the bar AC has a mass of 50 g. It slides frictionlessly on the metal strips 50 cm apart at the edges of the incline. A current I flows through these strips and the bar, as shown. There is a magnetic field $|B_y| = 0.02$ T directed in the $-y$ direction. How much must I be if the rod is to remain motionless? Neglect the slight overhang of the rod.



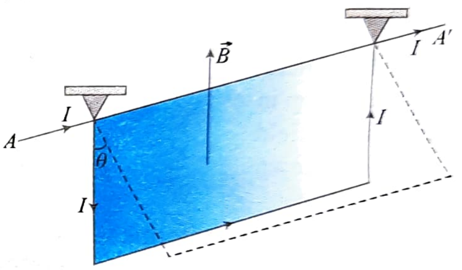
11. A metal rod of mass 10 g and length 25 cm is suspended on two springs as shown in figure. The springs are extended by 4 cm. When a 20 A current passes through the rod, it rises by 1 cm. Determine the magnetic field assuming acceleration due to gravity to be $10\ \text{ms}^{-1}$.



12. The figure shows a horizontal wire MN of length l and mass m is placed in a magnetic field B. The ends of wire are bent and dipped in two bowls containing Hg which are connected to an external circuit as shown. If the key is pressed for a short time δt due to which a charge q suddenly flows in the circuit, find the maximum height above initial level the wire MN will jump.



13. A copper wire with density ρ with cross-sectional area S bent to make three sides of a square frame which can turn about a horizontal axis OO' as shown in figure. The wire is located in uniform vertical magnetic field. Find the magnetic induction if on passing a current I through the wire the frame deflects by an angle θ in its equilibrium position.



ANSWERS

1. $BIl(\hat{k})$ **2.** $36\sqrt{2}$ N. Direction of this force is towards E

3. $IB(a + \sqrt{c^2 - b^2} + 2r + d)$ **4.** 1.0 T

5. (a) 3×10^{-5} N (b) zero **6.** 10 N

7. (a) $\dfrac{ilb}{mg}$ (b) The wire will slide towards right with acceleration $\dfrac{ilB}{2m}$

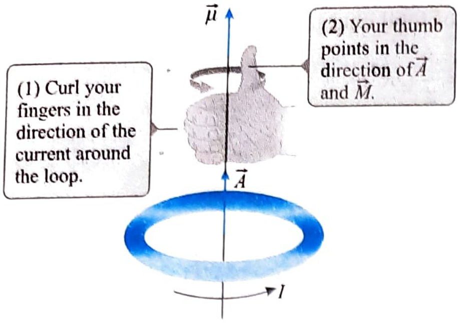
8. 0; $\dfrac{i\pi R^2}{2} B(-\hat{j})$ **9.** $2IRB$, 0, $2IBR^2$

10. 37.5 A **11.** 1.5×10^{-3} T **12.** $\dfrac{B^2q^2l^2}{2m^2g}$ **13.** $\dfrac{2aS\rho g}{Ia}\tan\theta$

MAGNETIC DIPOLE AND DIPOLE MOMENT

Magnetic dipole is the magnetic equivalent of electric dipole. The magnetic field pattern produced by a small current lo is similar to a bar magnet. Therefore, it also acts like a magnet dipole. The magnetic moment of a flat current loop is defin as the product of the current I and the area A enclosed by it, i. $\vec{M} = I\vec{A}$.

The direction of the magnetic moment coincides with th direction of the area vector (which is the direction of the magne field).

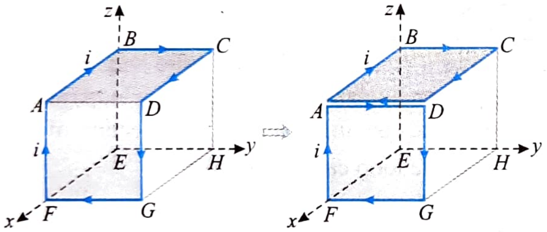

(1) Curl your fingers in the direction of the current around the loop.
(2) Your thumb points in the direction of $\vec{A}$ and $\vec{M}$.
$\vec{\mu}$
$\vec{A}$
I

If the loop contains N number of turns, the magnetic moment is given by $M = NIA$

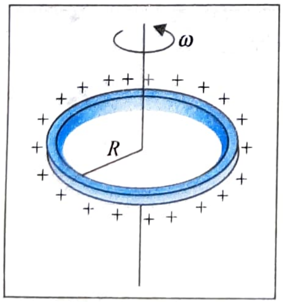
- Sometimes a current carrying loop does not lie in a single plane. But by assuming two equal and opposite currents in one branch (which obviously makes no change in the given circuit) two (or more) closed loops are completed in different planes. Now, the net magnetic moment of the given loop is the vector sum of individual loops. For example, in figure, six sides of a cube of side l carry a current i in the directions shown. By assuming two equal and opposite currents in wire AD, two loops in two different planes (xy and yz) are completed.

$$\vec{M}_{ABCDA} = -il^2\hat{k} \quad \text{and} \quad \vec{M}_{ADGFA} = -il^2\hat{i}$$

$$\Rightarrow \quad \vec{M}_{\text{net}} = -il^2(\hat{i} + \hat{k})$$



- Sometimes a non-conducting body is related with some angular speed. In this case, the ratio of magnetic moment and angular momentum is constant which is equal to $q/2m$, where q is the charge and m is the mass of the body. For example, in case of a ring of mass m, radius R, and charge q distributed on its circumference,



Angular momentum,

$$L = I\omega = (mR^2)(\omega) \qquad \text{...(i)}$$

Magnetic moment,

$$M = iA = (qf)(\pi R^2) \qquad \text{...(ii)}$$

Here, $f = \text{frequency} = \dfrac{\omega}{2\pi}$

$$\therefore \quad M = (q)\left(\frac{\omega}{2\pi}\right)(\pi R^2)$$

$$= q\frac{\omega R^2}{2}$$

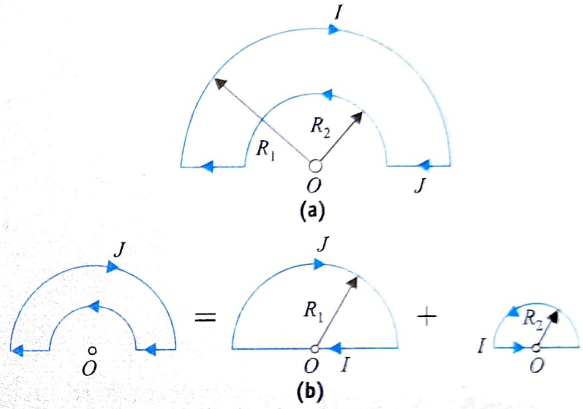
From Eqs (i) and (ii), $\dfrac{M}{L} = \dfrac{q}{2m}$

Although this expression is derived for simple case of a ring, it holds good for other bodies also.

Rigid body	Ring	Disc	Solid sphere	Spherical shell
Moment of inertia (I)	mR^2	$\dfrac{mR^2}{2}$	$\dfrac{2}{5}mR^2$	$\dfrac{2}{3}mR^2$
Magnetic moment $M = \dfrac{qI\omega}{2m}$	$\dfrac{q\omega R^2}{2}$	$\dfrac{q\omega R^2}{4}$	$\dfrac{q\omega R^2}{5}$	$\dfrac{q\omega R^2}{3}$

ILLUSTRATION 1.43

Compute the magnetic dipole moment of the loop shown in figure.



(a)

(b)

Sol. The given loop may be considered as the superposition of the two loops, as shown in the figure.

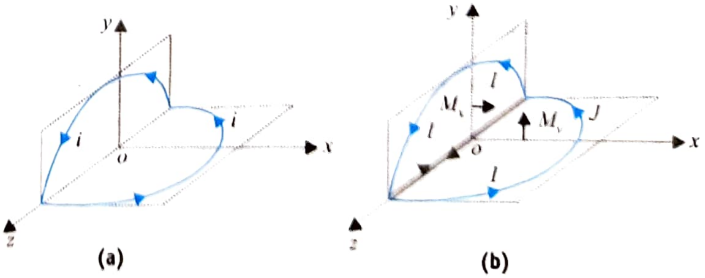
The resultant dipole moment is $M = \dfrac{\pi R_1^2 I}{2} - \dfrac{\pi R_2^2 I}{2}$

or $\quad M = \dfrac{\pi I}{2}\left(R_1^2 - R_2^2\right)$ (inward)

ILLUSTRATION 1.44

A circular loop of wire of radius R is bent about its diameter along two mutually perpendicular planes as shown in the figure. If the loop carries a current I, then determine its magnetic moment.

Sol. The given loop may be obtained by the superposition of two semicircular loops as shown in the figure.



(a)

(b)

The magnetic moment of the semicircle in the y-z plane is along the x-axis and that in the x-z plane is along the y-axis.

$$M_x = \frac{\pi R^2 I}{2}, \quad M_y = \frac{\pi R^2 I}{2}$$

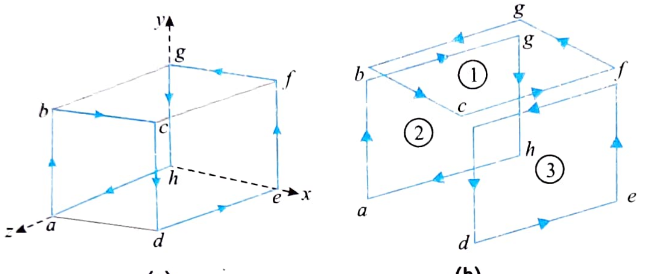
The total magnetic moment is $\vec{M} = M_x\hat{i} + M_y\hat{j}$

or $M = \frac{\pi R^2 I}{2}\left(\hat{i} + \hat{j}\right)$

ILLUSTRATION 1.45

A conductor carries a constant current I along the closed path *abcdefgha* involving 8 of the 12 edges of length l. Find the magnetic dipole moment of the closed path.

Sol. The closed path is a superposition of three loops: *bcfgb*, *abgha* and *cdefc*.



(a)

(b)

The magnetic moments of the three loops are:

Loop 1 (*bcfgb*): $\vec{M}_1 = l^2 I\hat{j}$

2 (*abgha*): $\vec{M}_2 = -l^2 I\hat{i}$

3 (*cdefc*): $\vec{M}_3 = l^2 I\hat{i}$

The total magnetic moment is

$$\vec{M} = \vec{M}_1 + \vec{M}_2 + \vec{M}_3 \text{ or } \vec{M} = l^2 I\hat{j}$$

ILLUSTRATION 1.46

A non-conducting disk of mass M and radius R has a surface charge density σ and rotates with an angular velocity ω about its axis. Show that magnetic dipole moment and angular momentum are related as $\vec{M} = \left(\frac{Q}{2M}\right)\vec{L}$.

Sol. The charge is distributed on the surface of the disk. We consider a differential ring of radius r and thickness dr.

The charge on the element is $dq = \sigma dA = \sigma(2\pi r dr)$

The magnetic moment of the ring $dM = (dI)A = (dI)\pi r^2$

The current in the differential ring

$$dI = (dq)f = (\sigma dA)\frac{\omega}{2\pi} = (\sigma 2\pi r dr)\frac{\omega}{2\pi} = \sigma\omega r dr$$

The magnetic moment of the differential ring,

$$dM = (\sigma\omega r dr)\,\pi r^2 = \pi\sigma\omega r^3 dr$$

$$M = \int dM = \int_0^R \pi\sigma\omega r^3 dr = \frac{1}{4}\pi\sigma\omega R^4$$

The magnetic moment vector $\vec{M}$ is parallel to $\vec{\omega}$ if charge is positive.

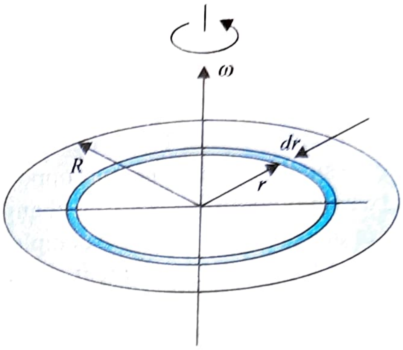
$$\vec{M} = \frac{1}{4}\pi\sigma R^4\vec{\omega}$$

In terms of total charge $Q = \sigma\pi R^2$, the magnetic moment is

$$\vec{M} = \frac{1}{4}QR^2\vec{\omega}$$

The angular momentum of disk is $\vec{L} = \left(\frac{1}{2}MR^2\right)\vec{\omega}$ an

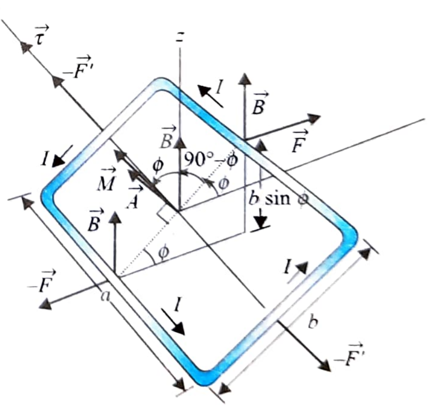
$$\vec{M} = \left(\frac{Q}{2M}\right)\vec{L}.$$

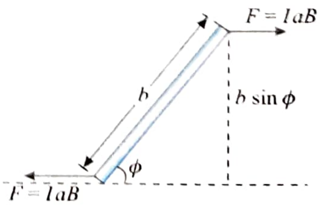


This is a general result for any rigid body of any arbitrary shape having mass M and charge Q.

TORQUE ON A CURRENT-CARRYING PLANAR LOOP IN A UNIFORM MAGNETIC FIELD

Figure shows a rectangular loop of wire with side lengths a an b. A line perpendicular to the plane of the loop (i.e., a normal t the plane) makes an angle ϕ with the direction of the magneti field $\vec{B}$, and the loop carries a current I.





The force $\vec{F}$ on the right side of the loop (length a) in the $+x$ direction as shown. On this side, $\vec{B}$ is perpendicular to the current direction and the force on this side has magnitude

$$F = IaB \quad \text{...(i)}$$

A force $-\vec{F}$ with the same magnitude but opposite direction acts on the opposite side of the loop, as shown in figure.

On sides of length b equal and opposite forces F' act whose lines of action are same.

The total force on the loop is zero because the forces on opposite sides cancel out in pairs. The net force on a current loop in a uniform magnetic field is zero. However, the net torque is not in general equal to zero. The two forces $\vec{F}$ and $-\vec{F}$ lie along different lines, and each gives rise to a torque about the y-axis. According to the right hand rule for determining the direction of torques, the vector torques due to $\vec{F}$ and $-\vec{F}$ are both in the $+y$ direction; hence the net vector torque $\vec{\tau}$ is in the $+y$ direction as well. The magnitude of the net torque is

$$\tau = F(b)\sin\phi = (IBa)(b\sin\phi) \qquad \text{...(ii)}$$

The area A of the loop is equal to ab, so we can rewrite Eq. (ii) as

$$\tau = IBA\sin\phi$$

(magnitude of torque on a current loop) ...(iii)

where ϕ is the angle between the normal to the loop (the direction of the vector area $\vec{A}$) and $\vec{B}$.

The product IA is called the magnetic dipole moment or magnetic moment of the loop

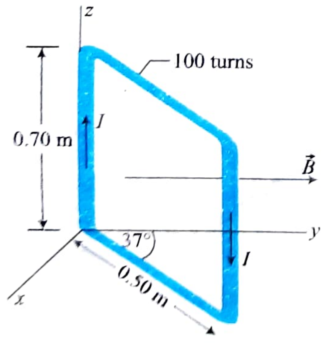
$$M = IA \qquad \text{...(iv)}$$

We can express this interaction in terms of the torque vector $\vec{\tau}$, which we used for electric–dipole interactions. The magnitude of $\vec{\tau}$ is equal to the magnitude of $\vec{M}\times\vec{B}$. So, we have

$$\vec{\tau} = \vec{M}\times\vec{B} \text{ (vector torque on a current loop)} \qquad \text{...(v)}$$

ILLUSTRATION 1.47

The rectangular loop in the drawing consists of 100 turns and carries a current of $I = 5.0$ A. Magnetic field of magnitude 2.0 T magnetic field is directed along the $+y$ axis. The loop is free to rotate about the z axis. **(a)** Determine the magnitude of the net torque exerted on the loop and **(b)** state whether the 37° angle will increase or decrease.



Sol. The torque on the loop is given by $\tau = NIAB\sin\phi$. From the given drawing, we see that the angle ϕ between the normal to the plane of the loop and the magnetic field is

$$90° - 37° = 53°$$

The area of the loop is $0.70\text{ m}\times 0.50\text{ m} = 0.35\text{ m}^2$.

(a) The magnitude of the net torque exerted on the loop is

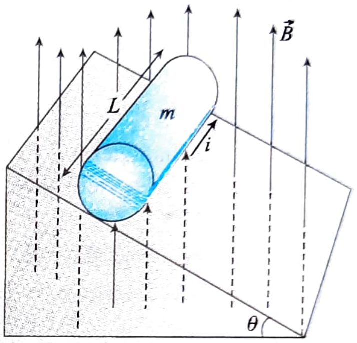
$$\tau = NIAB\sin\phi$$

$$= (100)(5.0)(0.35)(2.0)\sin 53° = 280\text{ N}\cdot\text{m}$$

(b) When a current-carrying loop is placed in a magnetic field, the loop tends to rotate such that its normal becomes aligned with the magnetic field. The normal to the loop makes an angle of 53° with respect to the magnetic field. Since this angle decreases as the loop rotates, the 37° angle increases.

ILLUSTRATION 1.48

The figure shows a wood cylinder of mass m and length L with N turns of wire wrapped around it longitudinally, so that the plane of the wire coil contains the long central axis of the cylinder. The cylinder is released on a plane inclined at an angle θ to the horizontal, with the plane of the coil parallel to the inclined plane. If there is a vertical uniform magnetic field of magnitude B, what is the least current i through the coil that keeps the cylinder from rolling down the plane?



Sol. As the plane of the loop is parallel to the incline the dipole moment is normal to the incline. The forces acting on the cylinder are the force of gravity mg, acting downward from the center of mass, the normal force of the incline N, acting perpendicularly to the incline through the center of mass, and the force of friction f, acting up the incline at the point of contact. For translational equilibrium of the cylinder

$$f = mg\sin\theta \qquad \text{...(i)}$$

Now let us consider rotational equlibrium, we take the axis of the cylinder to be the axis of rotation. The magnetic field produces a torque with magnitude $MB\sin\theta$, and the force of friction produces a torque with magnitude fr, where r is the radius of the cylinder. For rotational equilibrium of the cylinder

$$fr = MB\sin\theta \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$mg\sin\theta . r = MB\sin\theta \Rightarrow mgr = MB \qquad \text{...(iii)}$$

The loop is rectangular with two sides of length L and two of length $2r$, so its area is $A = 2rL$ and the dipole moment is $M = NiA = Ni(2rL)$. Hence Eq. (iii) becomes

$$mgr = 2NirLB \Rightarrow i = \frac{mg}{2NLB}$$

ILLUSTRATION 1.49

A coil in the shape of an equilateral triangle of side 0.02 m is suspended from a vertex such that it is hanging in a vertical plane between the pole pieces of a permanent magnet producing a horizontal magnetic field of 5×10^{-2} T. Find the couple acting on the coil when a current of 0.1 A is passed through it and the magnetic field is parallel to its plane.

Sol. The couple acting on a closed loop is given by

$$\tau = NiAB \sin\theta$$

where, N = number of loops, A = area of loop, B = magnetic field and θ = angle between magnetic field and normal to the surface of the loop.

Here $N = 1$, $i = 0.1$ A, $B = 5 \times 10^{-2}$ T, $\theta = 90°$ and

$$A = \frac{1}{2}\ (\text{Base} \times \text{height})$$

$$= \frac{1}{2}(0.02 \times 0.01732) = 1.732 \times 10^{-4}\,\text{m}^2$$

$$\therefore\ \tau = 1 \times 0.1 \times (1.732 \times 10^{-4}) \times (5 \times 10^{-2}) \times 1$$

$$= 8.66 \times 10^{-7}\ \text{Nm}$$

ILLUSTRATION 1.50

A circular coil of wire 8 cm in diameter has 12 turns and carries a current of 5 A. The coil is in a field where the magnetic induction is 0.6 T.

(a) What is the maximum torque on the coil?

(b) In what position would the torque be half as great as in (i)?

Sol.

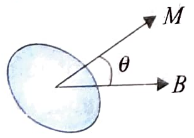
(a) The dipole moment of the coil: $M = NiA$

Here $N = 12$ turns, $i = 5$ A and $A = \pi r^2 = \pi(4 \times 10^{-2})^2$

$$\therefore\ M = 12 \times 5 \times \pi \times (4 \times 10^{-2})^2 = 0.3\ \text{Am}^2$$

Maximum torque = $MB = 0.3 \times 0.60 = 0.18$ Nm

(b) If θ be the angle between the axis of the coil and the field torque = $MB \sin\theta$



According to the problem,

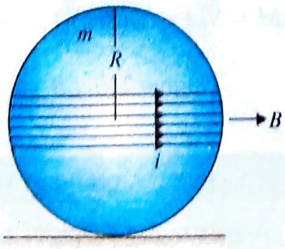
torque = $MB/2$

$$\text{So}\quad \frac{MB}{2} = MB\sin\theta \ \Rightarrow\ \sin\theta = \frac{1}{2} \quad \text{or} \quad \theta = 30°$$

Thus, the normal to the coil is at 30° to the field.

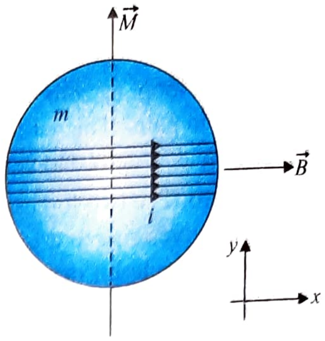
ILLUSTRATION 1.51

A wire is wrapped N times over a solid sphere of mass m which is placed on a smooth horizontal surface. A horizontal magnetic field of induction $\vec{B}$ is present. Find the **(a)** torque **(b)** angular acceleration experienced by the sphere. Assume that the mass of the wire is negligible compared to the mass of the sphere.



Sol.

(a) The net torque acting on the sphere is



$$\vec{\tau} = \vec{M} \times \vec{B} = (NiA\hat{j}) \times (B\hat{i}) = -NiAB\hat{k} \quad \text{where } A = \pi R$$

$$\text{or}\quad \vec{\tau} = -N\pi R^2 iB\hat{k}$$

(b) $\vec{\alpha} = \dfrac{\vec{\tau}}{I_c}$ ($\because$ the sphere is free to rotate, it must rota about the centroidal axis)

$$= \frac{-N\pi R^2 iB}{\frac{2}{5}mR^2}\hat{k} = \frac{5N\pi iB}{2m}\hat{k} \qquad \left(\because\ I_c = \frac{2}{5}mR^2\right)$$

ENERGY OF DIPOLE

When a magnetic dipole changes orientation in a magnetic fiel the field does work on it. In an infinitesimal angular displaceme $d\phi$, the work dW is given by $\tau\, d\phi$, and there is a correspondin change in potential energy.

Energy needed to rotate the loop through an angle $d\theta$ is

$$dU = \tau d\theta$$

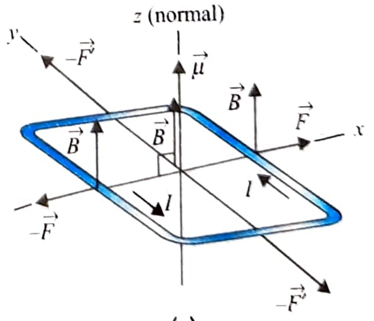
$$\Rightarrow\ \Delta U = \int_{\theta_1}^{\theta_2} \tau\, d\theta = \int_{\theta_1}^{\theta_2} MB\sin\theta\, d\theta$$

$$\Rightarrow\ \Delta U = MB(\cos\theta_1 - \cos\theta_2)$$

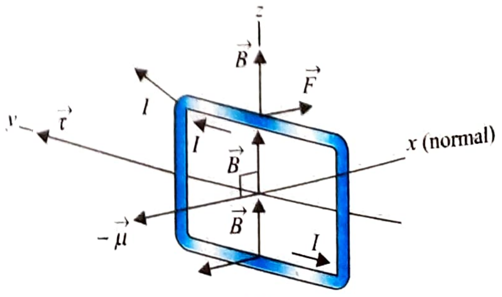
If we choose θ_1 such that at $\theta_1 = 90°$, $U_1 = 0$

$$U = -\vec{M}\cdot\vec{B}$$

This is the energy stored in the loop.



(a)



(b)

Important Points:

Forces on the sides of a current carrying loop in a uniform magnetic field.

- The resultant force is zero; the net torque has magnitude $\tau = IAB \sin \phi$.
- The torque is maximum when the normal to the loop is perpendicular to $\vec{B}$.
- When the normal to the loop is parallel to $\vec{B}$, the torque is zero and the equilibrium is stable. If the normal is antiparallel to $\vec{B}$, the torque is also zero but the equilibrium is unstable.

WORK DONE IN ROTATING A CURRENT-CARRYING COIL IN MAGNETIC FIELD

Let a current-carrying coil is rotated in a uniform magnetic field in such a way that its orientation changes with angle between magnetic field and magnetic moment of coil from θ_1 to θ_2. Then the potential energy of the coil in magnetic field in initial and final state is given as

$$U_i = -MB\cos\theta_1 \qquad ...(i)$$

and $$U_f = -MB\cos\theta_2 \qquad ...(ii)$$

To change the orientation of coil externally, we need to apply a torque against the magnetic torque of equal magnitude thus work done in in changing the orientation of the coil during its rotation is given as

$$W = U_f - U_i$$

$$\Rightarrow \quad W = (-MB\cos\theta_2) - (-MB\cos\theta_1)$$

$$\Rightarrow \quad W = MB(\cos\theta_1 - \cos\theta_2) \qquad ...(iii)$$

Important Point:

Above expression of work can be rewritten as

$$W = IAB(\cos\theta_1 - \cos\theta_2) = I(BA\cos\theta_1 - BA\cos\theta_2)$$

Here, $BA\cos\theta = \phi =$ Magnetic flux associated with the coil.

$$\Rightarrow \quad W = I(\phi_{\text{initial}} - \phi_{\text{final}})$$

$$\Rightarrow \quad W = -I(\phi_{\text{final}} - \phi_{\text{initial}})$$

$$\Rightarrow \quad W = -I\Delta\phi \qquad ...(v)$$

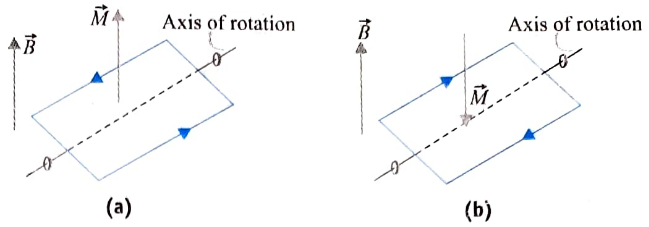
$\Delta\phi$ = change in magnetic flux associated with coil in two orientations.

Stable and Unstable Equilibrium of a Current-Carrying Loop in Magnetic Field

Let us consider a current-carrying circular loop placed in a plane perpendicular to the direction of magnetic field. The loop carries a current in clockwise direction due to which the direction vector of its magnetic moment is along the direction of magnetic induction due to which the angle between these two vectors is $\theta = 0$ and thus torque on loop is also $\vec{M} \times \vec{B} = 0$ and also net force acting on it will be zero as it is placed in uniform magnetic field. It means the loop is in equilibrium.

The interaction potential energy of the coil is given as

$$U = -MB\cos\theta = -MB \qquad ...(i)$$



(a) (b)

If we slightly tilt the loop about the axis of rotation it will experience net torque and this torque will have a tendency to bring back the loop in its initial position thus therea restoring torque will be developed, hence the equilibrium of coil is stable equilibrium.

We can also see from Eq. (i) that the potential energy of loop in this state is at its minimum value which corresponds to the case of stable equilibrium.

In Fig. (b), a similar situation is shown with the current direction in the loop is in anticlockwise direction. In this case the angle between magnetic moment and magnetic induction is $\theta = 180°$ and thus torque on loop is again $\vec{M} \times \vec{B} = 0$ and loop is in equilibrium.

The interaction potential energy of the coil in this situation is given as

$$U = -MB\cos\theta$$

$$= -MB(-1) = +MB \qquad ...(ii)$$

If in Fig. (b) we slightly tilt the loop about the axis of rotation the torque developed will have a tendency to further rotate the coil away from its initial position thus in this case the equilibrium of coil is unstable equilibrium.

We can also see from Eq. (ii) that the potential energy of loop in this state is at its maximum value which corresponds to the case of unstable equilibrium.

ILLUSTRATION 1.52

A circular coil of 100 turns and having a radius of 0.05 m carries a current of 0.1 A. Calculate the work required to turn the coil in an external field of 1.5 T through 180° about an axis perpendicular to the magnetic field? The plane of coil is initially at right angles to magnetic field.

Sol. Work done

$$W = MB(\cos\theta_1 - \cos\theta_2)$$

$$= NIAB(\cos\theta_1 - \cos\theta_2)$$

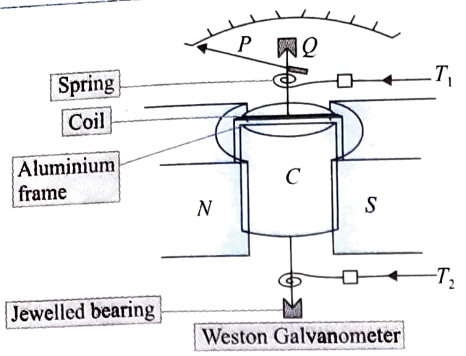
$$\Rightarrow \quad W = NI\pi r^2 B(\cos\theta_1 - \cos\theta_2)$$

$$= 100 \times 0.1 \times 3.14 \times (0.05)^2 \times 1.5(\cos 0° - \cos\pi)$$

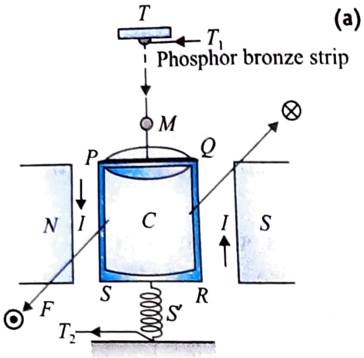
$$= 0.2355 \text{ J}$$

MOVING COIL GALVANOMETER

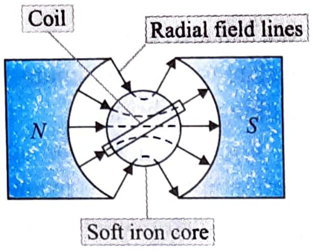
This is a device which is used for detection and measurement of small electric current. The principle of a moving coil galvanometer is based on the fact that when a current carrying coil is placed in a magnetic field, it experiences a torque.



(a)



(b)



(c)

CONSTRUCTION

A moving coil ballistic galvanometer is shown in Fig. (a). It essentially consists of a rectangular coil *PQRS* or a cylindrical coil of large number of turns of fine insulated wire wound over a non-conducting frame of ivory or bamboo [Fig. (b)].This coil is suspended by means of phosphor bronze wire between the pole pieces of a powerful horse shoe magnet NS. The poles of the magnet are curved to make the field radial as shown in Fig. (c). The lower end of the coil is attached to a spring of phosphorbronze wire. The spring and the free ends of phosphor bronze wire are joined to two terminals T_2 and T_1, respectively, on the top of the case of the instrument. *L* is a soft iron core. A small mirror *M* is attached on the suspension wire. Using lamp and scale arrangement, the deflection of the coil can be recorded. The whole arrangement is enclosed in a non-metallic case.

THEORY

Let the coil be suspended freely in the magnetic field.

Suppose, N = number of turns in the coil

A = area of the coil

B = magnetic field induction of radial magnetic field in which the coil is suspended.

Here, the magnetic field is radial, i.e., the plane of the coil always remains parallel to the direction of magnetic field, and hence the torque acting on the coil

$$\tau = NiAB \quad \text{...(i)}$$

Due to this torque, the coil rotates. As a result, the suspension wire gets twisted. Now a restoring torque is developed in the suspension wire. The coil will rotate till the deflecting torque acting on the coil due to flow of current through it is balanced by the restoring torque developed in the suspension wire due to twisting. Let C be the restoring couple per unit twist in the suspension wire and θ be the angle through which the coil has turned. The couple for this twist θ is $C\theta$.

In equilibrium, deflecting couple = restoring couple

$$NiAB = C\theta$$

or $i = C\theta/(NAB)$

or $i = K\theta$ (where $C/NAB = K$) ...(i

K is a constant for galvanometer and is known as galvanomete constant.

Hence, $i \propto \theta$

Therefore, the deflection produced in the galvanometer i directly proportional to the current flowing through it.

CURRENT SENSITIVITY OF THE GALVANOMETER

The current sensitivity of a galvanometer is defined as th deflection produced in the galvanometer when a unit current i passed through it.

We know that, $NiAB = C\theta$

$$\therefore \text{ Current sensitivity, } i_s = \frac{\theta}{i} = \frac{NAB}{C}$$

The unit of current sensitivity is radian per ampere or deflectio per ampere.

VOLTAGE SENSITIVITY OF THE GALVANOMETER

The voltage sensitivity of the galvanometer is defined as th deflection produced in the galvanometer when a unit voltage i applied across the terminals of the galvanometer.

$$\therefore \text{ Voltage sensitivity, } V_s = \frac{\theta}{V}$$

If R be the resistance of the galvanometer and a current i i passed through it, then

$$V = iR$$

$$\text{Voltage sensitivity, } V_s = \frac{\theta}{iR} = \frac{NAB}{CR}$$

The unit of voltage sensitivity is radian per volt or deflectio per volt.

CONDITIONS FOR SENSITIVE GALVANOMETER

A galvanometer is said to be more sensitive if it shows a larg deflection evern for a small value of current.

$$\text{We know that } \theta = \frac{NAB}{C} i$$

For a given value of i, θ will be large if

(1) N is large, (2) A is large, (3) B is large, and (4) C is small

The value of C for quartz and phosphor-bronze is very small s the suspension wire of quartz or phosphor-bronze is used. N an A cannot be increased beyond a certain limit, because otherwis size of the galvanometer increases. B can be increased usin strong magnets.

ILLUSTRATION 1.53

A galvanometer having 30 divisions has current sensitivity of 20 μA/division. Find the maximum current it can measure.

Sol. maximum current that can be measured:

I_g = current sensitivity × number of divisions

= (20 μA/division) × (30) = 600 μA

ILLUSTRATION 1.54

A rectangular coil of area 5.0×10^{-4} m^2 and 60 turns is pivoted about one of its vertical sides. The coil is in a radial horizontal field of 90 G (radial here means the field liners are in the plane of the coil for any rotation). What is the torsional constant of the hair spring connected to the coil if a current of 2.0 mA produces an angular deflection of 18°?

Sol. Torsional constant is given by: $C = \dfrac{INBA}{\theta}$

$$\Rightarrow \ C = \frac{2 \times 10^{-3} \times 60 \times 90 \times 10^{-4} \times 5 \times 10^{-4}}{18^\circ}$$

$$= 3 \times 10^{-8} \text{ Nm/degree}$$

ILLUSTRATION 1.55

The coil of a galvanometer is 0.02×0.8 m^2. It consists of 200 turns of the wire and is in a magnetic field of 0.20 T. The restoring torque constant of suspension fibre is 10^{-5} Nm/degree. Assuming magnetic field to be radial

(a) What is the maximum current that can be measured by this galvanometer if scale can accommodate 45° deflection?

(b) What is the smallest current that can be detected if minimum observed deflection is 0.1°?

Sol. We know that $I = \dfrac{C}{NBA}\theta$

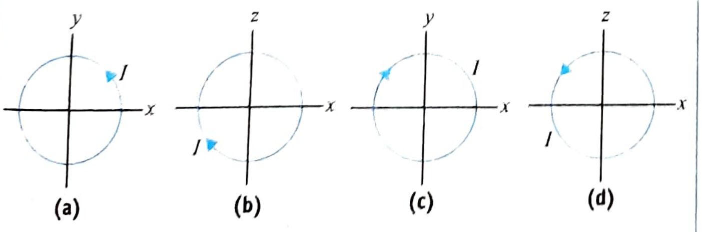
(a) $$I = \frac{10^{-5} \times 45^\circ}{200 \times 0.20 \times 0.02 \times 0.8} = 7.03 \times 10^{-4} \text{ A}$$

(b) $$I_{\min} = \frac{C}{NBA}\theta_{\min}$$

$$= \frac{10^{-5} \times 0.1^\circ}{200 \times 0.20 \times 0.02 \times 0.8} = 1.56 \times 10^{-6} \text{ A}$$

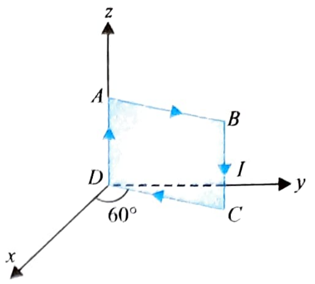
CONCEPT APPLICATION EXERCISE 1.4

1. A circular coil with area A and N turns is free to rotate about a diameter that coincides with the x-axis. Current I is circulating in the coil. There is a uniform magnetic field $\vec{B}$ in the positive y direction. Calculate the magnitude and direction of the torque $\vec{\tau}$ and the value of the potential energy U, when the coil is oriented as shown in parts (a) through (d) of figure.

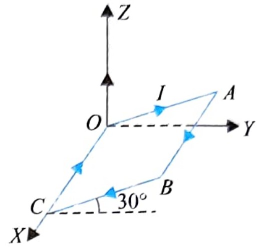


(a) (b) (c) (d)

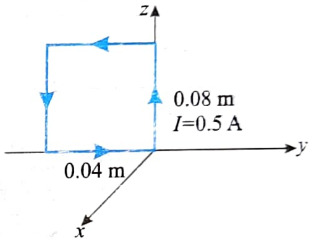
2. A square loop $OABCO$ of side l carries a current I. It is placed as shown in figure. Find the magnetic moment of the loop.



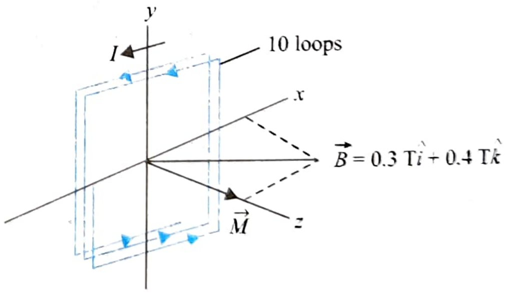
3. Find the magnetic moment of the current carrying loop $OABCO$ as shown in figure. Given that $i = 4$ A, $OA = 20$ cm, and $AB = 10$ cm.



4. The rectangular coil having 100 turns is placed in a uniform magnetic field of $(0.05/\sqrt{2})\hat{j}$ tesla as shown in figure. Find the torque acting on the loop.



5. A square 10-turn coil with sides of length 40 cm carries a current of 5 A. It lies in the x–y plane as shown in figure in a uniform magnetic field.

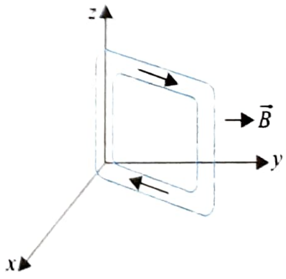


(a) Find the magnetic moment of the coil.

(b) Find the torque exerted on the coil.

(c) Find the potential energy of the coil.

6. The square loop in figure has sides of length 20 cm. It has 5 turns and carries a current of 2 A. The normal to the loop is at 37° to a uniform field $\vec{B} = 0.5\hat{j}$ T.

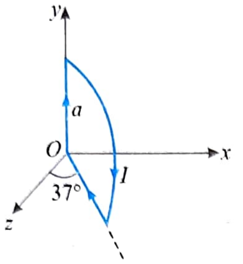


(a) Find the magnetic moment of the loop.

(b) Find the torque on the loop.

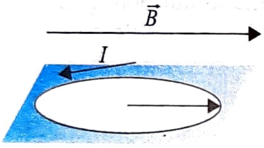
(c) Find the work needed to rotate the loop from its position of minimum energy to the given orientation.

7. The figure shows one quarter of a simple circular loop of wire that carries a current of 10 A. Its radius is $a = 5$ cm. A uniform magnetic field, $B = 400$ G, is directed in the $+x$ direction. Find the torque on the entire loop and the direction in which it will rotate.



8. A galvanometer coil is replaced by another coil of diameter one-fourth of the original diameter and the total number of turns as ten times the original number. What will be the new deflection if the same current is passed through it? Old deflection is θ.

9. A circular wire loop of radius R, mass m carrying current I lies on a rough surface (as shown in figure). There is a horizontal magnetic field $\vec{B}$. How large can the current I be before one edge of the loop will lift off the surface?

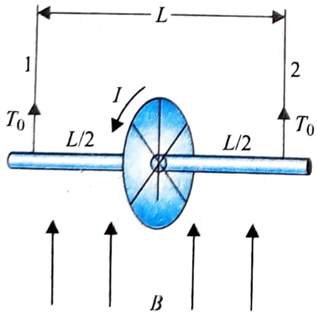


10. A wire is formed into a circle having a diameter of 10.0 cm and placed in a uniform magnetic field of 3.00 mT. The wire carries a current of 5.00 A. Find

(a) the maximum torque on the wire, and

(b) the range of potential energies of the wire–field system for different orientations of the circle.

11. The circular current loop of radius b shown in the figure is mounted rigidly on the axle, midway between the two supporting cords. In the absence of an external magnetic field, the tensions in the cords are equal and are T_0.

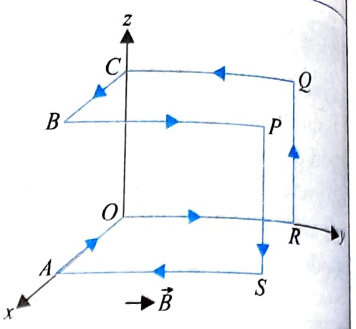


(a) What will be the tensions in the two cords when the vertical magnetic field B is present?

(b) Repeat if the field is parallel to the axis.

12. A circular coil of 100 turns and radius 5.0 cm, carrying a current of 10 A, is suspended vertically in a uniform magnetic field of magnitude 2.0 T. The field lines run horizontally in the plane of the coil. Calculate the force and torque on coil due to the magnetic field. In which direction should a balancing torque be applied to prevent the coil from turning?

13. The figure shows a coil bent with all edges of length 2 m and carrying a current of 2A. There exists in space a uniform magnetic field of 2T in positive y-direction. Find the torque on the loop.



14. A particle of charge q moves in a circle of radius r with speed v. Treating the circular path as a current loop with an average current, find the maximum torque exerted on the loop by a uniform field of magnitude B.

ANSWERS

1. (a) $NIAB(-\hat{i}), 0$ (b) $0, -NIAB$ (c) $NIAB(\hat{i}), 0$ (d) $0, NIAB$

2. $\frac{Il^2}{2}(-\sqrt{3}\hat{i} + \hat{j})$ **3.** $4 \times 10^{-2}(\hat{j} - \sqrt{3}\hat{k})$ Am^2

4. $4\sqrt{2} \times 10^{-3}$(Nm)$\hat{k}$

5. (a) $8.0\hat{k}$ Am^2 (b) $2.4\hat{j}$ N-m (c) -3.2 J

6. (a) $-0.24\hat{i} + 0.32\hat{j}$ Am^{-2} (b) $-0.12\hat{k}$ Nm (c) 0.04 J

7. $6\pi \times 10^{-4}$ Nm; loop will rotate about the y-axis, so as to decrease the angle labelled 37°.

8. $\frac{5}{8}\theta$ **9.** $\frac{mg}{\pi RB}$

10. (a) $375\pi \times 10^{-7}$ Nm (b) $U_{\max} = +MB$, $U_{\min} = -MB$

11. (a) $\frac{mg}{2} + \frac{\pi b^2 BI}{L}, \frac{mg}{2} - \frac{\pi b^2 BI}{L}$ (b) $mg/2$

12. 5π N **13.** 0 **14.** $\frac{1}{2}qvrB$

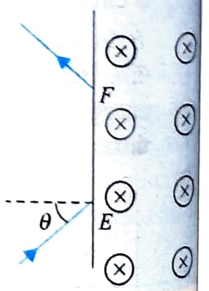
Solved Examples

EXAMPLE 1.1

A particle of mass m and charge $+q$ enters a region of magnetic field with a velocity v, as shown in figure.

(a) Find the angle subtended by the circular arc described by it in the magnetic field.

(b) How long does the particle stay inside the magnetic field?



Sol.

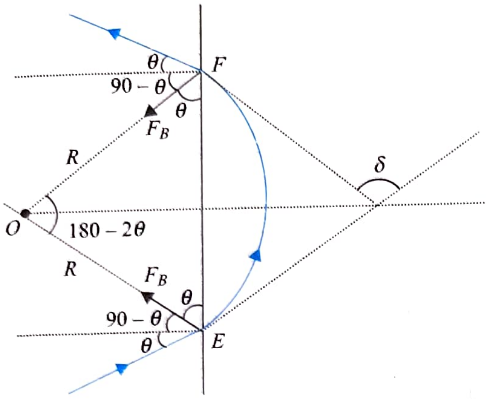
(a) The particle circulates under the influence of magnetic field. As the magnetic field is uniform, the charge comes out symmetrically. The angle subtended at the center is $(180° - 2\theta)$.

(b) The length of the arc traced by the particle, $l = R(\pi - 2\theta)$

Time spent in the field, $t = \frac{l}{v} = \frac{R(\pi - 2\theta)}{v}$

and $R = \dfrac{mv}{Bq}$ which gives $t = \dfrac{m}{Bq}(\pi - 2\theta)$

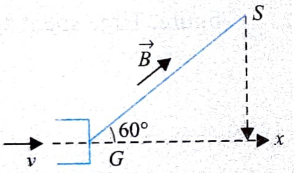
As time period: $T = \dfrac{2\pi m}{Bq}$, hence $t = \dfrac{T}{2\pi}(\pi - 2\theta)$



We can generalize this result. If ϕ is the angle subtended by the arc traced by the charged particle in the magnetic field, the time spent is $t = T\left(\dfrac{\phi}{2\pi}\right)$

EXAMPLE 1.2

An electron gun G emits electrons of energy 2 keV travelling in the positive x-direction. The electrons are required to hit the spot S where $GS = 0.1$ m, and the line GS makes an angle of 60° with the x axis as shown in figure. A uniform magnetic field $\vec{B}$ parallel to GS exists in the region outside the electron gun. Find the minimum value of B needed to make the electrons hit S.



Sol. Kinetic energy of electron, $K = \dfrac{1}{2}mv^2 = 2$ keV

$$\therefore \text{ Speed of electron, } v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2 \times 2 \times 1.6 \times 10^{-16}}{9.1 \times 10^{-31}}} \text{ m s}^{-1}$$

$$= 2.65 \times 10^7 \text{m s}^{-1}$$

Since the velocity ($\vec{v}$) of the electron makes an angle of $\theta = 60°$ with the magnetic field $\vec{B}$, the path will be a helix.

So, the particle will hit S if $GS = np$; Here, $n = 1, 2, 3, \ldots$

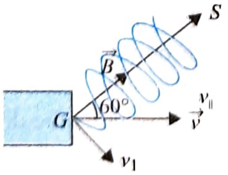
$$p = \text{pitch of helix} = \frac{2\pi m}{qB} v\cos\theta$$

$$\Rightarrow \quad GS = \frac{n\, 2\pi\, mv\cos\theta}{qB}$$

$$\Rightarrow \quad B = \frac{n\, 2\pi\, mv\cos\theta}{q(GS)}$$

But for B to be minimum, $n = 1$

$$B_{\min} = \frac{2\pi mv\cos\theta}{q(GS)}$$



Substituting the values, we have

$$B_{\min} = \frac{(2\pi)(9.1 \times 10^{-31})(2.65 \times 10^7)\left(\frac{1}{2}\right)}{(1.6 \times 10^{-19})(0.1)}$$

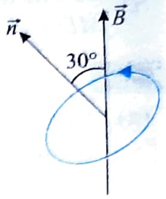
or $\quad B_{\min} = 4.73 \times 10^{-3}$ T

EXAMPLE 1.3

An electron in the ground state of hydrogen atom is revolving in anticlockwise direction in a circular orbit of radius R (see figure).

(a) Obtain an expression for the orbital magnetic moment of the electron.

(b) The atom is placed in a uniform magnetic induction $\vec{B}$ such that the normal to the plane of electron's orbit makes an angle of 30° with the magnetic induction. Find the torque experienced by the orbiting electron.



Sol.

(a) In ground state ($n = 1$) according to Bohr's theory:

$$mvR = \frac{h}{2\pi} \text{ or } v = \frac{h}{2\pi mR}$$

$$\text{Now time period, } T = \frac{2\pi R}{v} = \frac{2\pi R}{h/2\pi mR} = \frac{4\pi^2 mR^2}{h}$$

Magnetic moment $M = i\,A$

$$\text{where } I = \frac{\text{charge}}{\text{time period}} = \frac{e}{\frac{4\pi^2 mR^2}{h}} = \frac{eh}{4\pi^2 mR^2}$$

and $A = \pi R^2$

$$\therefore \quad M = (\pi R^2)\left(\frac{eh}{4\pi^2 mR^2}\right) \quad \text{or} \quad M = \frac{eh}{4\pi m}$$

Direction of magnetic moment $\vec{M}$ is perpendicular to the plane of orbit.

(b) $\quad \vec{\tau} = \vec{M} \times \vec{B}$

$|\vec{\tau}| = MB\sin\theta$

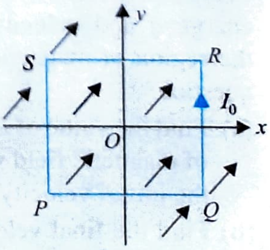
where θ is the angle between $\vec{M}$ and $\vec{B}$ $\theta = 30°$

$$\therefore \quad \tau = \left(\frac{eh}{4\pi m}\right)(B)\sin 30° = \frac{ehB}{8\pi m}$$

The direction of $\vec{\tau}$ is perpendicular to both $\vec{M}$ and $\vec{B}$.

EXAMPLE 1.4

A uniform, constant magnetic field $\vec{B}$ directed at an angle of 45° to the x-axis in the x–y plane. $PQRS$ is a rigid, square wire frame carrying a steady current I_0, with its center at the origin O. At time $t = 0$, the frame is at rest in the position shown in figure, with its sides parallel to the x- and y-axes. Each side of the frame is of mass M and length L.



(a) What is the torque $\vec{\tau}$ about O acting on the frame due to the magnetic field?

(b) Find the angle by which the frame rotates under the action of this torque in a short interval of time Δt, and the axis about which this rotation occurs. (Δt is so short that any variation on the torque during this interval may be neglected). Given, moment of inertia of the frame about an axis through its center perpendicular to its plane is (4/3) ML^2.

Sol.

(a) As magnetic field is in x–y plane and subtends an angle of 45° with x-axis,

$$B_x = B\cos 45° = B/\sqrt{2}$$

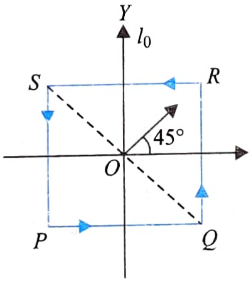
and $B_y = B\sin 45° = B/\sqrt{2}$

so, in vector form $\vec{B} = \hat{i}\left(B/\sqrt{2}\right) + \hat{j}\left(B/\sqrt{2}\right)$

and $\vec{M} = I_0 S\hat{k} = I_0 L^2\hat{k}$

$$\Rightarrow \quad \vec{\tau} = \vec{M}\times\vec{B} = I_0 L^2\hat{k}\times\left(\frac{B}{\sqrt{2}}\hat{i} + \frac{B}{\sqrt{2}}\hat{j}\right)$$

$$\Rightarrow \quad \vec{\tau} = \frac{I_0 L^2 B}{\sqrt{2}}\times(-\hat{i}+\hat{j})$$



i.e., torque has magnitude I_0L^2B and is directed along the line QS from Q to S.

(b) By the theorem of perpendicular axis, moment of inertia of the frame about QS,

$$I_{QS} = \frac{1}{2}I_z = \frac{1}{2}\left(\frac{4}{3}ML^2\right) = \frac{2}{3}ML^2$$

as $\tau = I\alpha \quad \Rightarrow \quad \alpha = \dfrac{\tau}{I} = \dfrac{I_0L^2B\times 3}{2L^2M} = \dfrac{3}{2}\dfrac{I_0B}{M}$

As here α is constant, equations of circular motion are valid. Hence, from $\theta = \omega_0 t + \dfrac{1}{2}\alpha t^2$, with $\omega_0 = 0$, we have

$$\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2}\left(\frac{3}{2}\frac{I_0B}{M}\right)(\Delta t)^2 = \frac{3}{4}\frac{I_0B}{M}\Delta t^2$$

EXAMPLE 1.5

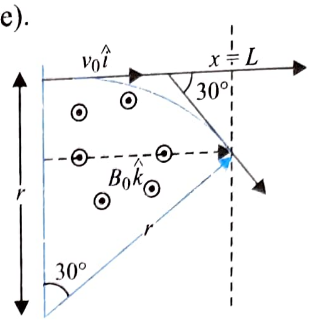
The region between $x = 0$ and $x = L$ is filled with uniform, steady magnetic field $B_0\hat{k}$. A particle of mass m, positive charge q and velocity $v_0\hat{i}$ travels along x-axis and enters the region of magnetic field. Neglect gravity throughout the question.

(a) Find the value of L if the particle emerges from the region of magnetic field with its final velocity at an angle 30° to the initial velocity.

(b) Find the final velocity of the particle and the time spent by it in the magnetic field, if the field now extends up to $x = 2.1L$.

Sol.

(a) As the initial velocity of the particle is perpendicular t the field, the particle will move along the arc of a circle a shown (figure).



If r is the radius of the circle, then $\dfrac{mv_0^2}{r} = qv_0B_0$

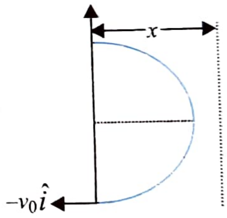
$$\Rightarrow r = \frac{mv_0}{qB_0}$$

Also, from geometry, $L = r\sin 30° \Rightarrow r = 2L$

or $\quad L = \dfrac{r}{2} = \dfrac{mv_0}{2qB_0}$

(b) In this case, $x = \dfrac{2.1mv_0}{2qB_0} > r$

Hence, the particle will complete semicircular path an emerge from the field with velocity $-v_0\ \hat{i}$ as shown i figure. Time spent by the particle in the magnetic field



$$T = \frac{\pi r}{v_0} = \frac{\pi m}{qB_0}$$

The speed of the particle does not change due to magnet field.

EXAMPLE 1.6

A rectangular loop $PQRS$ made from a uniform wire has leng a, width b and mass m. It is free to rotate about the arm P which remains hinged along a horizontal line taken as th y-axis. Take the vertically upward direction as the z-axis. uniform magnetic field $\vec{B} = (3\hat{i} + 4\hat{k})B_0$ exists in the regio The loop is held in the x–y plane and a current I is passe through it. The loop is now released and is found to stay in th horizontal position in equilibrium.

(a) What is the direction of the current I in PQ?

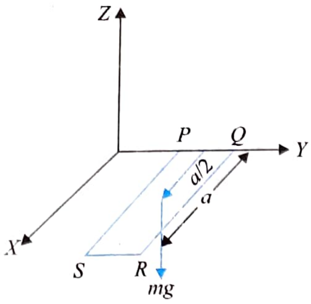
(b) Find the magnetic force on the arm RS.

(c) Find the expression for I in terms of B_0, a, b and m.

Sol.

(a) Torque due to weight of coil,

$$\vec{\tau} = \left(\frac{a}{2}\hat{i}\right)\times(-mg\hat{k}) = mg\frac{a}{2}(\hat{j})$$



For the equilibrium of loop, torque on it must be along negative y-axis. Let the magnetic moment of loop be $M\hat{k}$. As the loop lies in x–y plane, its magnetic moment vector (from right hand thumb rule) either points up or down.

Torque due to magnetic force,

$$\vec{\tau}_B = \vec{M} \times \vec{B} = M\hat{k} \times (3\hat{i} + 4\hat{k})B_0 = 3MB_0\hat{j}$$

If it is in negative direction, $\vec{M}$ must point downward. So, the current in the coil must be from P to Q.

(b) Force acting on arm

$$RS = I(\vec{l} \times \vec{B}) = I\,[(-b\hat{j}) \times (3\hat{i} + 4\hat{k})B_0]$$

$$= IB_0b\ (3\hat{k} - 4\hat{i})$$

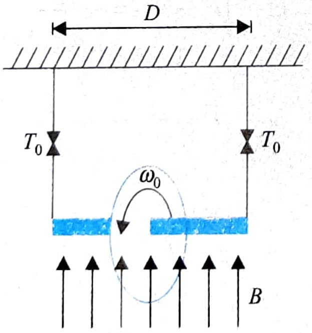
(c) In equilibrium $\vec{\tau}_{\text{gravity}} + \vec{\tau}_B = 0$

Hence, $3(abI)B_0 = \dfrac{mga}{2}$

or $I = \dfrac{mg}{6B_0b}$

EXAMPLE 1.7

A ring of radius R having uniformly distributed charge Q is mounted on a rod suspended by two identical strings. The tension in strings in equilibrium is T_0. Now a vertical magnetic field is switched on and ring is rotated at constant angular velocity ω. Find the maximum ω with which the ring can be rotated if the strings can withstand a maximum tension of $3T_0/2$.



Sol. In equilibrium,

$$2T_0 = mg$$

or $T_0 = \dfrac{mg}{2}$ ...(i)

magnetic moment, $M = iA = \left(\dfrac{\omega}{2\pi}Q\right)(\pi R^2)$

$$\tau = MB\sin 90° = \frac{\omega BQR^2}{2}$$

Let T_1 and T_2 be the tensions in the two strings when magnetic field is switched on ($T_1 > T_2$)

For translational equilibrium,

$$T_1 + T_2 = mg \qquad \text{...(ii)}$$

For rotational equilibrium

$$(T_1 - T_2)\frac{D}{2} = \tau = \frac{\omega BQR^2}{2}$$

or $T_1 - T_2 = \dfrac{\omega BQR^2}{D}$ ...(iii)

Solving Eqs. (ii) and (iii) we have

$$T_1 = \frac{mg}{2} + \frac{\omega BQR^2}{2D}$$

As $T_1 > T_2$ and maximum values of T_1 can be $3T_0/2$, we have

$$\frac{3T_0}{2} = T_0 + \frac{\omega_{\max}BQR^2}{2D} \qquad \left(\frac{mg}{2} = T_0\right)$$

$$\therefore\quad \omega_{\max} = \frac{DT_0}{BQR^2}$$

EXAMPLE 1.8

A moving coil galvanometer has a coil of area A and number of turns N. A magnetic field B is applied on it. The torque acting on it is given by $\tau = ki$ where i is current through the coil. If moment of inertia of the coil is I about the axis of rotation

(a) Find the value of k in terms of galvanometer parameters (N, B, A).

(b) Find the value of torsional constant if current i_0 produces angular deflection of $\pi/2$ rad.

(c) If a charge Q is passed almost instantaneously through coil, find the maximum angular deflection in it.

Sol.

(a) The torque acting on the coil of moving coil galvanometer is

$$\tau = NiAB$$

Given $\tau = ki$

$$ki = NiAB \quad \Rightarrow \quad k = NBA$$

(b) If C is torsional constant of the spring of galvanometer, then

$$\tau = C\theta$$

$$Ni_0AB = C\left(\frac{\pi}{2}\right) \quad \Rightarrow \quad C = \frac{2NBAi_0}{\pi}$$

(c) If q_m is the maximum deflection, then from conservation of energy

$$\frac{1}{2}C\theta_m^2 = \frac{1}{2}I\omega^2 \quad \Rightarrow \quad \theta_m = \sqrt{\frac{I}{C}}\,\omega \qquad \text{...(i)}$$

We have $\tau = NiAB$

Put $\tau = \dfrac{dL}{dt}$ where L is angular momentum.

$$\frac{dL}{dt} = N\left(\frac{dQ}{dt}\right)AB \quad \text{or} \quad dL = NAB\,dQ$$

Integrating $\displaystyle\int_0^L dL = NAB\int_0^Q dQ \quad \Rightarrow \quad L = NABQ$

If ω is angular velocity,

put $L = I\omega$

$$I\omega = NABQ$$

$$\omega = \frac{NABQ}{I} \qquad \text{...(ii)}$$

Substituting this value in Eq. (i), we get

$$\theta = \sqrt{\frac{I}{C}}\cdot\frac{NABQ}{I} = \frac{NABQ}{\sqrt{\left\{\frac{2NAB\,i_0 I}{\pi}\right\}}} = Q\sqrt{\frac{\pi NAB}{2Ii_0}}$$

EXAMPLE 1.9

A slightly divergent beam of charged particles accelerated by a potential difference V propagates from a point A along the axis of a solenoid. The beam is brought into focus at a distance l from the point A at two successive values of magnetic induction B_1 and B_2. Find the specific charge q/m of the particles.

Sol. Let us first calculate the velocity of the particles from the energy equation,

$$\frac{1}{2}mv^2 = Vq \Rightarrow v = \sqrt{\frac{2Vq}{m}}$$

Since the charged particles are slightly divergent, they will follow a helical path. Let θ be the small angle made by a particle with B cos $\theta = 1$

$\therefore$ p (pitch of the particle) $= v_{\|} \times T$

$$= v\cos\theta \times \frac{2\pi m}{qB} = \frac{2\pi vm}{qB}$$

Particles are focussed if l contains integral number of pitches.

$$l = np \Rightarrow p = l/n = l,\ l/2,\ l/3,\ ...$$

$\therefore$ For two consecutive focussings (as B increases, p decreases)

$$l = \frac{2\pi mv}{qB_1} \quad\text{and}\quad \frac{l}{2} = \frac{2\pi mv}{qB_2}$$

$$\text{or}\quad B_1 = \frac{2\pi mv}{ql} \quad\text{and}\quad B_2 = \frac{4\pi mv}{ql},$$

$$\text{or}\quad B_2 - B_1 = \frac{2\pi mv}{ql}$$

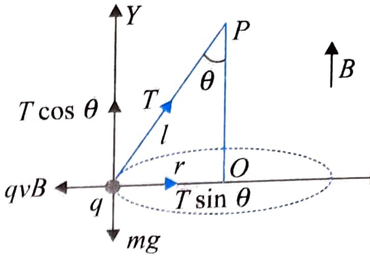
$$\text{or}\quad B_2 - B_1 = \frac{2\pi m}{ql}\sqrt{\frac{2Vq}{m}},$$

$$\text{or}\quad \frac{q}{m} = \frac{8\pi^2 V}{l^2(B_2 - B_1)^2}$$

EXAMPLE 1.10

A small charged ball having mass m and charge q is suspended from a rigid support by means of an inextensible thread of length l. It is made to rotate on a horizontal circular path in a uniform, time independent magnetic field of induction B which is directed upward. The time period of revolution of the ball is T_0. If the thread is always stretched, calculate the radius of circular path on which the ball moves.

Sol. The situation is shown in figure.



$$T\cos\theta = mg \qquad \text{...(i)}$$

$$\text{and}\quad T\sin\theta - qvB = \left(\frac{mv^2}{r}\right) \qquad \text{...(ii)}$$

From Eq. (ii),

$$\left(\frac{mg}{\cos\theta}\right)\sin\theta - qBr\omega = mr\omega^2$$

$$\left[\because\ v = r\omega \text{ and } T = \frac{mg}{\cos\theta} \text{ from Eq. (i)}\right.$$

From figure, $\sin\theta = \frac{r}{l}$ and $\cos\theta = \frac{\sqrt{(l^2 - r^2)}}{l}$

$$\therefore \quad \frac{mgr}{\sqrt{(l^2 - r^2)}} - qBr\omega = mr\omega^2$$

$$\text{or}\quad \frac{1}{\sqrt{(l^2 - r^2)}} = \frac{\omega^2}{g} + \frac{qB\omega}{mg}$$

$$\text{or}\quad (l^2 - r^2) = \frac{(1/\omega)^2}{[(\omega/g) + (qB/mg)]^2}$$

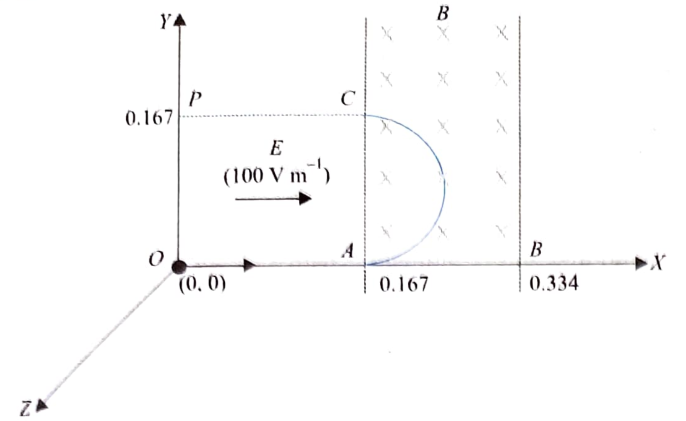
$$\text{or}\quad (l^2 - r^2) = \frac{(T_0/2\pi)^2}{[(2\pi/gT_0) + (qB/mg)]^2}$$

$$\therefore \quad r = \left[l^2 - \frac{(T_0/2\pi)^2}{\{(2\pi/gT_0) + (qB/mg)\}^2}\right]^{1/2}$$

EXAMPLE 1.11

There is a constant homogeneous electric field of 100 V m within the region $x = 0$ and $x = 0.167$ m pointing in x-directio There is a constant homogeneous magnetic field B within th region $x = 0.167$ m and $x = 0.334$ m pointing in the z-directio A proton at rest at the origin is released in positive x-directio Find the minimum strength of the magnetic field B, so that th proton is detected back at $x = 0$, $y = 0.167$ m. (Mass of proton $= 1.67 \times 10^{-27}$ kg)

Sol. First of all the proton is accelerated in the electric fiel Then it enters in magnetic field and describes a circular pat After that it leaves the magnetic field in negative direction. I moiton is retarded in electric field. Finally, it strikes y-axis at th same distance 0.167 m.



Let us first calculate the velocity of the proton when it enters in magnetic field after traversing in electric field.

Force acting on proton in electric field = eE

$\therefore$ Acceleration of proton $a = \left(\dfrac{eE}{m}\right)$

Using the formula $v^2 + u^2 = 2as$, we have

$$v^2 = 2\left(\frac{eE}{m}\right)s = 2\left(\frac{eE}{m}\right)(0.167) \qquad \text{...(i)}$$

Now consider the motion in magnetic field. The proton describes a circular path of radius ($AC/2$).

$$\text{Hence, } \frac{mv^2}{r} = evB \quad \text{or} \quad B = \frac{mv}{er} \qquad \text{...(ii)}$$

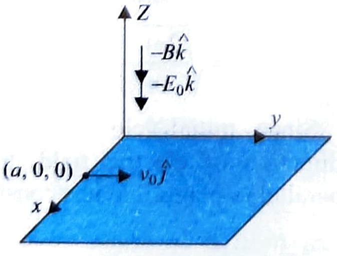
$$\therefore \quad B = \frac{m}{e(0.167/2)} \times \sqrt{\left[\frac{2eE}{m}(0.167)\right]}$$

$$= 2\sqrt{\left(\frac{2m}{e} \times \frac{E}{0.167}\right)} = 2\sqrt{\left[\frac{2(1.67\times10^{-27})(100)}{(1.6\times10^{-19})(0.167)}\right]}$$

$$= \frac{10^{-2}}{\sqrt{2}} = 7.07 \times 10^{-3}\ \text{T}$$

EXAMPLE 1.12

A positively charged particle of mass m and charge q is projected on a rough horizontal x–y plane surface with z-axis in the vertically upward direction. Both electric and magnetic fields are acting in the region and given by $\vec{E} = -E_0\hat{k}$ and $\vec{B} = -B_0\hat{k}$, respectively. The particle enters into the field at $(a_0, 0, 0)$ with velocity $\vec{v} = v_0\hat{j}$. The particle starts moving in some curved path on the plane. If the coefficient of friction between the particle and the plane is μ. Then calculate the



(a) time when the particle will come to rest

(b) distance travelled by the particle when it comes to rest.

Sol.

(a) Force of electric field will act in downward direction, so normal force between particle and surface will be as

$$N = mg + qE_0 \qquad \text{...(i)}$$

Let at any time velocity of the particle is v, then

$$-m\frac{dv}{dt} = \mu N \qquad \text{...(ii)}$$

From Eqs (i) and (ii),

$$-m.\frac{dv}{dt} = \mu(mg + qE_0)$$

$$-m\int_{v_0}^{0} dv = \mu(mg + qE_0)\int_0^t dt$$

$$\text{Thus, } t = \frac{mv_0}{\mu(mg + qE_0)}$$

(b) From Eq. (ii),

$$-m\frac{dv}{dl}\frac{dl}{dt} = \mu N$$

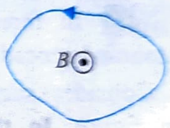
$$\Rightarrow \quad -mv\frac{dv}{d\ell} = \mu(mg + qE_0)$$

$$\Rightarrow \quad -m\int_{v_0}^{0} v\,dv = \mu(mg + qE_0)\int_0^l dl$$

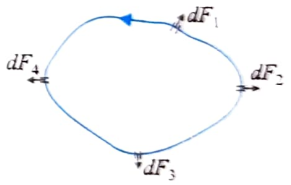
$$\Rightarrow \quad l = \frac{mv_0^2}{2\mu(mg + qE_0)}$$

EXAMPLE 1.13

A loop of flexible conducting wire of length 0.5 m lies in a magnetic field of 1.0 T perpendicular to the plane of the loop. Show that when a current as shown in figure is passed through the loop, it opens into a circle. Also calculate the tension developed in the wire if the current is 1.57 A.

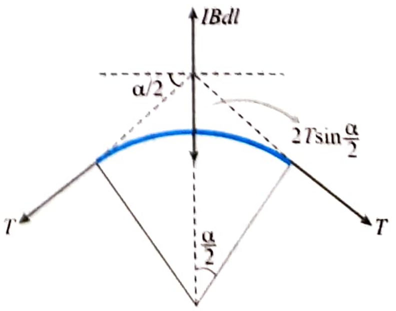


Sol. The loop may be divided into a large number of small length elements. When a current I is passed through the loop placed in the magnetic field such that the plane of the loop is perpendicular to field, then force on each element is



$$d\vec{F} = Id\vec{l} \times \vec{B} = IdlB\sin 90^\circ$$
$$= I\,d\,l\,B$$

Perpendicular to current element $Id\vec{l}$ as well as magnetic field $\vec{B}$. Hence, the loop opens into a circle.



Consider an element of length dl of circle of radius r, making an angle α at center. If T is the tension in the wire, then force toward center

$$2T\sin\frac{\alpha}{2} = IB\,dl$$

For small angle α, $\sin\dfrac{\alpha}{2} = \dfrac{\alpha}{2}$

$$2T\cdot\frac{\alpha}{2} = IB\,dl$$

$$T = \frac{IB\,dl}{\alpha} = IBr \qquad \left(\because\ \alpha = \frac{dl}{r}\right)$$

$$= IB\left(\frac{l}{2\pi}\right) \qquad (\because\ l = 2\pi r)$$

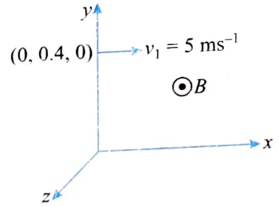
$$= 1.57\times 1\times\left(\frac{0.5}{2\times 3.14}\right) = 0.125\ \text{N}$$

EXAMPLE 1.14

A positively charged particle having charge $q_1 = 1$ C and mass $m_1 = 40$ g is revolving along a circle of radius $R = 40$ cm with velocity $v_1 = 5\ \text{m s}^{-1}$ in a uniform magnetic field with center of circle at origin O of a three-dimensional system.

At $t = 0$, the particle was at (0, 0.4 m, 0) and velocity was directed along positive x direction. Another particle having charge $q_2 = 1$ C and mass $m_2 = 10$ g moving uniformly parallel to positive z-direction with velocity $v_2 = 40/\pi\ \text{m s}^{-1}$ collides with revolving particle at $t = 0$ and gets stuck to it. Neglecting gravitational force and coulomb force, calculate x-, y- and z-coordinates of the combined particle at $t = \pi/40$ s.

Sol. In three dimensional co-ordinates system, axes are assumed according to right hand screw rule.



Consider such a system shown in figure. In this system positive z-direction is normal to plane of paper and is directed toward the reader.

The particle is positively charged, centre of its circular path is at origin and when the particle is on positive y-axis, its velocity is directed along positive x-direction, it means that the particle is moving clockwise in figure.

Since the particle is positively charged, therefore, current associated with its motion is also clockwise or along positive x-direction at (0, 0.4, 0).

Lorentz's force is toward origin. Therefore, according to Fleming's left hand rule, magnetic induction B must be along positive z-direction.

Radius of circular path followed by this particle is $r_1 = 0.4$ m

$$\text{But}\ \ r_1 = \frac{m_1 v_1}{q_1 B} \ \Rightarrow\ B = \frac{m_1 v_1}{q_1 r_1} = 0.5\ \text{T}$$

Now the second particle collides and gets struck with it. Velocity of combined particle can be found out by applying law of conservation of momentum.

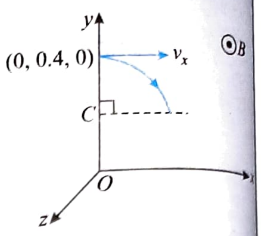
$$(m_1 + m_2)\vec{v} = m_1\vec{v}_1 + m_2\vec{v}_2$$

$$(40\times 10^{-3} + 10\times 10^{-3})\vec{v}$$

$$= (40\times 10^{-3})\times 5\hat{i} + (10\times 10^{-3})\frac{40}{\pi}\hat{k}$$

$\therefore$ x-direction of velocity of combined particle is $v_x = 4\ \text{ms}^{-1}$

and z-direction, $v_z = \dfrac{8}{\pi}\ \text{ms}^{-1}$



Due to v_x, combined particle tries to move clockwise along a circular path and due to v_z it ties to move uniformly along z-axis.

Therefore, its ultimate path becomes helix. Radius of this helix is

$$R = \frac{(m_1 + m_2)v_x}{(q_1 + q_2)B} = 0.2\ \text{m}$$

and period of revolution $T = \dfrac{2\pi(m_1 + m_2)}{(q_1 + q_2)B} = \dfrac{\pi}{10}$ s

The particle turns by angle $\pi/2$ in time $\pi/4$ s.

$\therefore$ x-coordinate of new position of combined particle = $R = 0.2$ m

→ y-coordinate $= r_1 - R = 0.2$ m

and z-coordinate $= v_z t = 0.2$ m

EXAMPLE 1.15

An electron accelerated by a potential difference $V = 3$ volt firs enters into a uniform electric field of a parallel plate capacito whose plates extend over a length $l = 6$ cm in the direction of initial velocity.

The electric field is normal to the direction of initial velocity and its strength varies with time as $E = \alpha t$, where $\alpha = 3600$ $\text{Vm}^{-1}\ \text{s}^{-1}$.

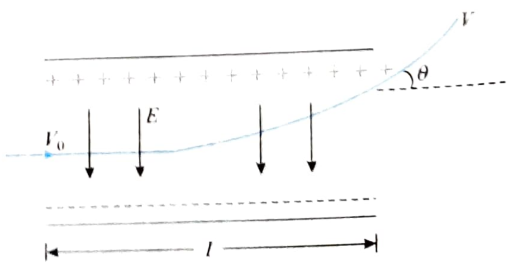
Then the electron enters into a uniform magnetic field of induction $B = \pi\times 10^{-9}$ T. Direction of magnetic field is same as that of the electric field.

Calculate pitch of helical path traced by the electron in the magnetic field. (Mass of electron, $m = 9\times 10^{-31}$ kg)

Sol. Since, electron is accelerated through a potential differen V, therefore, its initial velocity v_0 is given by $\dfrac{1}{2}mv_0^2 = eV$

$$v_0 = \sqrt{\frac{2eV}{m}} \quad \text{...(}$$

Since, initial velocity is parallel to plates or normal to th direction of electric field, therefore, component of velocit parallel to plates remains constant as v_0.



Hence, time taken by the electron to cross the electric field is

$$t_0 = \frac{l}{v_0}$$

Now consider motion of electron, normal to plates.

At some instant t, its acceleration $= \frac{eE}{m} = \frac{e\alpha t}{m}$

Let velocity component normal to plates to v_y.

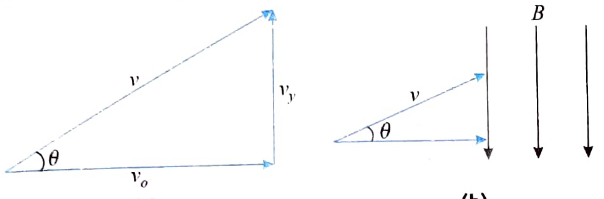
Then this acceleration is equal to $\frac{d}{dt} v_y$.

$$\frac{d}{dt} v_y = \frac{e\alpha t}{m} \quad \text{or} \quad dv_y = \frac{e\alpha t}{m} dt$$

But at initial moment $t = 0$, $v_y = 0 \Rightarrow \int_0^{v_y} dv_y = \frac{e\alpha}{m} \int_0^{t_0} t\, dt$

$$v_y = \frac{e\alpha}{2m} t_0^2 = \frac{\alpha l^2}{4V} \quad \text{...(ii)}$$

Angular deviation θ of electron from its initial direction of motion is shown in figure.



(a) (b)

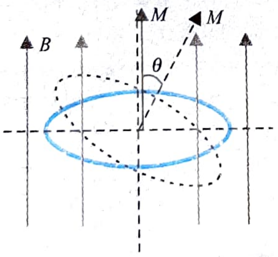
Now electron enters into magnetic field. Pitch of its helical path

$$P = \frac{2\pi m}{eB} v\cos(90° - \theta)$$

$$= \frac{2\pi m}{eB} v\sin\theta = \frac{2\pi m}{eB} v_y$$

$$= \frac{\pi m\alpha l^2}{2eBV} = 1.215 \text{ cm}$$

EXAMPLE 1.16



A small circular coil with a mass of m consists of N turns of fine wire and carries a current of I. The coil is located in a uniform magnetic field of B with the coil axis originally parallel to the field direction.

(a) What is the period of small angle oscillations of the coil axis around its equilibrium position, assuming no friction and no mechanical force?

(b) If the coil is rotated through an angle of θ from its equilibrium position and then released, what will be its angular speed when it passes through equilibrium position?

Sol.

(a) Magnetic moment of coil:

$$M = NIA = NI\pi R^2$$

Let the coil is displaced by small angle θ, the torque on the coil: $\tau = MB \sin\theta$

$\Rightarrow \quad -I\alpha = MB\theta \qquad$ (for small angle $\sin\theta \approx \theta$)

$$\Rightarrow \quad -\frac{1}{2}mR^2\alpha = NI\pi R^2 B\theta$$

$$\Rightarrow \quad \alpha = -\left(\frac{2NI\pi B}{m}\right)\theta$$

Comparing with $\alpha = -\omega^2\theta$ we get $\omega = \sqrt{\frac{2NI\pi B}{m}}$

Time period:

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{2NI\pi B}}$$

(b) Apply conservation of energy: $(KE + PE)_i = (KE + PE)_f$

$$\Rightarrow \quad 0 - MB\cos 10° = \frac{1}{2}I\omega^2 - MB\cos 0°$$

$$\Rightarrow \quad \frac{1}{2}\left(\frac{1}{2}mR^2\right)\omega^2 = MB(1 - \cos\theta)$$

$$\Rightarrow \quad \frac{1}{4}mR^2\omega^2 = NI\pi R^2 B(1 - \cos\theta)$$

$$\Rightarrow \quad \omega = \sqrt{\frac{4NI\pi B(1 - \cos\theta)}{m}}$$

EXAMPLE 1.17

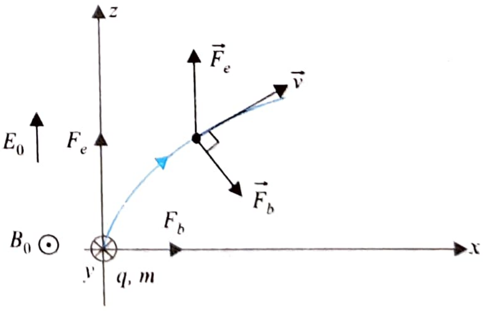
A particle of mass m and charge q is released from the origin in a region occupied by electric field E and magnetic field B, such that $\vec{B} = -B_0\hat{j}$; $\vec{E} = E_0\hat{k}$.

(a) Find the subsequent path of the particle.

(b) Find the speed of the particle as a function of the z-coordinate.

Sol.

(a) In figure shown B_0 is outward. As soon as we release the particle at origin. Electric field will apply force $F_e = qE_0$ on the particle. Due to this force, the charge particle will gain acceleration and hence velocity along z-axis. And as it gains velocity along z-axis, then magnetic field will apply the force F_b on charge particle. Now both the forces are always in x–z plane, so path of charge will be in x–z plane as shown. Let at any time its velocity is $\vec{v} = v_x\hat{i} + v_z\hat{k}$. Let $\vec{a} = a_x\hat{i} + a_z\hat{k}$ is the acceleration at any time.



Then $\vec{F}_{\text{net}} = m\vec{a}$

$$\Rightarrow \quad qE_0\hat{k} - q(v_x\hat{i} + v_z\hat{k}) \times B_0\hat{j} = m(a_x\hat{i} + a_z\hat{k})$$

$$\Rightarrow \quad qE_0\hat{k} - qv_xB_0\hat{k} + qv_zB_0\hat{i} = m(a_x\hat{i} + a_z\hat{k})$$

$$\Rightarrow \quad ma_x = qv_zB_0 \quad \text{and} \quad ma_z = q(E_0 - v_xB_0) \quad \text{...(i)}$$

$$\Rightarrow \quad m\frac{dv_x}{dt} = qv_zB_0 \quad \text{and} \quad m\frac{dv_z}{dt} = q(E_0 - v_xB_0) \quad \text{...(ii)}$$

From Eq. (ii),

$$\frac{dv_z}{dt}=\frac{q}{m}(E_0-v_xB_0) \Rightarrow \frac{d^2v_z}{dt^2}=\frac{-qB_0}{m}\frac{dv_x}{dt}$$

$$\Rightarrow \frac{d^2v_z}{dt^2}=\frac{-qB_0}{m}\frac{qB_0}{m}v_z=-\left(\frac{qB_0}{m}\right)^2 v_z$$

Solution of the above equation is

$$v_z=v_{z0}\sin(\omega t+\phi) \qquad \text{...(iii)}$$

$$\left(\text{where } \omega=\frac{qB_0}{m}\right)$$

Putting at $t=0$, $v_z=0$, we get $\phi=0$

Differentiating Eq. (iii),

$$\frac{dv_z}{dt}=v_{z0}\omega\cos(\omega t+\phi) \Rightarrow a_z=\omega v_{z0}\cos\omega t$$

$$\text{at} \quad t=0,\ a_z=\frac{F_e}{m}=\frac{qE_0}{m}$$

$$\Rightarrow \frac{qE_0}{m}=\frac{qB_0}{m}v_{z0}\cos 0° \Rightarrow v_{z0}=\frac{E_0}{B_0}$$

So Eq. (iii) can be written as: $v_z=\dfrac{E_0}{B_0}\sin(\omega t)$ ...(iv)

Putting the value of v_z in Eq. (ii), $m\dfrac{dv_x}{dt}=\dfrac{qE_0}{B_0}(\sin\omega t)B_0$

$$\Rightarrow \int_0^{v_x}dv_x=\int_0^t\frac{qE_0}{m}\sin\omega t\,dt \Rightarrow v_x=\frac{qE_0}{m\omega}[1-\cos\omega t]$$

$$\Rightarrow v_x=\frac{qE_0}{m(qB_0/m)}[1-\cos\omega t] \Rightarrow v_x=\frac{E_0}{B_0}[1-\cos\omega t]$$

...(v)

Equations (iv) and (v) give velocity as a function of time.

From Eq. (v), $\dfrac{dx}{dt}=\dfrac{E_0}{B_0}(1-\cos\omega t)$

$$\Rightarrow \int_0^x dx=\frac{E_0}{B_0}\int_0^t(1-\cos\omega t)\,dt$$

$$\Rightarrow x=\frac{E_0}{\omega B_0}[\omega t-\sin\omega t] \qquad \text{...(vi)}$$

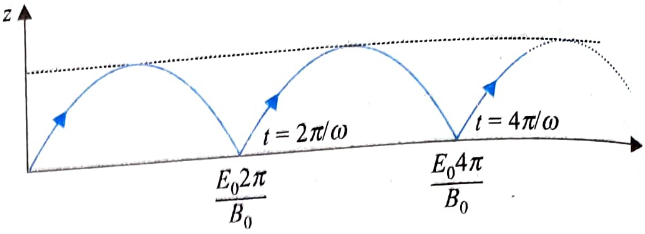
From Eq. (iv)

$$\frac{dz}{dt}=\frac{E_0}{B_0}\sin\omega t \Rightarrow \int_0^z dz=\frac{E_0}{B_0}\int_0^t\sin\omega t\,dt$$

$$\Rightarrow z=\frac{E_0}{\omega B_0}[1-\cos\omega t] \qquad \text{...(vii)}$$

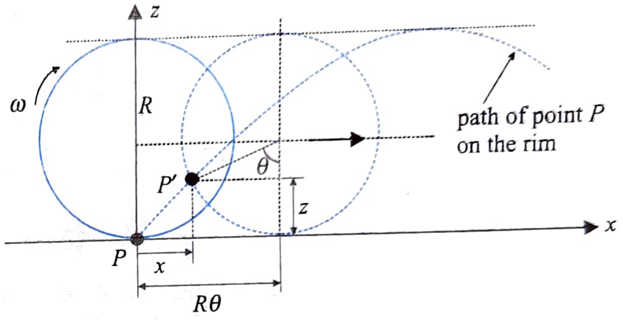
Equations (vi) and (vii) give the path of the verticle. We see that at $t=2\pi$

$$x=\frac{E_0}{B_0}2\pi, \quad z=0$$



This path is known as cycloidal path. This is the same kin of path as followed by a point on the rim of a purely rollin wheel, which is explained as below.

Let a wheel is rolling purely with angular velocity ω a shown. Let at $t=0$, a point P on the rim is at the bottom most position of the wheel. After time t, the wheel turns b an angle θ as shown and point P comes at point P'. Let u find the coordinates of P'.



$$x=R\theta-R\sin\theta=R\omega t-R\sin\omega t$$

$$=R[\omega t-\sin\omega t] \qquad \text{...(vii}$$

$$z=R-R\cos\theta=R[1-\cos\omega t] \qquad \text{...(i}$$

Equations (viii) and (ix) resemble Eqs. (vi) and (vii). Hence, the resulting path followed by charge is cycloida

(b) To find velocity as function of z-coordinate.

From Eqs. (iv) and (v)

$$v=\sqrt{v_x^2+v_z^2}=\frac{E_0}{B_0}\sqrt{(1-\cos\omega t)^2+(\sin\omega t)^2}$$

$$=\frac{E_0}{B_0}\sqrt{(1-\cos\omega t)^2+(1-\cos\omega t)(1+\cos\omega t)}$$

$$=\frac{E_0}{B_0}\sqrt{(1-\cos\omega t)2}$$

$$=\frac{E_0}{B_0}\sqrt{\frac{z\omega B_0}{E_0}2}$$

[From Eq. (vii

$$=\sqrt{\frac{E_0^2}{B_0^2}\frac{zqB_0}{m}\frac{B_0}{E_0}2}=\sqrt{\frac{2qE_0z}{m}}$$

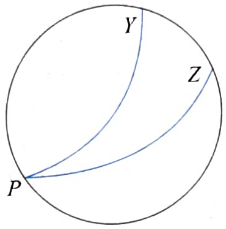
Alternate method to find velocity: Since the magne field does not perform any work, therefore, whatev has been the gain in kinetic energy it is only because the work done by electric field. Applying work–ener theorem, $W_E=\Delta K$

$$\Rightarrow qE_0z=\frac{1}{2}mv^2-0 \quad \text{or} \quad v=\sqrt{\frac{2qE_0z}{m}}$$

Exercises

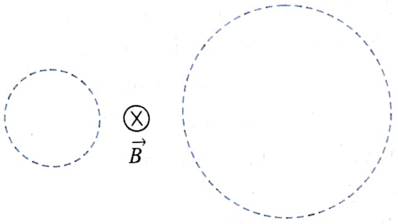
Single Correct Answer Type

1. Two particles Y and Z emitted by a radioactive source at P made tracks in a chamber as illustrated in the figure. A magnetic field acts downward into the paper. Careful measurements showed that both tracks were circular, the radius of Y track being half that of the Z track. Which one of the following statements is certainly true?



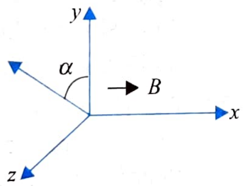
(1) Both particles Y and Z carried a positive charge
(2) The mass of particle Z was one-half that of particle Y
(3) The mass of particle Z was twice that of particle Y
(4) The charge of particle Z was twice that of particle Y

2. An electron and a proton each travel with equal speeds around circular orbits in the same uniform magnetic field as indicated (not to scale) in figure. The field is into the page on the diagram. The electron travels _______ around the _______ circle and the proton travels _______ around the _______ circle.



(1) clockwise, smaller, counterclockwise, larger
(2) counterclockwise, larger, counterclockwise, smaller
(3) clockwise, larger, counterclockwise, smaller
(4) counterclockwise, larger, clockwise, smaller

3. In a region of space, a uniform magnetic field B exists in the x-direction. An electron is fired from the origin with its initial velocity u making an angle α with the y direction in the y-z plane. In the subsequent motion of the electron,



(1) y-coordinate of the electron will never be negative
(2) z-coordinate of the electron will never be negative
(3) x-coordinate of the electron will never be negative
(4) trajectory of the electron would be helical

4. An electron is moving along positive x-axis. To get it moving on an anticlockwise circular path in x–y plane, a magnetic field is applied
(1) along positive y-axis (2) along positive z-axis
(3) along negative y-axis (4) along negative z-axis

5. An electron accelerated through a potential difference V passes through a uniform transverse magnetic field and experiences a force F. If the accelerating potential is increased to 2V, the electron in the same magnetic field will experience a force
(1) F (2) $F/2$
(3) $\sqrt{2}\,F$ (4) $2F$

6. A charged particle moves along a circle under the action of possible constant electric and magnetic fields. Which of the following are possible?
(1) $E = 0, B = 0$ (2) $E = 0, B \neq 0$
(3) $E \neq 0, B = 0$ (4) $E \neq 0, B \neq 0$

7. A charged particle is whirled in a horizontal circle on a frictionless table by attaching it to a string fixed at one end. If a magnetic field is switched on in the vertical direction, the tension in the string
(1) will increase (2) will decrease
(3) remains same (4) may increase or decrease

8. An electron is launched with velocity $\vec{v}$ in a uniform magnetic field $\vec{B}$. The angle θ between $\vec{v}$ and $\vec{B}$ lies between 0 and $\frac{\pi}{2}$. Its velocity vector $\vec{v}$ returns to its initial value in a time interval of
(1) $\frac{2\pi m}{eB}$ (2) $\frac{2 \times 2\pi m}{eB}$
(3) $\frac{\pi m}{eB}$
(4) depends upon angle between $\vec{v}$ and $\vec{B}$

9. A charged particle moves with velocity $\vec{v} = a\hat{i} + d\hat{j}$ in a magnetic field $\vec{B} = A\hat{i} + D\hat{j}$. The force acting on the particle has magnitude F. Then,
(1) $F = 0$, if $aD = dA$. (2) $F = 0$, if $aD = -dA$.
(3) $F = 0$, if $aA = -dD$. (4) $F \propto (a^2 + b^2)^{1/2} \times (A^2 + D^2)^{1/2}$

10. A particle with a specific charge s is fired with a speed v toward a wall at a distance d, perpendicular to the wall. What minimum magnetic field must exist in this region for the particle not to hit the wall?
(1) v/sd (2) $2v/sd$
(3) $v/2sd$ (4) $v/4sd$

11. A charged particle begins to move from the origin in a region which has a uniform magnetic field in the x-direction and a uniform electric field in the y-direction. Its speed is v when it reaches the point (x, y, z). Then, v will depend
(1) only on x
(2) only on y
(3) on both x and y, but not z
(4) on x, y and z

12. A proton and an α-particle enter a uniform magnetic field moving with the same speed. If the proton takes 25 μs to make 5 revolutions, then the periodic time for the α-particle would be
(1) 50 μs (2) 25 μs
(3) 10 μs (4) 5 μs

13. Two particles X and Y having equal charges, after being accelerated through the same potential difference, enter a region of uniform magnetic field and describe circular paths of radii R_1 and R_2, respectively. The ratio of masses of X and Y is

(1) $(R_1/R_2)^{1/2}$ (2) (R_2/R_1)

(3) $(R_1/R_2)^2$ (4) (R_1/R_2)

14. An electron of mass 0.90×10^{-30} kg under the action of a magnetic field moves in a circle of 2.0 cm radius at a speed of 3.0×10^6 ms^{-1}. If a proton of mass 1.8×10^{-27} kg was to move in a circle of the same radius in the same magnetic field, then its speed will be

(1) 3.0×10^6 m s^{-1}

(2) 1.5×10^3 m s^{-1}

(3) 6.0×10^4 m s^{-1}

(4) cannot be estimated from the given data

15. A charged particle of mass 10^{-3} kg and charge 10^{-5} C enters a magnetic field of induction 1 T. If $g = 10$ m s^{-2}, for what value of velocity will it pass straight through the field without deflection?

(1) 10^{-3} m s^{-1} (2) 10^3 m s^{-1}

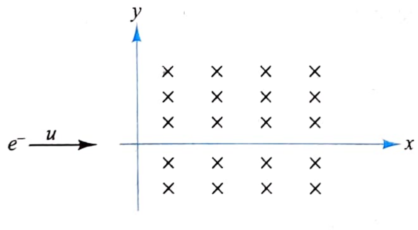
(3) 10^6 m s^{-1} (4) 1 m s^{-1}

16. A particle of mass 2×10^{-5} kg moves horizontally between two horizontal plates of a charged parallel plate capacitor between which there is an electric field of 200 NC^{-1} acting upward. A magnetic induction of 2.0 T is applied at right angles to the electric field in a direction normal to both $\vec{B}$ and $\vec{v}$. If g is 9.8 m s^{-2} and the charge on the particle is 10^{-6} C, then find the velocity of charge particle so that it continues to move horizontally

(1) 2 m s^{-1} (2) 20 m s^{-1}

(3) 0.2 m s^{-1} (4) 100 m s^{-1}

17. An electron moving with a speed u along the positive x-axis at $y = 0$ enters a region of uniform magnetic field which exists to the right of y-axis. The electron exits from the region after some time with the speed v at coordinate y, then



(1) $v > u, y < 0$ (2) $v = u, y > 0$

(3) $v > u, y > 0$ (4) $v = u, y < 0$

18. In a moving coil galvanometer, we use a radial magnetic feild so that the galvanometer scale is

(1) logarithmic (2) exponential

(3) linear (4) none of the above

19. The current that must flow through the coil of a galvanometer so as to produce a deflection of one division on its scale is called

(1) charge sensitivity of the galvanometer

(2) current sensitivity of the galvanometer

(3) micro-volt sensitivity

(4) none of the above

20. In a moving coil galvanometer, the deflection of the coil θ related to the electric current i by the relation

(1) $i \propto \tan \theta$ (2) $i \propto \theta$

(3) $i \propto \theta^2$ (4) $i \propto \sqrt{\theta}$

21. A current carrying loop lies on a smooth horizontal plan Then,

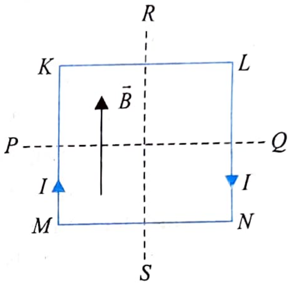
(1) it is possible to establish a uniform magnetic field in region so that the loop starts rotating about its own ax

(2) it is possible to establish a uniform magnetic field in region so that the loop will tip over about any of the poi

(3) it is not possible that loop will tip over about any of point whatever be the direction of established magne field (uniform).

(4) both (1) and (2) are correct.

22. A square loop of wire carrying current I is lying in the pla of paper as shown in figure. The magnetic field is present the region as shown. The loop will tend to rotate



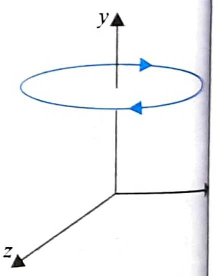
(1) about PQ with KL coming out of the page

(2) about PQ with KL going into the page

(3) about RS with MK coming out of the page

(4) about RS with MK going into the page

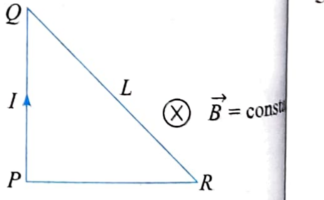
23. A circular coil having mass m is kept above ground (x–z plane) at some height. The coil carries a current i in the direction shown in figure. In which direction a uniform magnetic field $\vec{B}$ be applied so that the magnetic force balances the weight of the coil?



(1) Positive x-direction (2) Negative x-direction

(3) Positive z-direction (4) None of these

24. The rigid conducting thin wire frame carries an electric current I and this frame is inside a uniform magnetic field $\vec{B}$ as shown in figure. Then,



(1) the net magnetic force on the frame is zero but the torq is not zero.

(2) the net magnetic force on the frame and the torque due magnetic field are both zero.

(3) the net magnetic force on the frame is not zero and th torque is also not zero.

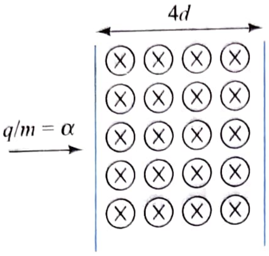
(4) none of these

25. Three particles, an electron (e), a proton (p) and a heliu atom (He) are moving in circular paths with constant spee

in the x–y plane in a region where a uniform magnetic field B exists along z-axis. The times taken by e, p and He inside the field to complete one revolution are t_e, t_p and t_{He} respectively. Then,

(1) $t_{He} > t_p = t_e$
(2) $t_{He} > t_p > t_e$
(3) $t_{He} = t_p = t_e$
(4) none of these

26. If a charged particle of charge to mass ratio $(q/m) = \alpha$ enters in a magnetic field of strength B at a speed $v = (2\alpha d)(B)$, then



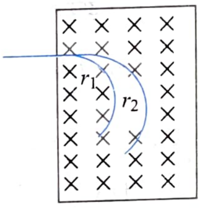
(1) angle subtended by the path of charged particle in magnetic field at the center of circular path is 2π

(2) the charge will move on a circular path and then will come out from magnetic field at some distance from the point of insertion

(3) the time for which particle will be in the magnetic field is $\dfrac{2\pi}{\alpha B}$

(4) angle subtended by the path of charged particle in magnetic field at the center of circular path is $\pi/2$

27. A beam of mixture of α particles and protons are accelerated through same potential difference before entering into the magnetic field of strength B. If $r_1 = 5$ cm, then r_2 is



(1) 5 cm
(2) $5\sqrt{2}$ cm
(3) $10\sqrt{2}$ cm
(4) 20 cm

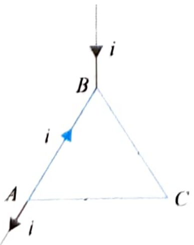
28. An electron is projected at an angle θ with a uniform magnetic field. If the pitch of the helical path is equal to its radius, then the angle of projection is

(1) $\tan^{-1} \pi$
(2) $\tan^{-1} 2\pi$
(3) $\cot^{-1} \pi$
(4) $\cot^{-1} 2\pi$

29. A charged particle moves in a uniform magnetic field perpendicular to it, with a radius of curvature 4 cm. On passing through a metallic sheet it loses half of its kinetic energy. Then, the radius of curvature of the particle is

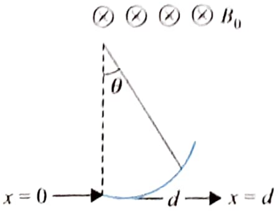
(1) 2 cm
(2) 4 cm
(3) 8 cm
(4) $2\sqrt{2}$ cm

30. Figure shows an equilateral triangle ABC of side l carrying currents as shown, and placed in a uniform magnetic field B perpendicular to the plane of triangle. The magnitude of magnetic force on the triangle is



(1) ilB
(2) $2ilB$
(3) $3\,ilB$
(4) zero

31. A charged particle moving along +ve x-direction with a velocity v enters a region where there is a uniform magnetic field $B_0(-\hat{k})$, from $x = 0$ to $x = d$. The particle gets deflected at an angle θ from its initial path. The specific charge of the particle is



(1) $\dfrac{v\cos\theta}{Bd}$
(2) $\dfrac{v\tan\theta}{Bd}$
(3) $\dfrac{v}{Bd}$
(4) $\dfrac{v\sin\theta}{Bd}$

32. A particle of specific charge $\dfrac{q}{m} = \pi$ C kg^{-1} is projected from the origin toward positive x-axis with a velocity of 10 m s^{-1} in a uniform magnetic field $\vec{B} = -2\hat{k}$ T. The velocity $\vec{v}$ of particle after time $t = 1/12$ s will be (in ms^{-1})

(1) $5[\hat{i} + \sqrt{3}\,\hat{j}]$
(2) $5[\sqrt{3}\,\hat{i} + \hat{j}]$
(3) $5[\sqrt{3}\,\hat{i} - \hat{j}]$
(4) $5[\hat{i} + \hat{j}]$

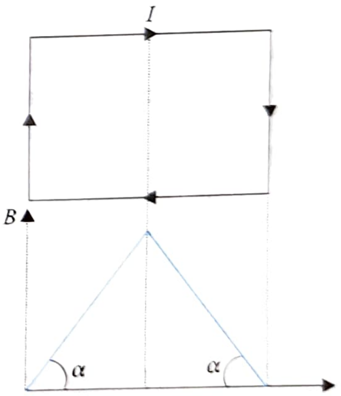
33. There exist uniform magnetic and electric fields of magnitudes 1 T and 1 V m^{-1}, respectively, along positive y-axis. A charged particle of mass 1 kg and charge 1 C is having velocity 1 m s^{-1} along x-axis and is at origin at $t = 0$. Then, the coordinates of the particle at time π s will be

(1) (0, 1, 2) m
(2) $(0, -\pi^2, -2)$ m
(3) $(2, \pi^2/2, 2)$ m
(4) $(0, \pi^2/2, 2)$ m

34. A uniform magnetic field exists in a region which forms an equilateral triangle of side a. The magnetic field is perpendicular to the plane of the triangle. A charge q enters into this magnetic field perpendicular to a side with speed v. The charge enters from midpoint and leaves the field from mid-point of other side. Magnetic induction in the triangle is

(1) $\dfrac{mv}{qa}$
(2) $\dfrac{2mv}{qa}$
(3) $\dfrac{mv}{2qa}$
(4) $\dfrac{mv}{4qa}$

35. A current carrying loop is placed in the non-uniform magnetic field whose variation in space is shown in figure. Direction of magnetic field is into the plane of paper. The magnetic force experienced by the loop is



(1) non-zero
(2) zero
(3) cannot say anything
(4) none of the above

36. A proton of mass 1.67×10^{-27} kg and charge 1.67×10^{-19} C is projected with a speed of 2×10^6 m s^{-1} at an angle of 60° to the X-axis. If a uniform magnetic field of 0.10 T is applied along Y-axis, the path of proton is

(1) a circle of radius 0.2 m and time period $\pi \times 10^{-7}$ s
(2) a circle of radius 0.1 m and time period $2\pi \times 10^{-7}$ s
(3) a helix of radius 0.1 m and time period $2\pi \times 10^{-7}$ s
(4) a helix of radius 0.2 m and time period $4\pi \times 10^{-7}$ s

37. An electron is accelerated from rest through a potential difference V. This electron experiences a force F in a uniform magnetic field. On increasing the potential difference to V', the force experienced by the electron in the same magnetic field becomes $2F$. Then, the ratio (V'/V) is equal to

(1) 1/4
(2) 2/1
(3) 1/2
(4) 1/4

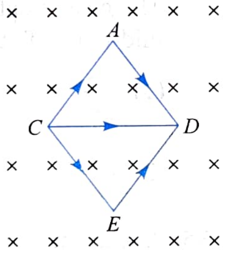
38. A particle of charge q and mass m starts moving from the origin under the action of an electric field $\vec{E} = E_0\hat{i}$ and $\vec{B} = B_0\hat{i}$ with a velocity $\vec{v} = v_0\hat{j}$. The speed of the particle will become $2v_0$ after a time

(1) $t = \dfrac{2mv_0}{qE}$
(2) $t = \dfrac{2Bq}{mv_0}$
(3) $t = \dfrac{\sqrt{3}Bq}{mv_0}$
(4) $t = \dfrac{\sqrt{3}mv_0}{qE}$

39. A conducting rod of mass m and length l is placed over a smooth horizontal surface. A uniform magnetic field B is acting perpendicular to the rod. Charge q is suddenly passed through the rod and it acquires an initial velocity v on the surface, then q is equal to

(1) $\dfrac{2mv}{Bl}$
(2) $\dfrac{Bl}{2mv}$
(3) $\dfrac{mv}{Bl}$
(4) $\dfrac{Blv}{2m}$

40. Let current $i = 2$ A be flowing in each part of a wire frame as shown in figure. The frame is a combination of two equilateral triangles ACD and CDE of side 1 m. It is placed in uniform magnetic field $B = 4$ T acting perpendicular to the plane of frame. The magnitude of magnetic force acting on the frame is



(1) 24 N
(2) zero
(3) 16 N
(4) 8 N

41. A charged particle enters a uniform magnetic field with velocity vector at an angle of 45° with the magnetic field. The pitch of the helical path followed by the particle is p. The radius of the helix will be

(1) $\dfrac{p}{\sqrt{2}\pi}$
(2) $\sqrt{2}p$
(3) $\dfrac{p}{2\pi}$
(4) $\dfrac{\sqrt{2}p}{\pi}$

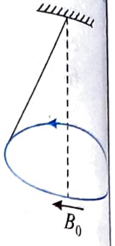
42. The plane of a rectangular loop of wire with sides 0.05 m and 0.08 m is parallel to a uniform magnetic field of induction 1.5×10^{-2} T. A current of 10.0 ampere flows through the loop. If the side of length 0.08 m is normal and the side of length 0.05 m is parallel to the lines of induction, then the torque acting on the loop is

(1) 6000 Nm
(2) zero
(3) 1.2×10^{-2} Nm
(4) 6×10^{-4} Nm

43. A loop of flexible conducting wire of length l lies in magnetic field B which is normal to the plane of loop. A current I is passed through the loop. The tension developed in the wire to open up is

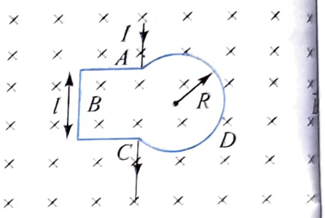
(1) $\dfrac{\pi}{2}BIl$
(2) $\dfrac{BIl}{2}$
(3) $\dfrac{BIl}{2\pi}$
(4) BIl

44. A uniform current carrying ring of mass m and radius R is connected by a massless string as shown in figure. A uniform magnetic field B_0 exists in the region to keep the ring in horizontal position, then the current in the ring is (l = length of string)



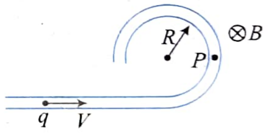
(1) $\dfrac{mg}{\pi RB_0}$
(2) $\dfrac{mg}{RB_0}$
(3) $\dfrac{mg}{3\pi RB_0}$
(4) $\dfrac{mg\,l}{\pi R^2 B_0}$

45. Figure shows a conducting loop $ABCDA$ placed in a uniform magnetic field (strength B) perpendicular to its plane. Th part ABC is the (three-fourth) portion of the squar of side length l. The part ADC is a circular arc of radius R. The points A and C are connected to a battery which supplies a current I to the circuit. The magnetic force on the loop due to the field B is



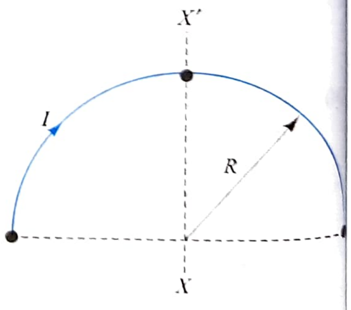
(1) zero
(2) BIl
(3) $2BIR$
(4) $\dfrac{BIlR}{I+R}$

46. A charged particle moves insides a pipe which is bent a shown in figure. If $R < \dfrac{mv}{qB}$, then force exerted by the pip on charged particle at P is (Neglect gravity)



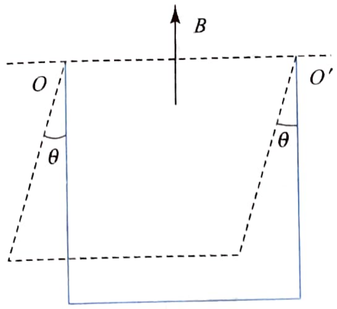
(1) toward center
(2) away from center
(3) zero
(4) none of these

47. A semicircular wire of radius R, carrying current I, is place in a magnetic field as shown in figure. On left side of $X'X$, magnetic field strength is B_0, and on right side of $X'X$, magnetic field strength is $2B_0$. Both fields are directed inside the page. The magnetic force experienced by the wire would be



(1) $3IB_0R$
(2) $2IB_0R$
(3) $\sqrt{10}\ IB_0R$
(4) $\sqrt{5}\ IB_0R$

48. A wire of cross-sectional area A forms three sides of a square and is free to rotate about axis OO'. If the structure is deflected by an angle θ from the vertical when current is passed through it in a magnetic field B acting vertically upward and density of the wire is ρ, then the value of θ is given by



(1) $\frac{2A\rho g}{iB} = \cot\theta$ (2) $\frac{2A\rho g}{iB} = \tan\theta$

(3) $\frac{A\rho g}{iB} = \sin\theta$ (4) $\frac{A\rho g}{2iB} = \cos\theta$

49. A uniform magnetic field of 1.5 T exists in a cylindrical region of radius 10.0 cm, its direction being parallel to the axis along east to west. A current carrying wire in north–south direction passes through this region. The wire intersects the axis and experiences a force of 1.2 N downward. If the wire is turned from north south to northeast–southwest direction, then magnitude and direction of force is

(1) 1.2 N, upward (2) $1.2\sqrt{2}$, downward

(3) 1.2 N, downward (4) $\frac{1.2}{\sqrt{2}}$ N, downward

50. A particle of positive charge q and mass m enters with velocity $V\hat{j}$ at the origin in a magnetic field $B(-\hat{k})$ which is present in the whole space. The charge makes a perfectly inelastic collision with an identical particle (having same charge) at rest but free to move at its maximum positive y-coordinate. After collision, the combined charge will move on trajectory

$\left(\text{where } r = \frac{mV}{qB}\right)$

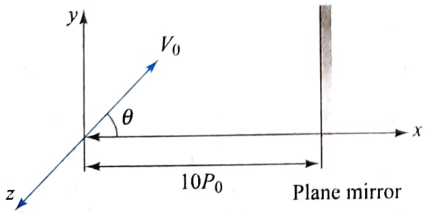
(1) $y = \frac{mv}{qB}x$

(2) $(x + r)^2 + (y - r/2)^2 = r^2/4$

(3) $(x + r)^2 + (y - r/2)^2 = r^2/8$

(4) $(x - r)^2 + (y + r/2)^2 = r^2/4$

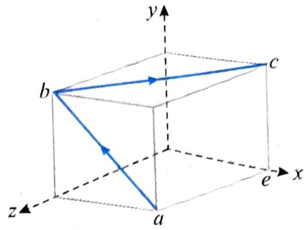
51. In the plane mirror, the coordinates of image of a charged particle (initially at origin as shown in figure) after two and a half time periods are (initial velocity of charge particle is V_0 in the x-y plane and the plane mirror is perpendicular to the x-axis. A uniform magnetic field $B\hat{i}$ exists in the space. P_0 is pitch of helix, R_0 is radius of helix.)



(1) $17P_0, 0, -2R_0$ (2) $3P_0, 0, -2R_0$

(3) $17.5P_0, 0, -2R_0$ (4) $3P_0, 0, 2R_0$

52. Two straight segments of wire ab and bc each carrying current I, are placed as shown in figure. The cube edge is 50 cm and magnetic field is uniform along Y-axis having magnitude 0.4 T. If $I = 3$ A, the force experienced by wire abc in the presence of magnetic field is



(1) $0.6\hat{i}$ (2) $1.2(\hat{i} + \hat{k})$

(3) $0.6\left(\sqrt{2}\hat{i} + \hat{j} - \sqrt{2}\hat{k}\right)$ (4) $0.6\left(\sqrt{2}\hat{i} - \hat{k}\right)$

53. A particle of specific charge α is projected from origin with velocity $\vec{v} = v_0\hat{i} - v_0\hat{k}$ in a uniform magnetic field $\vec{B} = -B_0\hat{k}$. Find time dependence of velocity of the particle.

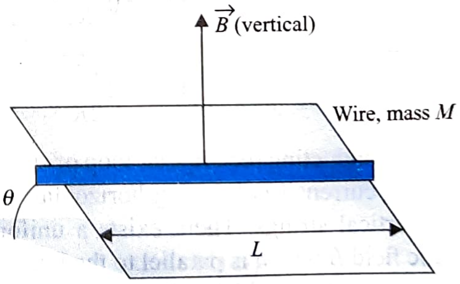
(1) $\vec{v}_{(t)} = v_0\cos(\alpha B_0 t)\hat{i} + v_0\sin(\alpha B_0 t)\hat{j} - v_0\hat{k}$

(2) $\vec{v}_{(t)} = -v_0\cos(\alpha B_0 t)\hat{i} + v_0\sin(\alpha B_0 t)\hat{j} + v_0\hat{k}$

(3) $\vec{v}_{(t)} = -v_0\cos(\alpha B_0 t)\hat{i} + v_0\sin(\alpha B_0 t)\hat{j} - v_0\hat{k}$

(4) $\vec{v}_{(t)} = v_0\cos(\alpha B_0 t)\hat{i} + v_0\sin(\alpha B_0 t)\hat{j} + v_0\hat{k}$

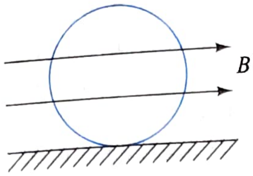
54. A straight piece of conducting wire with mass M and length L is placed on a frictionless incline tilted at an angle θ from the horizontal (as shown in figure). There is a uniform, vertical magnetic field at all points (produced by an arrangement of magnets not shown in figure). To keep the wire from sliding down the incline, a voltage source is attached to the ends of the wire. When just the right amount of current flows through the wire, the wire remains at rest. Determine the magnitude and direction of the current in the wire that will cause the wire to remain at rest.



(1) $\frac{Mg\tan\theta}{2LB}$ to the left (2) $\frac{Mg\tan\theta}{LB}$ to the right

(3) $\frac{Mg\tan\theta}{LB}$ to the left (4) $\frac{3Mg\tan\theta}{2LB}$ to the left

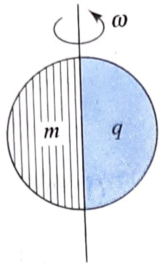
55. A conducting ring of mass 2 kg and radius 0.5 m is placed on a smooth horizontal plane. The ring carries a current of $i = 4$ A. A horizontal magnetic field $B = 10$ T is switched on at time $t = 0$ as shown in figure. The initial angular acceleration of the ring will be



(1) 40π rad s^{-2} (2) 20π rad s^{-2}

(3) 5π rad s^{-2} (4) 15π rad s^{-2}

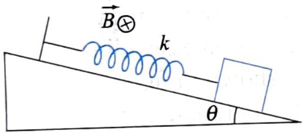
56. Consider a hypothetic spherical body. The body is cut into two parts about the diameter. One of hemispherical portion has mass distribution m while the other portion has identical charge distribution q. The body is rotated about the axis with constant speed ω. Then, the ratio of magnetic moment to angular momentum is



(1) $\frac{q}{2m}$ (2) $> \frac{q}{2m}$

(3) $< \frac{q}{2m}$ (4) cannot be calculated

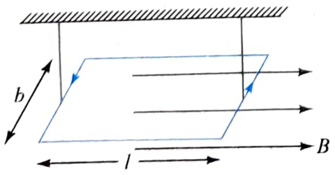
57. A small block of mass m, having charge q, is placed on a frictionless inclined plane making an angle θ with the horizontal. There exists a uniform magnetic field B parallel to the inclined plane but perpendicular to the length of spring. If m is slightly pulled on the incline in downward direction, the time period of oscillation will be (assume that the block does not leave contact with the plane)



(1) $2\pi\sqrt{\frac{m}{K}}$ (2) $2\pi\sqrt{\frac{2m}{K}}$

(3) $2\pi\sqrt{\frac{qB}{K}}$ (4) $2\pi\sqrt{\frac{qB}{2K}}$

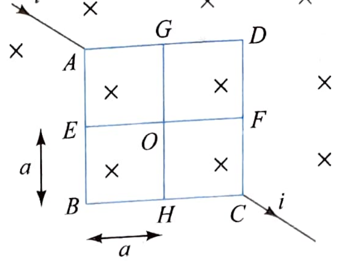
58. A uniform conducting rectangular loop of sides l, b and mass m carrying current i is hanging horizontally with the help of two vertical strings. There exists a uniform horizontal magnetic field B which is parallel to the longer side of loop. The value of tension which is least is



(1) $\frac{mg - Bb}{2}$ (2) $\frac{mg + Bb}{2}$

(3) $\frac{mg - 2iBb}{2}$ (4) $\frac{mg + 2Bb}{2}$

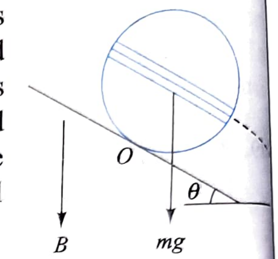
59. In figure, there is a uniform conducting structure in which each small square has side a. The structure is kept in a uniform magnetic field B. Then the magnetic force on the structure will be



(1) $2\sqrt{2}\,iBa$ (2) $\sqrt{2}\,iBa$

(3) $2\,iBa$ (4) iBa

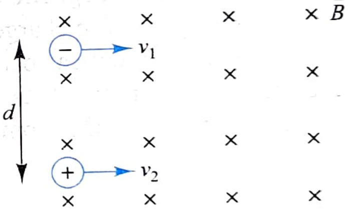
60. In figure, a coil of single turn is wound on a sphere of radius r and mass m. The plane of the coil is parallel to the inclined plane and lies in the equatorial plane of the sphere. If the sphere is in rotational equilibrium, the value of B is [Current in the coil is i]



(1) $\frac{mg}{\pi i r}$ (2) $\frac{mg\sin\theta}{\pi i}$

(3) $\frac{mg\sin\theta}{\pi i}$ (4) none of these

61. Two identical particles having the same mass m and charge $+q$ and $-q$ separated by a distance d enter a uniform magneti field B directed perpendicular to paper inwards with speed v_1 and v_2 as shown in figure. The particles will not collide i



(1) $d > \frac{m}{Bq}(v_1 + v_2)$ (2) $d < \frac{m}{Bq}(v_1 + v_2)$

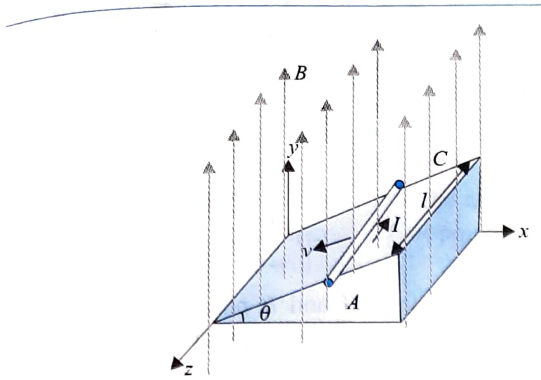
(3) $d > \frac{2m}{Bq}(v_1 + v_2)$ (4) $v_1 = v_2$

62. A charged particle of specific charge (charge/mass) is released from origin at time $t = 0$ with velocity $\vec{v} =$ $(\hat{i} + \hat{j})$ in uniform magnetic field $\vec{B} = B_0\hat{i}$. Coordinates the particle at time $t = \pi/(B_0\alpha)$ are

(1) $\left(\frac{v_0}{2B_0\alpha}, \frac{\sqrt{2}v_0}{\alpha B_0}, \frac{-v_0}{B_0\alpha}\right)$ (2) $\left(\frac{-v_0}{2B_0\alpha}, 0, 0\right)$

(3) $\left(0, \frac{2v_0}{B_0\alpha}, \frac{v_0\pi}{2B_0\alpha}\right)$ (4) $\left(\frac{v_0\pi}{B_0\alpha}, 0, \frac{-2v_0}{B_0\alpha}\right)$

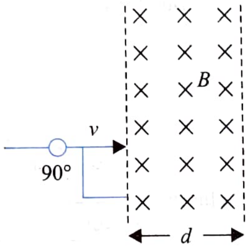
63. A conducting rod of length l and mass m is moving down a smooth inclined plane of inclination θ with constant velocity v. A current i is flowing in the conductor in a direction perpendicular to paper inwards. A vertically upward magnetic field $\vec{B}$ exists in space. Then, magnitude of magnetic field $\vec{B}$ is



(1) $\dfrac{mg}{il}\sin\theta$　　(2) $\dfrac{mg}{il}\tan\theta$

(3) $\dfrac{mg\cos\theta}{il}$　　(4) $\dfrac{mg}{il\sin\theta}$

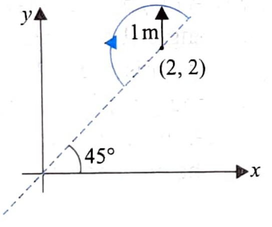
64. A positive charge particle of mass m and charge q is projected with velocity v as shown in figure. If radius of curvature of charge particle in magnetic field region is greater than d, then find the time spent by the charge particle in magnetic field.



(1) $\dfrac{2m}{qB}\sin^{-1}\left(\dfrac{dqB}{mv}\right)$　　(2) $\dfrac{2m}{qB}\sin^{-1}\left(\dfrac{mv}{dqB}\right)$

(3) $\dfrac{m}{qB}\sin^{-1}\left(\dfrac{dqB}{mv}\right)$　　(4) $\dfrac{m}{qB}\sin^{-1}\left(\dfrac{mv}{dqB}\right)$

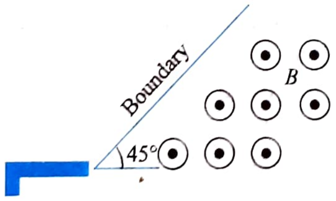
65. A uniform magnetic field $\vec{B}=3\hat{i}+4\hat{j}+\hat{k}$ exists in region of space. A semicircular wire of radius 1 m carrying current 1 A having its centre at (2, 2, 0) is placed in x-y plane as shown in figure. The force on semicircular wire will be



(1) $\sqrt{2}(\hat{i}+\hat{j}+\hat{k})$　　(2) $\sqrt{2}(\hat{i}-\hat{j}+\hat{k})$

(3) $\sqrt{2}(\hat{i}+\hat{j}-\hat{k})$　　(4) $\sqrt{2}(-\hat{i}+\hat{j}+\hat{k})$

66. An electron gun ejects electrons at an angle of 45° with magnetic field boundary as shown in figure. Find the angular deviation of electrons as it comes out of field.



(1) 45°　　(2) 90°

(3) 60°　　(4) None of these

67. In a certain region of space, there exists a uniform and constant electric field of strength E along x-axis and uniform constant magnetic field of induction B along z-axis. A charged particle having charge q and mass m is projected with speed v parallel to x-axis from a point $(a, b, 0)$. When the particle reaches a point $2a$, $b/2$, 0 its speed becomes $2v$. Find the value of electric field strength in terms of m, v and co-ordinates.

(1) $\dfrac{3}{2}\dfrac{mv^2}{qa}$　　(2) $\dfrac{mv^2}{qB}$

(3) $\dfrac{2mv^2}{qBa}$　　(4) $\dfrac{3}{2}vB$

68. A particle of specific charge $q/m=\pi$ C/kg is projected from the origin towards positive x-axis with a velocity of 10 m/s in a uniform magnetic field $\vec{B}=-2\hat{k}$ Tesla. The velocity $\vec{V}$ of the particle after time $t = 1/6$ s will be

(1) $(5\hat{i}+5\sqrt{3}\hat{j})$ m/s　　(2) $10\hat{j}$ m/s

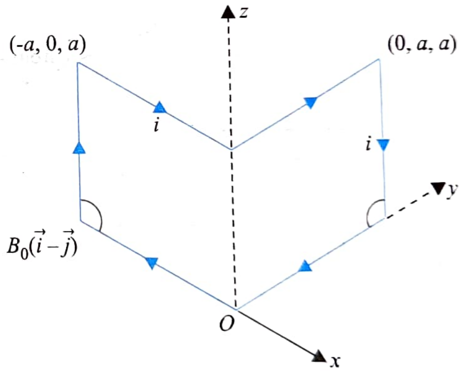
(3) $(5\sqrt{3}\hat{i}-5\hat{j})$ m/s　　(4) $-10\hat{j}$ m/s

69. An α-particle and a proton are both simultaneously projected in opposite directions into a region of constant magnetic field perpendicular to the direction of the field. After some time it is found that the velocity of the α-particle has changed in a direction by 45°. Then at this time, the angle between velocity vectors of α-particle and proton is

(1) 90°　　(2) 45°

(3) 135°　　(4) none

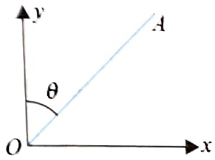
70. The torque experienced by a given current carrying loop in a uniform magnetic field $\vec{B}$ given by $B_0(\vec{i}-\vec{j})$ would have magnitude



(1) $\sqrt{2}B_0a^2I$　　(2) zero

(3) $2B_0a^2I$　　(4) B_0a^2I

71. A uniform magnetic field B and electric field E exist along y and negative z axis respectively. Under the influence of these fields a charge particle moves along OA undeflected. If electric field is swithed off, find the pitch of helical trajectory in which the particle will move.

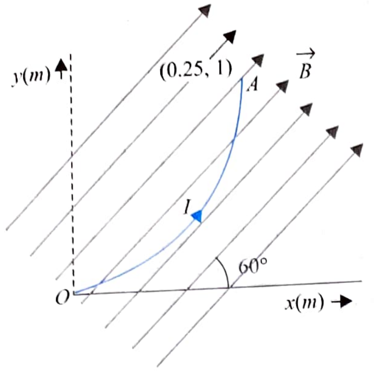


(1) $\dfrac{2\pi mE}{qB^2\cot\theta}$　　(2) $\dfrac{4\pi mE}{qB^2\tan\theta}$

(3) $\dfrac{4\pi mE}{qB^2\cot\theta}$　　(4) $\dfrac{2\pi mE}{qB^2\tan\theta}$

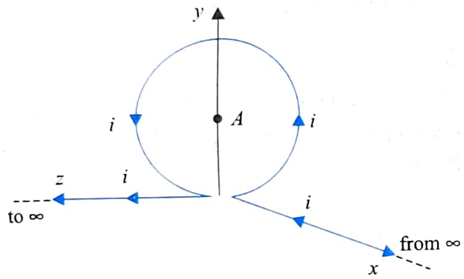
72. A parabolic section of wire OA is located in the x-y plane and carries current I = 12 A. A uniform magnetic field $B = 4.0$ T making an angle 60° with x axis exists in x-y plane.

Calculate the magnetic force on the wire OA. Co-ordinates of A are (0.25 m, 1 m).



(1) $6(\sqrt{3}-4)\hat{k}$ N
(2) $6(4-\sqrt{3})\hat{k}$ N
(3) $3(\sqrt{3}-4)\hat{k}$ N
(4) $3(4-\sqrt{3})\hat{k}$ N

73. Find the magnitude of the magnetic induction B of a magnetic field generated by a system of thin conductors along which a current i is flowing at a point A (O, R, O), that is the centre of a circular conductor of radius R. The ring is in yz plane.



(1) $B=\dfrac{\mu_0 i}{4\pi R}\sqrt{\left(2\pi^2-2\pi+1\right)}$

(2) $B=\dfrac{\mu_0 i}{4\pi R}\sqrt{2\left(2\pi^2-2\pi+1\right)}$

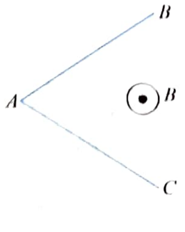
(3) $B=\dfrac{\mu_0 i}{2\pi R}\sqrt{\left(2\pi^2-2\pi+1\right)}$

(4) None of these

74. A charged particle (charge q, mass m) has velocity v_0 at origin in $+x$ direction. In space there is a uniform magnetic field B in $-z$ direction. Find the y coordinate of particle when is crosses y axis.

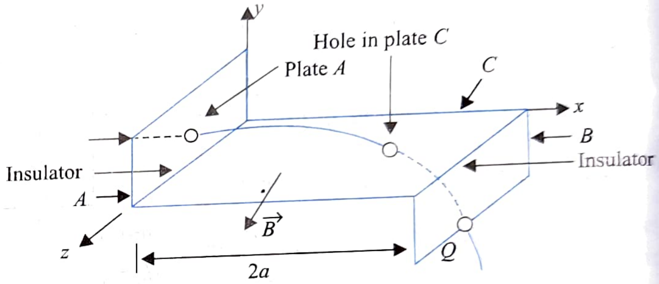
(1) $\dfrac{mv_0}{qB}$
(2) $\dfrac{2mv_0}{qB}$
(3) $\dfrac{mv_0}{2qB}$
(4) None of these

75. AB and AC are the boundary lines within which a magnetic field B exists. If the magnetic field is absent, a charged particle of mass m and charge q must have passed through a point P on angle bisector of $-BAC$, at a distance $r\sqrt{2}$ from A, if it has fallen on AB normally at a point Q such that $QP = r$. If the magnetic field is presen how much time the charged particle will take to come out o the magnetic field



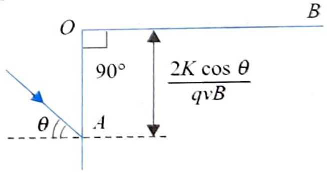
(1) $\dfrac{m\pi}{Bq}$
(2) $\dfrac{2m\pi}{Bq}$
(3) $\dfrac{4m\pi}{Bq}$
(4) $\dfrac{m\pi}{2Bq}$

76. Two rectangular plates A and B placed at a distance 2 apart, are connected to a battery to produce an electric fiel There are insulators between plates C and other two plate A magnetic field exists along z-axis. A charged particle o mass m and charge q passes through a hole at the middl of the plate A with velocity v and strikes at Q which is th middle of the bottom edge of plate B after passing through hole inplate C. If $E = mv^2/qa$, what will be the speed of th particle at Q?



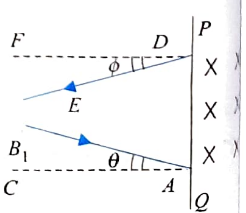
(1) $v\sqrt{2}$
(2) $2v$
(3) $v\sqrt{5}$
(4) $v\sqrt{3}$

77. A magnetic field B exists between OA and OB. Inclined at a angle θ, a charged particle strikes at point A on surface O at a distance $(2K\cos\theta)/(qvB)$ from O, where K is kinet energy, q is the charge and v is the velocity of the partic At what angle with horizontal (measured from end B) wi the charged particle emerge from OB?



(1) θ
(2) $90° - \theta$
(3) $90° + \theta$
(4) 90°

78. To the right of line PQ is a uniform magnetic field $\vec{B}$. B_1A i the line of incidence of a charged particle, which comes ou of the field along DE; CA and DF are the normals at A an D. $B_1AC = \theta$. Angle measured from CA in clockwis direction is taken as positive. What will be the value of θ s that angle subtended by the part of the circle (along which charged particle moves in the field) at its centre and facing the circle is less than π?



(1) θ is positive
(2) $\theta = 0$
(3) θ is negative
(4) θ depends upon $\vec{B}$ and charge

79. In the previous problem, if O is the point on AD and OO_1 is the perpendicular from the centre of the circle, O_{-1}, then OA/OD will be

(1) 1 (2) > 1

(3) < 1 (4) depend upon $\vec{B}$ andcharge

80. In the previous problem, if ϕ is the angle between line of emergence DE and normal DF at point D, ratio of ϕ/θ for positive value of θ will be

(1) 1 (2) >1

(3) < 1 (4) depends upon $\vec{B}$ and charge

81. In the previous problem, if time period, $T = 2\pi \dfrac{m}{BQ}$ where Q is the charge of the particle and m is its mass, the ratio of time spent by the particle in the field when θ is positive to when θ is negative is given by

(1) $\left(\dfrac{\pi/2+\theta}{\pi/2-\theta}\right)$ (2) $\left(\dfrac{\pi+\theta}{\pi-\theta}\right)$

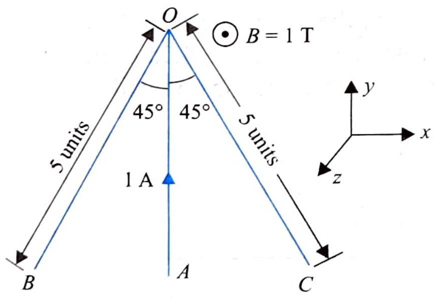
(3) $\left(\dfrac{\pi-\theta}{\pi+\theta}\right)$ (4) $\left(\dfrac{\pi/2-\theta}{\pi/2+\theta}\right)$

82. In the previous problem, the maximum range of movement of the centre of the part of the circle from line AD in which charged particle of charge Q moves with a velocity v when θ is positive to when θ is negative is given by

(1) $\pm\dfrac{mv}{2QB}$ (2) $\pm\dfrac{mv}{QB}$

(3) $\pm\dfrac{2mv}{QB}$ (4) $\pm\dfrac{2mv}{3QB}$

83. A triangular system of mass 100 g consisting of 3 wires of lengths, as shown in figure and length of AO as 4 units, are placed in the magnetic field of 1T. The current of 1A flows through wire AO. The wires are of same material and cross-sectional area. In which direction will the system move?



(1) at $\tan^{-1}(3/4)$ with $(-x)$ axis.

(2) along x-axis

(3) along y-axis

(4) along $(-x)$ axis

84. In the previous problem, what will be force with which system moves? (Use $\sqrt{2} = 1.4$)

(1) 3.05 N (2) 4.0 N

(3) 0.5 N (4) 1.0 N

85. In the previous problem, instead of being stationary, the system enters the magnetic field with a velocity 0.5 m/sec along y-direction. Along which direction the system will move now?

(1) along $a\hat{i} + b\hat{j}$

(2) along a circular path

(3) system will become stationary

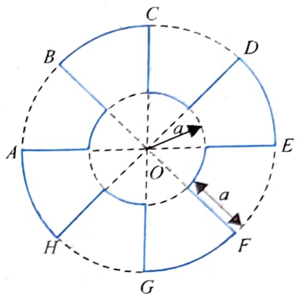
(4) along a direction in x-y plane

86. In the previous problem, what will be acceleration of the system (in m/sec^2)?

(1) $5\sqrt{2}$ (2) $0.5\sqrt{2}$

(3) zero (4) 2.5

87. Current I flows through the circuit, as shown in figure. Find the magnetic moment of the figure, if $AB = BC = CD = DE = EF = FG = GH = HA$.



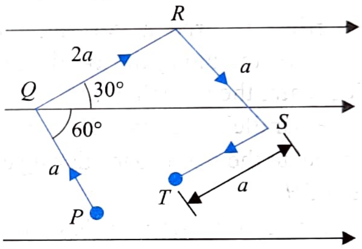
(1) $\dfrac{7}{2} I \pi a^2$

(2) $\dfrac{5}{2} I \pi a^2$

(3) $4 I \pi a^2$

(4) $\dfrac{5}{3} I \pi a^2$

88. Find the force acting on rectangular loop $PQRST$.



(1) $\dfrac{I a B}{2}$ (2) $\dfrac{I a B}{\sqrt{2}}$

(3) $\dfrac{I a B\sqrt{3}}{2}$ (4) $I a B\sqrt{3}$

Multiple Correct Answers Type

1. A charged particle goes undeflected in a region containing electric and magnetic fields. It is possible that

(1) $\vec{E} \parallel \vec{B}, \vec{v} \parallel \vec{E}$

(2) $\vec{E}$ is not parallel to $\vec{B}$

(3) $\vec{v} \parallel \vec{B}$ but $\vec{E}$ is not parallel to $\vec{B}$

(4) $\vec{E} \parallel \vec{B}$ but $\vec{v}$ is not parallel to $\vec{E}$

2. If a charged particle goes unaccelerated in a region containing electric and magnetic fields, then

(1) $\vec{E}$ must be perpendicular to $\vec{B}$

(2) $\vec{v}$ must be perpendicular to $\vec{E}$

(3) $\vec{v}$ must be perpendicular to $\vec{B}$

(4) E must be equal to vB

3. The force $\vec{F}$ experienced by a particle of charge q moving with a velocity $\vec{v}$ in a magnetic field $\vec{B}$ is given by

$\vec{F} = q(\vec{v} \times \vec{B})$. Which pairs of vectors are at right angles to each other?

(1) $\vec{F}$ and $\vec{v}$ (2) $\vec{F}$ and $\vec{B}$

(3) $\vec{B}$ and $\vec{v}$ (4) $\vec{F}$ and $(\vec{v} \times \vec{B})$

4. Which of the following statements is correct?

(1) If a moving charged particle enters into a region of magnetic field from outside, it does not complete a circular path.

(2) If a moving charged particle traces a helical path in a uniform magnetic field, the axis of the helix is parallel to the magnetic field.

(3) The power associated with the force exerted by a magnetic field on a moving charged particle is always equal to zero.

(4) If in a region a uniform magnetic field and a uniform electric field both exist, a charged particle moving in this region cannot trace a circular path.

5. An electron is moving along the positive x-axis. You want to apply a magnetic field for a short time so that the electron may reverse its direction and move parallel to the negative x-axis. This can be done by applying the magnetic field along

(1) y-axis (2) z-axis

(3) y-axis only (4) z-axis only

6. A charged particle is fired at an angle θ to a uniform magnetic field directed along the x-axis. During its motion along a helical path, the particle will

(1) never move parallel to the x-axis

(2) move parallel to the x-axis once during every rotation for all values of θ

(3) move parallel to the x-axis at least once during every rotation if $\theta = 45°$

(4) never move perpendicular to the x-direction

7. In previous problem, if the pitch of the helical path is equal to the maximum distance of the particle from the x-axis, then which of the following are not correct?

(1) $\cos\theta = \frac{1}{\pi}$ (2) $\sin\theta = \frac{1}{\pi}$

(3) $\tan\theta = \frac{1}{\pi}$ (4) $\tan\theta = \pi$

8. A proton is fired from origin with velocity $\vec{v} = v_0\hat{j} + v_0\hat{k}$ in a uniform magnetic field $\vec{B} = B_0\hat{j}$. In the subsequent motion of the proton

(1) its z-coordinate can never be negative

(2) its x-coordinate can never be positive

(3) its x- and z-coordinates cannot be zero at the same time

(4) its y-coordinate will be proportional to its time of flight

9. Velocity and acceleration vector of a charged particle moving in a magnetic field at some instant are $\vec{v} = 3\hat{i} + 4\hat{j}$ and $\vec{a} = 2\hat{i} + x\hat{j}$. Select the correct options:

(1) $x = -1.5$

(2) $x = 3$

(3) Magnetic field is along z-direction

(4) Kinetic energy of the particle is constant

10. Let $\vec{E}$ and $\vec{B}$ denote the electric and magnetic fields in a certain region of space. A proton moving with a velocity along a straight line enters the region and is found to pass through it undeflected. Indicate which of the following statements are consistent with the obserations:

(1) $\vec{E} = 0$ and $\vec{B} = 0$

(2) $\vec{E} \neq 0$ and $\vec{B} = 0$

(3) $\vec{E} \neq 0$ and $\vec{B} \neq 0$ and both $\vec{E}$ and $\vec{B}$ are parallel to $\vec{v}$

(4) $\vec{E}$ is parallel to $\vec{v}$ but $\vec{B}$ is perpendicular to $\vec{v}$

11. When a current carrying coil is placed in a uniform magnetic field with its magnetic moment anti-parallel to the field.

(1) torque on it is maximum

(2) torque on it is zero

(3) potential energy is maximum

(4) dipole is in unstable equilibrium

12. A charged particle P leaves the origin with speed $v = v_0$ at some inclination with the x-axis. There is a uniform magnetic field B along the x-axis. P strikes a fixed target T on the x-axis for a minimum value of $B = B_0$. P will also strike T if

(1) $B = 2B_0$, $v = 2v_0$ (2) $B = 2B_0$, $v = v_0$

(3) $B = B_0$, $v = 2v_0$ (4) $B = \frac{B_0}{2}$, $v = 2v_0$

13. A particle having a mass of 0.5 g carries a charge of 2.5×10^{-8} C. The particle is given an initial horizontal velocity of 6×10^4 ms^{-1}. To keep the particle moving in a horizontal direction

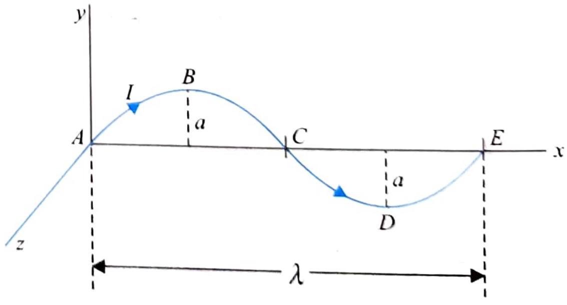
(1) the magnetic field may be perpendicular to the direction of the velocity

(2) the magnetic field should be along the direction of the velocity

(3) magnetic field should have a minimum value of 3.27 T

(4) no magnetic field is required

14. A conductor $ABCDE$, shaped as shown, carries current I. It is placed in the x–y plane with the ends A and E on the x-axis. A uniform magnetic field of magnitude B exists in the region. The force acting on it will be



(1) zero, if B is in the x-direction

(2) λBI in the z-direction, if B is in the y-direction

(3) λBI in the negative y-direction, if B is in the z-direction

(4) λaBI, if B is in the x-direction

15. In previous problem, if the current is I and the magnetic field at D has magnitude B, then

(1) $B = \frac{\mu_0 I}{2\sqrt{2}\,\pi}$

(2) $B = \dfrac{\mu_0 I}{2\sqrt{3}\,\pi}$

(3) B is parallel to the z-axis

(4) B makes an angle of 45° with the x–y plane

16. A charged particle moves in a gravity free space where an electric field of strength E and a magnetic field of induction B exist. Which of the following statement is/are correct?

(1) If $E \neq 0$ and $B \neq 0$, velocity of the particle may remain constant

(2) If $E = 0$, the particle cannot trace a circular path

(3) If $E = 0$, kinetic energy of the particle remains constant

(4) None of these.

17. A charged particle of unit mass and unit charge moves with velocity $\vec{v} = (8\hat{i} + 6\hat{j})$ m s^{-1} in a magnetic field of $\vec{B} = 2\hat{k}$ T. Choose the correct altenative(s).

(1) The path of the particle may be $x^2 + y^2 - 4x - 21 = 0$

(2) The path of the particle may be $x^2 + y^2 = 25$

(3) The path of the particle may be $y^2 + z^2 = 25$

(4) The time period of the particle will be 3.14 s

18. A charged particle of specific charge α moves with a velocity $\vec{v} = v_0\hat{i}$ in a magnetic field $\vec{B} = \dfrac{B_0}{\sqrt{2}}\left(\hat{j} + \hat{k}\right)$.

Then (specific charge = charge per unit mass)

(1) path of the particle is a helix

(2) path of the particle is circle

(3) distance moved by the particle in time $t = \dfrac{\pi}{B_0\alpha}$ is $\dfrac{\pi v_0}{B_0\alpha}$

(4) velocity of the particle after time $t = \dfrac{\pi}{B_0\alpha}$ is $\left(\dfrac{v_0}{2}\hat{i} + \dfrac{v_0}{2}\hat{j}\right)$

19. A particle of charge $-q$ and mass m enters a uniform magnetic field $\vec{B}$ (perpendicular to paper inward) at P with a velocity v_0 at an angle α and leaves the field at Q with velocity v at angle β as shown in figure.

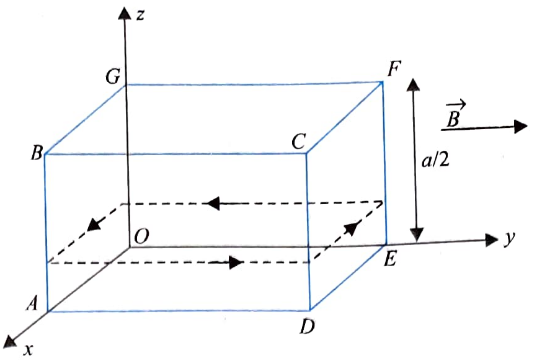
(1) $\alpha = \beta$

(2) $v = v_0$

(3) $PQ = \dfrac{2mv_0 \sin\alpha}{Bq}$

(4) The particle remains in field for time $t = \dfrac{2m(\pi - \alpha)}{Bq}$

20. A wooden cubical block $ABCDEFG$ of mass m and side a is wrapped by a square wire loop of perimeter $4a$, carrying current I. The whole system is placed at frictionless horizontal surface in a uniform magnetic field $\vec{B} = B_0\hat{j}$ as shown in figure. In this situation, normal force between horizontal surface and block passes through a point at a distance x from center. Choose correct statement(s).



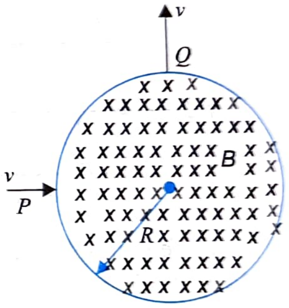
(1) The block must not topple if $I < \dfrac{mg}{aB_0}$

(2) The block must not topple if $I < \dfrac{mg}{2aB_0}$

(3) $x = \dfrac{a}{4}$ if $I = \dfrac{mg}{2aB_0}$

(4) $x = \dfrac{a}{4}$ if $I = \dfrac{mg}{4aB_0}$

21. A particle of charge q and mass m enters normally (at point P) in a region of magnetic field with speed v. It comes out normally from Q after time T as shown in figure. The magnetic field B is present only in the region of radius R and is uniform. Initial and final velocities are along radial direction and they are perpendicular to each other. For this to happen, which of the following expression(s) is/are correct?



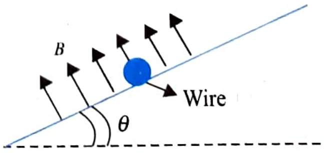
(1) $B = \dfrac{mv}{qR}$

(2) $T = \dfrac{\pi R}{2v}$

(3) $T = \dfrac{\pi m}{2qB}$

(4) None of these

22. A wire of mass m and length l is placed on a smooth incline making an angle θ with the horizontal, whose front view is shown in figure. When a finite amount of charge is passed through it in an infinitesimal time, the wire immediately acquires some velocity and then ascends the incline by a distance s. For this small duration, we can neglect the gravitational force because the current can be considered very large due to small time duration. The amount of charge passed through the wire is



(1) $\dfrac{m\sqrt{2gs\sin\theta}}{Bl}$ (2) $\dfrac{mv}{Bl}$

(3) $\dfrac{m\sqrt{2gs\sin\theta}}{Bl\cos\theta}$ (4) information insufficient

23. A particle is released from the origin with a velocity $v\hat{i}$. The electric field in the region is $E\hat{i}$ and magnetic field is $B\hat{k}$. Then

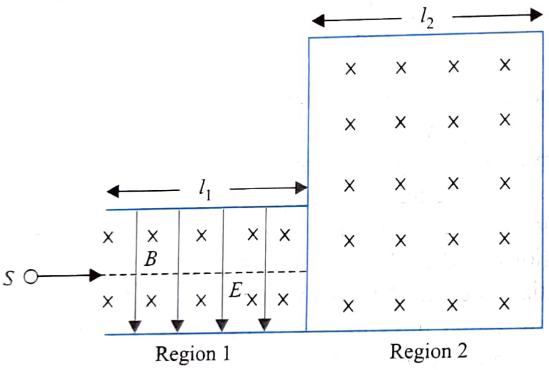
(1) If $v = 0$ and $E = 0$ then particle will execute a circular path.

(2) If $v = 0$, $E \neq 0$ and particle is positively charged then it executes clockwise circle as seen from $+z$ direction.

(3) If $v = 0$ and $E \neq 0$ then the particle will execute a cycloidal motion in the $x - y$ plane.

(4) If $v > 0$ and $E = 0$ and the particle is positively charged and $B > 0$ then the particle will execute anticlockwise circle as seen from $+z$ direction.

24. A charged particle of specific charge s passes undeviated through region 1 as shown in figure.



(1) Velocity of particle in region 1 is $v = E/B$

(2) Work done to move the charged particle in region 1 and region 2 is zero

(3) The radius of the trajectory of the charged particle in Region 2 is (E/sBB_0)

(4) The particle emerges from region 2 with a velocity $\vec{v}'$ where $\vec{v}' = -\vec{v}$ for $l_2 > \dfrac{E}{sBB_0}$

25. A proton is fired from origin with velocity $\vec{v} = v_0\hat{j} + v_0\hat{k}$ in a uniform magnetic field $\vec{B} = B_0\hat{j}$. In the subsequent motion of the proton

(1) its z co-ordinate can never be negative

(2) its x co-ordinate can never be positive

(3) its x and z co-ordinates cannot be zero at the same time

(4) its y co-ordinate will be proportional to its time of flight

Linked Comprehension Type

For Problems 1–3

A particle of mass m and charge q is accelerated by a potential difference V volt and made to enter a magnetic field region at an angle θ with the field. At the same moment, another particle of same mass and charge is projected in the direction of the field from the same point. Magnetic field of induction is B.

1. What would be the speed of second particle so that both particles meet again and again after a regular interval o time, which should be minimum?

(1) $\sqrt{\dfrac{qV}{m}}\cos\theta$ (2) $\sqrt{\dfrac{2qV}{m}}\cos\theta$

(3) $\sqrt{\dfrac{qV}{m}}\sin\theta$ (4) $\sqrt{\dfrac{qV}{2m}}\cos\theta$

2. Find the time interval after which they meet.

(1) $\dfrac{2\pi m}{qB}$ (2) $\dfrac{\pi m}{2qB}$

(3) $\dfrac{\pi m}{qB}$ (4) $\dfrac{3\pi m}{2qB}$

3. Find the distance travelled by the second particle during tha interval mentioned in the above problem.

(1) $\sqrt{\dfrac{Vm}{q}}\dfrac{2\pi}{B}\cos\theta$ (2) $\sqrt{\dfrac{2Vm}{3q}}\dfrac{2\pi}{B}\cos\theta$

(3) $\sqrt{\dfrac{2Vm}{q}}\dfrac{2\pi}{B}\cos\theta$ (4) $\dfrac{2}{3}\sqrt{\dfrac{Vm}{q}}\dfrac{\pi}{m}\cos\theta$

For Problems 4–5

A charged particle carrying charge $q = 10$ μC moves wi velocity $v_1 = 10^6$ m s^{-1} at angle 45° with x-axis in the xy plan and experiences a force $F_1 = 5\sqrt{2}$ mN along the negative z-axi When the same particle moves with velocity $v_2 = 10^6$ m s^{-1} alo the z-axis, it experiences a force F_2 in y-direction.

4. Find the magnetic field $\vec{B}$.

(1) $(10^{-3}\,\mathrm{T})(\hat{i} + \hat{j})$ (2) $(2 \times 10^{-3}\,\mathrm{T})\hat{i}$

(3) $(10^{-3}\,\mathrm{T})\hat{i}$ (4) $(2 \times 10^{-3}\,\mathrm{T})(\hat{i} + \hat{j})$

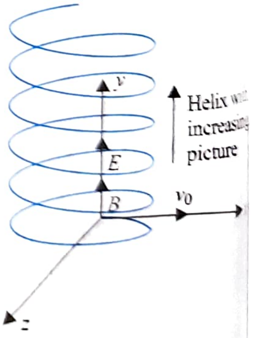
5. Find the magnitude of the force F_2.

(1) 10^{-2} N (2) 10^{-3} N

(3) 10^{-4} N (4) 10^{-5} N

For Problems 6–7

Uniform electric and magnetic fields with strength E and induction B, res-pectively, are along y-axis as shown in figure. A particle with specific charge q/m leaves the origin O in the direction of x-axis with an initial non-relativistic velocity v_0.



6. The coordinate y_n of the particle when it crosses the y-ax for the n^{th} time is

(1) $\dfrac{2\pi^2 mn^2 E}{qB^2}$ (2) $\dfrac{\pi^2 mn^2 E}{qB^2}$

(3) $\dfrac{2\pi^2 mn^2 E}{3qB^2}$ (4) $\dfrac{\sqrt{3}\pi^2 mn^2 E}{qB^2}$

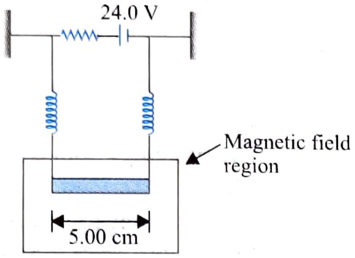
7. The angle α between the particle's velocity vector an y-axis at that moment is

(1) $\tan^{-1}\left(\frac{3v_0B}{2\pi nE}\right)$ (2) $\tan^{-1}\left(\frac{v_0B}{\pi nE}\right)$

(3) $\tan^{-1}\left(\frac{v_0B}{\sqrt{2\pi nE}}\right)$ (4) $\tan^{-1}\left(\frac{v_0B}{2\pi nE}\right)$

For Problems 8–9

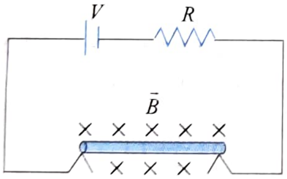
The circuit in figure consists of wires at the top and bottom and identical metal springs at the left and right sides. The wire at the bottom has a mass of 10.0 g and is 5.00 cm long. The wire is hanging as shown in the figure. The springs stretch 0.500 cm under the weight of the wire, and the circuit has a total resistance of 12.0 Ω. When a magnetic field is turned on, the springs stretch an additional 0.300 cm.



8. From the above statements we can conclude that
   (1) the magnetic field is directed into the plane of page
   (2) the magnetic field is directed out of the page
   (3) the magnetic field is toward left in the plane of page
   (4) the magnetic field is toward right in the plane of page
9. The magnitude of magnetic field is
   (1) 1.2 T (2) 6 T
   (3) 0.6 T (4) 12 T

For Problems 10–11

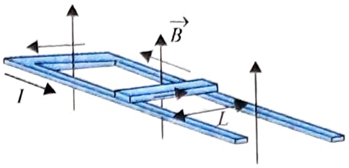
A thin, 50 cm long metal bar with mass 750 g rests on, but is not atta-ched to, two metallic supports in a uniform 0.450 T magnetic field, as shown in figure. A battery and a 25 Ω resistor in series are connected to the supports.



10. What is the largest voltage the battery can have without breaking the circuit at the supports?
    (1) 817 V (2) 412 V
    (3) 325 V (4) 160 V
11. The battery voltage has the maximum value calculated in above question. If the resistor suddenly gets partially short-circuited, decreasing its resistance to 2 Ω, find the initial acceleration of the bar.
    (1) 113 m s^{-2} (2) 55 m s^{-2}
    (3) 180 m s^{-2} (4) 12.4 m s^{-2}

For Problems 12–13

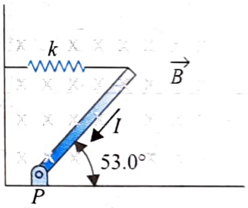
A conducting bar with mass m and length L slides over horizontal rails that are connected to a voltage source. The voltage source maintains a constant current I in the rails and bar, and a constant, uniform, vertical magnetic field $\vec{B}$ fills the region between the rails (as shown in figure).



12. Find the magnitude and direction of the net force on the conducting bar. Ignore friction, air resistance and electrical resistance.
    (1) ILB, to the right. (2) ILB, to the left.
    (3) $2ILB$, to the right. (4) $2ILB$, to the left.
13. If the bar has mass m, find the distance d that the bar must move along the rails from rest to attain speed v.
    (1) $\frac{3v^2m}{2ILB}$ (2) $\frac{5v^2m}{2ILB}$
    (3) $\frac{v^2m}{ILB}$ (4) $\frac{v^2m}{2ILB}$

For Problems 14–15

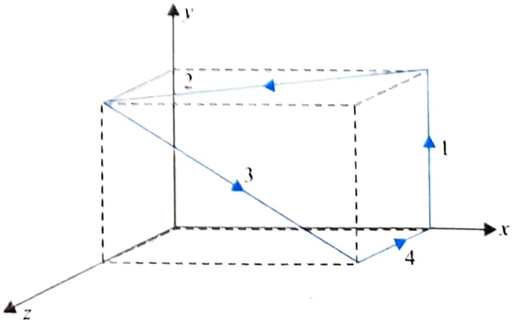
A thin, uniform rod with negligible mass and length 0.200 m is attached to the floor by a frictionless hinge at point P (as shown in figure). A horizontal spring with force constant $k = 4.80$ N m^{-1} connects the other end of the rod to a vertical wall. The rod is in a uniform magnetic field $B = 0.340$ T directed into the plane of the figure. There is current $I = 6.50$ A in the rod, in the direction shown.



14. Calculate the torque due to the magnetic force on the rod, for an axis at P.
    (1) 0.0442 N m^{-1}, clockwise
    (2) 0.0442 N m^{-1}, anticlockwise
    (3) 0.022 N m^{-1}, clockwise
    (4) 0.022 N m^{-1}, anticlockwise
15. When the rod is in equilibrium and makes an angle of 53.0° with the floor, is the spring stretched or compressed?
    (1) 0.05765 m, stretched (2) 0.05765 m, compressed
    (3) 0.0242 m, stretched (4) 0.0242 m, compressed

For Problems 16–18

A wire carrying a 10 A current is bent to pass through various sides of a cube of side 10 cm as shown in figure. A magnetic field $\vec{B} = (2\hat{i} - 3\hat{j} + \hat{k})$T is present in the region.



16. Find the net force on the loop shown.

(1) $\vec{F}_{\text{net}} = 0$ (2) $\vec{F}_{\text{net}} = (0.1\hat{i} - 0.2\hat{k})$ N

(3) $\vec{F}_{\text{net}} = (0.3\hat{i} + 0.4\hat{k})$ N (4) $\vec{F}_{\text{net}} = (0.36\hat{k})$ N

17. Find the magnetic moment vector of the loop.

(1) $(0.1\hat{i} + 0.05\hat{j} - 0.05\hat{k})\,\text{Am}^2$

(2) $(0.1\hat{i} + 0.05\hat{j} + 0.05\hat{k})\,\text{Am}^2$

(3) $(0.1\hat{i} - 0.05\hat{j} + 0.05\hat{k})\,\text{Am}^2$

(4) $(0.1\hat{i} - 0.05\hat{j} - 0.05\hat{k})\,\text{Am}^2$

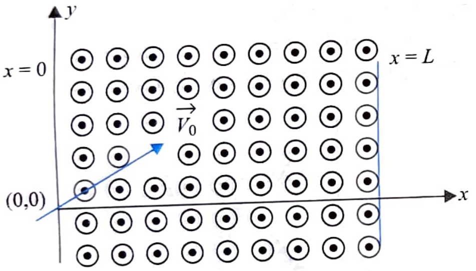
18. Find the net torque on the loop.

(1) $-0.1\hat{i} + 0.4\hat{k}$ Nm (2) $-0.1\hat{i} - 0.4\hat{k}$ Nm

(3) $0.2\hat{i} - 0.4\hat{k}$ Nm (4) $0.1\hat{i} - 0.4\hat{k}$ Nm

For Problems 19–20

The region between $x = 0$ and $x = L$ is filled with uniform constant magnetic field $25(\text{T})\hat{k}$. A particle of mass m = 50 g having positive charge q = 1 C and velocity $\vec{V}_0 = 50\sqrt{3}\,\hat{i} + 50\,\hat{j}\ \text{m s}^{-1}$ enters the region of the magnetic field. Neglect gravity throughout the question.



19. The value of L if the particle emerges from the region of magnetic field with its velocity $100(\text{m s}^{-1})\hat{i}$ is

(1) 20 cm (2) 10 cm

(3) 30 cm (4) None of these

20. The maximum and minimum values of L such that velocity of emerging particle makes angle $\pi/3$ with the y-axis are

(1) $L_{\min}$ = 10 cm, $L_{\max} = \infty$

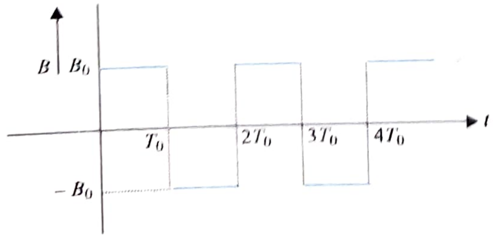
(2) $L_{\min}$ = 20 cm, $L_{\max}$ = 40 cm

(3) $L_{\min}$ = 10 cm, $L_{\max}$ = 20 cm

(4) $L_{\min}$ = 20 cm, $L_{\max} = \infty$

For Problems 21–22

In a region, magnetic field along x-axis changes according to the graph given in figure:



If time period, pitch and radius of helix path are T_0, P_0 and R_0, respectively, and if the particle is projected at an angle θ_0 with the positive x-axis toward positive y-axis in x–y plane, then

21. Select the correct statement:

(1) At $t = \dfrac{T_0}{2}$, coordinates of charge are $\left(\dfrac{P_0}{2}, 0, -2R_0\right)$

(2) At $t = \dfrac{3T_0}{2}$, coordinates of charge are $\left(\dfrac{3P_0}{2}, 0, 2R_0\right)$

(3) At $t = \dfrac{T_0}{2}$, coordinates of charge are $(P_0, 0, -2R_0)$

(4) At $t = \dfrac{3T_0}{2}$, coordinates of charge are $(3P_0, 0, 2R_0)$

22. Select the correct statement:

(1) Two extremes (positions of charge particle during th motion) from x-axis are at a distance $2R_0$ from each othe

(2) Two extremes from x-axis are at a distance $4R_0$ from each other

(3) Two extremes (positions of charge particle durin the motion) from x-axis are at a distance R_0 from eac other

(4) Two extremes from x-axis are at a distance $3R_0$ from eac other

For Problems 23–25

In a certain region of space, there exists a uniform and constan electric field of magnitude E along the positive y-axis of coordinate system. A charged particle of mass m and charge $-$ ($q > 0$) is projected from the origin with speed $2v$ at an angle o 60° with the positive x-axis in x–y plane. When the x-coordinat of particle becomes $\sqrt{3}\,mv^2/qE$ a uniform and constant magnet field of strength B is also switched on along positive y-axis.

23. Velocity of the particle just before the magnetic field i switched on is

(1) $v\hat{i}$ (2) $v\hat{i} + \dfrac{\sqrt{3}v}{2}\hat{j}$

(3) $v\hat{i} - \dfrac{\sqrt{3}v}{2}\hat{j}$ (4) $2v\hat{i} - \dfrac{\sqrt{3}v}{2}\hat{j}$

24. x-coordinate of the particle as a function of time after th magnetic field is switched on is

(1) $\dfrac{\sqrt{3}mv^2}{qE} - \dfrac{mv}{qB}\sin\left(\dfrac{qB}{m}t\right)$

(2) $\dfrac{\sqrt{3}mv^2}{qE} + \dfrac{mv}{qB}\sin\left(\dfrac{qB}{m}t\right)$

(3) $\dfrac{\sqrt{3}mv^2}{qE} - \dfrac{mv}{qB}\cos\left(\dfrac{qB}{m}t\right)$

(4) $\dfrac{\sqrt{3}mv^2}{qE} + \dfrac{mv}{qB}\cos\left(\dfrac{qB}{m}t\right)$

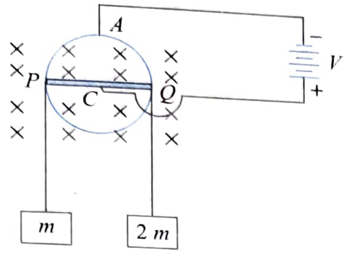
25. z-coordinate of the particle as a function of time after th magnetic field is switched on is

(1) $\dfrac{mv}{qB}\left[1 - \cos\left(\dfrac{qB}{m}t\right)\right]$ (2) $-\dfrac{mv}{qB}\left[1 + \cos\left(\dfrac{qB}{m}t\right)\right]$

(3) $-\dfrac{mv}{qB}\left[1 - \cos\left(\dfrac{qB}{m}t\right)\right]$ (4) $\dfrac{mv}{qB}\left[1 + \cos\left(\dfrac{qB}{m}t\right)\right]$

For Problems 26–28

A conducting ring of mass m and radius r has a weightless conducting rod PQ of length $2r$ and resistance $2R$ attached to it along its diameter. It is pivoted at its center C with its plane vertical, and two blocks of mass m and $2m$ are suspended by means of a light inextensible string passing over it as shown in figure. The ring is free to rotate about C and the system is placed in a magnetic field B (into the plane of the ring). A circuit is now completed by connecting the ring at A and C to a battery of emf V. It is found that for certain value of V, the system remains static.



26. In static condition, find the current through rod PC.

(1) V/R (2) $V/2R$

(3) $4V/R$ (4) $2V/R$

27. Net torque applied by the tension in strings can be related as

(1) $\frac{3BVr^2}{R}$ (2) $\frac{BVr^2}{R}$

(3) $\frac{BVr^2}{3R}$ (4) $\frac{BVr^2}{2R}$

28. The value of V can be related with m, B and r as

(1) $2mgR/Br$ (2) mgR/Br

(3) $mgR/2Br$ (4) $3mgR/Br$

For Problems 29–31

A charged particle with charge to mass ratio $\left(\frac{q}{m}\right) = \frac{10^3}{19}$ C kg^{-1} enters a uniform magnetic field $\vec{B} = 20\hat{i} + 30\hat{j} + 50\hat{k}$ T at time $t = 0$ with velocity $\vec{V} = (20\hat{i} + 50\hat{j} + 30\hat{k})$ m/s. Assume that magnetic field exists in large space.

29. During the further motion of the particle in the magnetic field, the angle between the magnetic field and velocity of the particle

(1) remains constant

(2) increases

(3) decreases

(4) may increase or decrease

30. The frequency (in Hz) of the revolution of the particle in cycles per second will be

(1) $\frac{10^3}{\pi\sqrt{38}}$ (2) $\frac{10^4}{\pi\sqrt{38}}$

(3) $\frac{10^4}{\pi\sqrt{19}}$ (4) $\frac{10^4}{2\pi\sqrt{19}}$

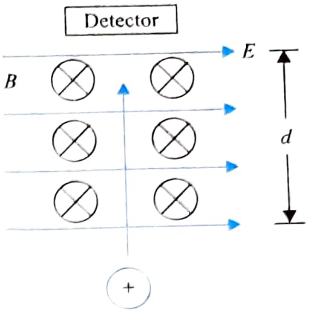
31. The pitch of the helical path of the motion of the particle will be

(1) $\pi/100$ m (2) $\pi/125$ m

(3) $\pi/215$ m (4) $\pi/250$ m

For Problems 32–34

A velocity filter uses the properties of electric and magnetic field to select particles that are moving with a specific velocity. Charged particles with varying speeds are directed into the filter as shown. The filter consists of an electric field E and a magnetic field B, each of constant magnitude, directed perpendicular to each other as shown. The charge particles will experience a force due to electric field given by $F = qE$. If positively charged particles are used, this force is towards right. The moving particle will also experience a force due to magnetic field given by $F = qvB$. When forces due to the two fields are of equal magnitude, the net force on the particle will be zero, the particle will pass through centre with its path unaltered. The electric and magnetic fields can be adjusted to choose the specific velocity to be filtered. The effect of gravity can be neglected.



32. The electric and magnetic fields are adjusted to detect particles with positive charge q of certain speed v_0. Which of the following expressions is equal to this speed?

(1) $\frac{B}{q^2E}$ (2) $\left(\frac{E}{q^2B}\right)$

(3) $\left(\frac{B}{E}\right)$ (4) $\left(\frac{E}{B}\right)$

33. Which of the following is true about the velocity filter shown in figure?

(1) it would not work with negatively charged particles.

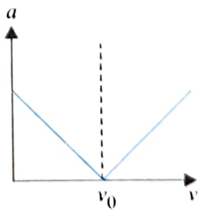
(2) wider the detector entrance, the narrower the range of speed detected.

(3) greater the distance d, the narrower the range of speed detected.

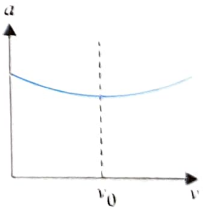
(4) The detector may not detect a charged particle with desired filter speed if its charge is too high.

34. If the filter is set to detect particles of speed v_0, which one of the following diagrams best illustrates how the magnitude of the particle acceleration (a) depends on velocity v?

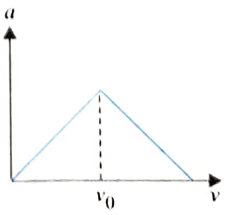
(1)



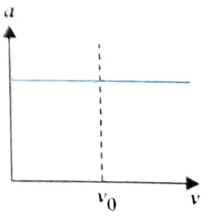
(2)



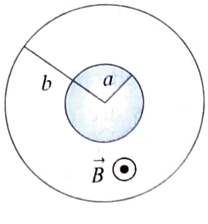
(3)



(4)



For Problems 35–37



In the given arrangement, the space between a pair of co-axial cylindrical conductors is evacuated. The outer cylinder, called anode, may be given a positive potential V relative to the inner cylinder.

A static homogeneous magnetic field $\vec{B}$ parallel to the cylinder axis, directed out of plane of figure is also present. Induced charges in the conductors are neglected.

We study the dynamics of electrons with rest mass m and charge e. The electrons are released at the surface of inner cylinder. Consider the following two cases:

Case 1: Firstly the potential V is turned on, but $\vec{B} = 0$. An electron with negligible velocity is ejected at the surface of inner cylinder. It is found to hit the anode.

Case 2: Now $V = 0$ but $\vec{B}$ is present. An electron starts out with an initial velocity $\vec{v}_0$ in radial direction. For magnetic field larger than critical value B_c the electron will not reach the anode.

35. Considering the case 1, the trajectory of electron will be
(1) Straight line (2) Circular
(3) Parabolic (4) Helical

36. Considering case 2, the trajectory of electron will be
(1) Straight line (2) Circular
(3) Parabolic (4) Helical

37. The critical magnetic field B_C is given by
(1) $\frac{2mv_0}{eb}$ (2) $\frac{mv_0}{eb}$
(3) $\frac{2bmv_0}{e(b^2-a^2)}$ (4) $\frac{2amv_0}{e(b^2-a^2)}$

For Problems 38–40

Magnetic force on a charged particle is given by $\vec{F}_m = q(\vec{v} \times \vec{B})$ and electrostatic force $\vec{F}_e = q\vec{E}$. A particle having charge $q = 1C$ and mass 1 kg is released from rest at origin. There are electric and magnetic fields given by $\vec{E} = (10\hat{i})$N/C for x = 1.8 m and $\vec{B} = -(5\hat{k})$T for 1.8 m $\le x \le$ 2.4 m.

A screen is placed parallel to $y - z$ plane at x = 3 m. Neglect gravity forces.

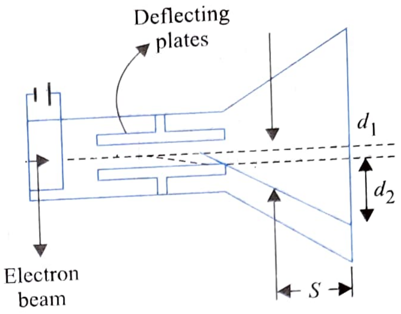
38. The speed with which the particle will collide the screen is
(1) 3 m s^{-1} (2) 6 m/s^{-1}
(3) 9 m/s^{-1} (4) 12 m/s^{-1}

39. y-coordinate of particle where it collides with screen (in meters) is
(1) $\frac{0.6(\sqrt{3}-1)}{\sqrt{3}}$ (2) $\frac{0.6(\sqrt{3}+1)}{\sqrt{3}}$
(3) $1.2(\sqrt{3}+1)$ (4) $\frac{1.2(\sqrt{3}-1)}{\sqrt{3}}$

40. Time after which the particle will collide the screen is (in seconds)
(1) $\frac{1}{5}\left(3+\frac{\pi}{6}+\frac{1}{\sqrt{3}}\right)$ (2) $\frac{1}{5}\left(6+\frac{\pi}{3}+\sqrt{3}\right)$
(3) $\frac{1}{3}\left(5+\frac{\pi}{6}+\frac{1}{\sqrt{3}}\right)$ (4) $\frac{1}{3}\left(6+\frac{\pi}{18}+\sqrt{3}\right)$

For Problems 41–43

Following experiment was performed by J.J. Thomson in order to measure ratio of charge e and mass m of electron.



Electrons emitted from a hot filament are accelerated by a potential difference V. As the electrons pass through deflecting plates, they encounter both electric and magnetic fields. The entire region in which electrons leave the plates they enter a field free region that extends to fluorescent screen. The entire region in which electrons travel is evacuated.

Firstly, electric and magnetic fields were made zero and position of undeflected electron beam on the screen was noted. The electric field was turned on and resulting deflection was noted. Deflection is given by $d_1 = \frac{eEL^2}{2mV^2}$ where L = length of deflecting plate and v = speed of electron.

In second part of experiment, magnetic field was adjusted so as to exactly cancel the electric force leaving the electron beam undeflected. This gives $eE = evB$. Using expression for d_1, we can find out $\frac{e}{m} = \frac{2d_1E}{B^2L^2}$

41. If the electron is deflected downward when only electric field is turned on, in what direction do the electric and magnetic fields point in second part of experiment
(1) The electric field points to the top, while the magnetic field points into the page
(2) The electric field points to the top, while the magnetic field points out of page
(3) Electric field points to the bottom, while the magnetic field points out of page
(4) Electric field points to the bottom, while the magnetic field points into the page

42. A beam of electron with velocity 3×10^7 m s^{-1} is deflected 2mm while passing through 10 cm in an electric field of 1800 V/m perpendicular to its path. e/m for electron is
(1) 1.5×10^{11} C kg^{-1} (2) 2×10^{11} C kg^{-1}
(3) 2.5×10^{11} C kg^{-1} (4) 3×10^{11} C kg^{-1}

43. If the electron speed were doubled by increasing the potential difference V, which of the following would be true in order to correctly measure e/m.
(1) Magnetic field would have to be halved.
(2) Magnetic field would have to be doubled.
(3) Length L of the plates would have to be doubled.
(4) Length L of the plates would have to be halved.

Matrix Match Type

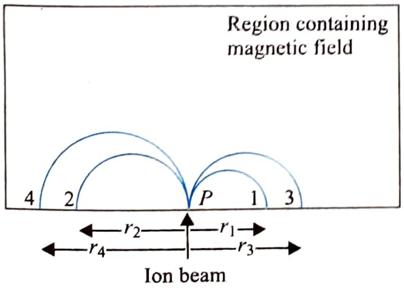
1. A charged particle passes through a region that could have electric field only or magnetic field only or both electric and magnetic fields or none of the fields. Match Column I with Column II.

Column I	Column II
i. Kinetic energy of the particle remains constant	**a.** Under special conditions, this is possible when both electric and magnetic fields are present
ii. Acceleration of the particle is zero	**b.** The region has electric field only
iii. Kinetic energy of the particle changes and it also suffers deflection	**c.** The region has magnetic field only
iv. Kinetic energy of the particle changes but it suffers no deflection	**d.** The region contains no field

2. Match Column I with Column II.

Column I	Column II
i. A charge particle is moving in uniform electric and magnetic fields in gravity free space.	**a.** Velocity of the particle may be constant.
ii. A charge particle is moving in uniform electric, magnetic and gravitational fields.	**b.** Path of the particle may be straight line.
iii. A charge particle is moving in uniform magnetic and gravitational fields (where electric field is zero).	**c.** Path of the particle may be circular.
iv. A charge particle is moving in only uniform electric field.	**d.** Path of the particle may be helical.

3. A beam consisting of four types of ions A, B, C, and D enters a region at P that contains a uniform magnetic field as shown in the figure. The field is perpendicular to the plane of the paper, but its precise direction is not given. All ions in the beam travel with the same speed. The table on the next page shows the masses and charges of the ions.



$r_4 > r_3 = r_2 > r_1$

Ion	Mass	Charge
A	$2m$	e
B	$4m$	$-e$
C	$2m$	$-e$
D	m	$+e$

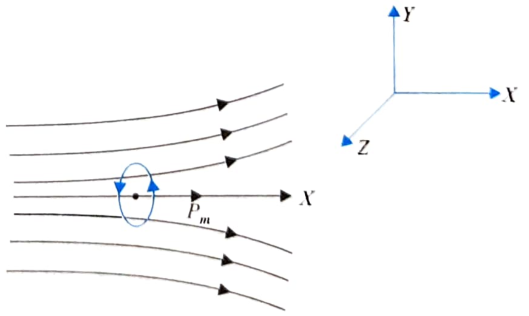
The ions fall at different positions 1, 2, 3, and 4 as shown. Correctly match the ions with their respective positions of fall.

Column I	Column II
i. A	**a.** 4
ii. B	**b.** 2
iii. C	**c.** 1
iv. D	**d.** 3

4. A charged particle with some initial velocity is projected in a region where non-zero electric and/or magnetic fields are present. In column I, information about the existence of electric and/or magnetic field and direction of initial velocity of charged particle are given, while in Column II the probable path of the charged particle is mentioned. Match the entries of Column I with the entries of Column II.

Column I	Column II
i. $\vec{E}=0, \vec{B}\neq 0,$ and initial velocity may be at any angle with $\vec{B}$	**a.** Straight line
ii. $\vec{E}\neq 0, \vec{B}=0$ and initial velocity may be at any angle with $\vec{E}$	**b.** Parabola
iii. $\vec{E}\neq 0, \vec{B}\neq 0, \vec{E}\parallel\vec{B}$ and initial velocity is $\perp$ to both	**c.** Circular
iv. $\vec{E}\neq 0, \vec{B}\neq 0, \vec{E}$ is perpendicular to $\vec{B}$ and non-uniform pitch $\vec{v}$ perpendicular to both $\vec{E}$ and $\vec{B}$	**d.** Helical path

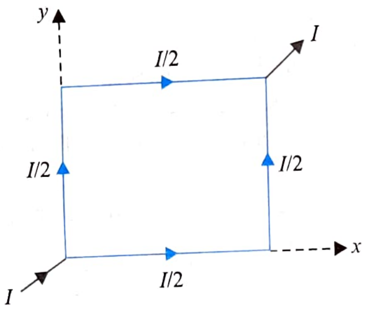
5. An elementary current loop is placed in a non-uniform magnetic field as shown in figure. In Column I, different orientations of loop are described and in Column II, the corresponding forces experienced by the loop. P_m is magnetic moment of loop.



Column I	Column II
i. In the given situation,	**a.** resultant force is acting along $\vec{P}_m$

ii.	If loop is rotated such that $\vec{P}_m$ is along +ve Z direction	**b.**	resultant force is acting opposite to $\vec{P}_m$
iii.	If loop is rotated such that $\vec{P}_m$ is along –ve Z direction	**c.**	$F_x = 0, F_y = 0$
iv.	If loop is rotated such that $\vec{P}_m$ is along +ve Y direction	**d.**	$F_x = 0, F_z = 0$

6. A square loop of uniform conducting wire is as shown in figure. A current I (in amperes) enters the loop from one end and exits the loop from opposite end as shown. The length of one side of the square loop is l meter. The wire has uniform cross-section area and uniform linear mass density. In four situations of Column I, the loop is subjected to four different magnetic fields. Under the conditions of Column I, match the Column I with corresponding results of Column II (B_0 in Column I is a positive non-zero constant)



	Column I		Column II
i.	$\vec{B} = B_0\hat{i}$ in tesla	**a.**	Magnitude of net force on the loop is $\sqrt{2}\,B_0Il$ newton
ii.	$\vec{B} = B_0\hat{j}$ in tesla	**b.**	Magnitude of net force on the loop is zero
iii.	$\vec{B} = B_0(\hat{i} + \hat{j})$ in tesla	**c.**	Magnitude of net torque of the loop about its center is zero
iv.	$\vec{B} = B_0\hat{k}$ in tesla	**d.**	Magnitude of net force on the loop is B_0Il newton

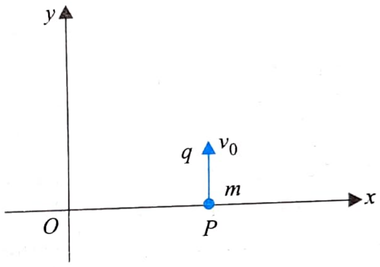
7. A charged particle having a charge q and mass m is to be subjected to a combination of constant uniform magnetic field $(\vec{B})$ and a constant uniform gravitational field $(\vec{E})$. Apart from these field forces there exists no other force. Now match the column.

	Column I		Column II
i.	The charged particle moves without change in its direction.	**a.**	It is possible that both $\vec{B}$ and $\vec{E}$ are zero.
ii.	The charged particle moves without change in its velocity.	**b.**	It is possible that both $\vec{B}$ and $\vec{E}$ are non zero.
iii.	The charged particle takes a circular path.	**c.**	It is possible that $\vec{B}$ is zero and $\vec{E}$ is not zero.
iv.	The charged particle take a parabolic path.	**d.**	It is possible that $\vec{B}$ is non-zero and $\vec{E}$ is zero.

8. Column I shows the state of motion of a charged particle. Column II shows the possible combination of electric field and magnetic field under which the path in column I is possible. Match appropriately

	Column I		Column II
i.	Charge at rest experiences a force.	**a.**	$E = 0, B = 0$
ii.	A charge in motion goes undeviated with same velocity.	**b.**	$E \neq 0, B \neq 0$
iii.	A charge in motion goes undeviated with varying speed.	**c.**	$E = 0, B \neq 0$
iv.	A charged particle undergoes helical motion.	**d.**	$E \neq 0, B = 0$

9. A negatively charged particle of mass 'm' having magnitude of charge q enters a magnetic field $\vec{B} = B_0\hat{k}$ T at point $P(3\text{ m}, 0, 0)$ with velocity $\vec{v}_0 = 3\hat{j} + 4\hat{k}$ m/s at $t = 0$ as shown in the figure $\left[\text{Given } \frac{qB_0}{m} = 1 \text{ rad/s}\right]$ [No other field is present]

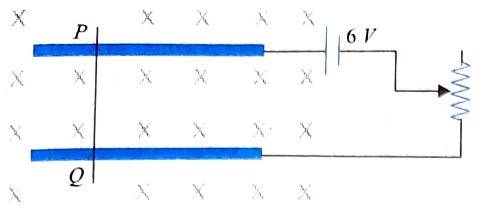


Now match the following:

	Column I		Column II
i.	Pitch of the motion of the particle	**a.**	$(-3\sin t\hat{i} + 3\cos t\hat{j})$ unit
ii.	$\frac{24\pi}{25}$ × Radius of curvature of particle during motion at any time $t = t$ sec	**b.**	$(-3\cos t\hat{i} - 3\sin t\hat{j})$ unit
iii.	Velocity component of particle in x-y plane.	**c.**	8π unit
iv.	Acceleration of particle.	**d.**	Constant

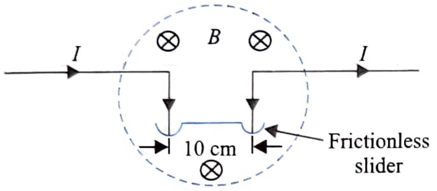
Numerical Value Type

1. A metal wire PQ of mass 10 g lies at rest on two horizontal metal rails separated by 5 cm as shown in figure. A vertically downward magnetic field of magnitude 0.80 T exists in the space. The resistance of the circuit is slowly decreased and it is found that when the resistance goes below 20.0 Ω, the wire PQ starts sliding on the rails. The coefficient of friction between wires and rails is found to be $n/25$. Find n.



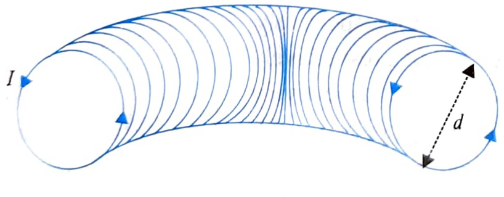
2. A current $I = 10$ A flows in a ring of radius $r_0 = 15$ cm made of a very thin wire. The tensile strength of the wire is equal to $T = 1.5$ N. The ring is placed in a magnetic field, which is perpendicular to the plane of the ring so that the forces tend to break the ring. Find B (in T) at which the ring is broken.

3. A 10 cm length of wire with a mass of 20 g is attached frictionlessly to the vertical segments of a wire in which a current I flows. The surrounding has uniform horizontal field $B = 10^4$ G and the direction is shown in figure. What must be the current I (in A) to maintain the 10 cm wire in an equilibrium position?

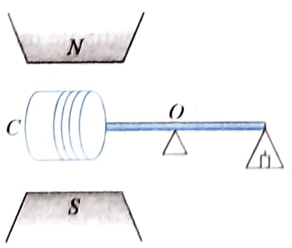


4. A current I flows in a rectangularly shaped wire whose center lies at $(x_0, 0, 0)$ and whose vertices are located at the points A $(x_0 + d, -a, -b)$, B $(x_0 - d, a, -b)$, C $(x_0 - d, a, +b)$, and D $(x_0 + d, -a, +b)$ respectively. Assume that $a, b, d \ll x_0$. Find the magnitude of magnetic dipole moment vector of the rectangular wire frame in J/T. (Given: $b = 10$ m, $I = 0.01$ A, $d = 4$ m, $a = 3$ m.)

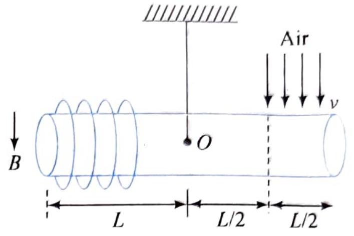
5. Calculate the magnetic moment (in Am^2) of a thin wire with a current $I = 8$ A, wound tightlly on a half a tore (see figure). The diameter of the cross section of the tore is equal to $d = 5$ cm, and the number of turns is $N = 500$.



6. A small coil C with $N = 200$ turns is mounted on one end of a balance beam and introduced between the poles of an electromagnet as shown in figure. The area of the coil is $S = 1$ cm^2, the length of the right arm of the balance beam is $l = 30$ cm. When there is no current in the coil the balance is in equilibrium. On passing a current $I = 22$ mA through the coil, equilibrium is restored by putting an additional weight of mass $m = 60$ mg on the balance pan. Find the magnetic induction field (in terms of $\times 10^{-1}$ T) between the poles of the electromagnet, assuming it to be uniform.

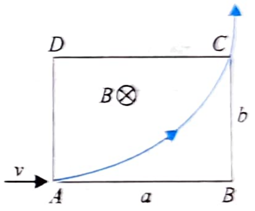


7. A non-conducting non-magnetic rod having circular cross section of radius R is suspended from a rigid support as shown in the figure. A light and small coil of 300 turns is wrapped tightly at the left end of the rod where uniform magnetic field B exists in vertically downward direction. Air of density ρ hits the half of the right part of the rod with velocity V as shown in the figure. What should be current in clockwise direction (as seen from O) in the coil so that rod remains horizontal? Give answer in mA. Given $\frac{2}{Lv}\sqrt{\frac{\pi RB}{\rho}} = \frac{1}{\sqrt{5}}\,\text{A}^{-1/2}$.



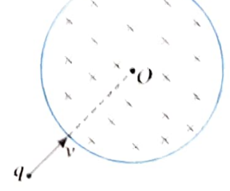
8. A charged particle enters a uniform magnetic field with velocity $v_0 = 4$ m/s perpendicular to it, the length of magnetic field is $x = \frac{\sqrt{3}}{2}R$, where R is the radius of the circular path of the particle in the field. Find the magnitude of change in velocity (in m/s) of the particle when it comes out of the field.

9. A charged particle of mass $m = 1$ mg and charge $q = 1$ μC enters along AB at point A in a uniform magnetic field $B = 1.2$ T existing in the rectangular region of size $a \times b$, where $a = 4$ m and $b = 3$ m. The particle leaves the region exactly at corner point C. What is the speed v (in m s^{-1}) of the particle?



10. A magnetic field $\vec{B} = -B_0\hat{i}$ exists within a sphere of radius $R = v_0 T\sqrt{3}$ where T is the time period of one revolution of a charged particle starting its motion from origin and moving with a velocity $\vec{v}_0 = \frac{v_0}{2}\sqrt{3}\,\hat{i} - \frac{v_0}{2}\hat{j}$. Find the number of turns that the particle will take to come out of the magnetic field.

11. The figure shows a circular region of radius $R = \sqrt{3}$ m which has a uniform magnetic field $B = 0.2$ T directed into the plane of the figure. A particle having mass $m = 2$ g, speed $v = 0.3$ m/s and charge $q = 1$ mC is projected along the radius of the circular region as shown in figure. Calculate the angular deviation (in degrees) produced in the path of the particle as it comes out of the magnetic field. Neglect any other force apart from the magnetic force.



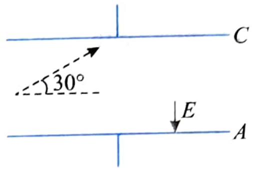
12. A charged particle carrying charge $q = 1$ mC moves in uniform magnetic field with velocity $v_1 = 10^6$ m/s at angle 45° with x-axis in the xy-plane and experiences a force $F_1 = 5\sqrt{2}$ mN along the negative z-axis. When the same particle moves with velocity $v_2 = 10^6$ m/s along the z-axis, it experiences a force F_2 in y-direction. Find the magnitude of the force F_2 (in N).

13. What is the value of B (in × 10^{-8} T) that can be set up at the equator to permit a proton of speed 10^7 m/s to circulate around the earth?

$[R = 6.4 \times 10^6 \text{ m}, \; m_P = 1.67 \times 10^{-27} \text{ kg}]$

14. A proton beam passes without deviation through a region of space where there are uniform transverse mutually perpendicular electric and magnetic fields with E = 120 kV/m and B = 50 mT. Then the beam strikes a grounded target. Find the force (in $\times 10^{-5}$ N) imparted by the beam on the target if the beam current is equal to $I = 0.80$ A.

15. A charged particle having charge 10^{-6} C and mass of 10^{-10} kg is fired at the middle of the plate making an angle 30° with plane of the plate. Length of the plate is 0.17 m and it is separated by 0.1 m. Electric field $E = 10^{-3}$ N/C is present between the plates. Just outside the plates magnetic field is present. Find the magnitude of the magnetic field (in mT) perpendicular to the plane of the figure, if it has to graze the plate at C and A (in mT) parallel to the surface of the plate. (Neglect gravity)



Archives

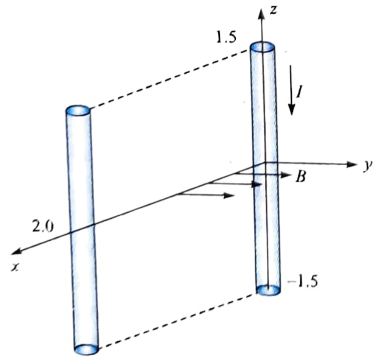
JEE MAIN

Single Correct Answer Type

1. Proton, deuteron and alpha particle of same kinetic energy are moving in circular trajectories in a constant magnetic field. The radii of proton, deuteron and alpha particle are, respectively, r_p, r_d and r_α. Which one of the following relation is correct?

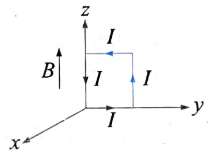
(1) $r_\alpha = r_p = r_d$
(2) $r_\alpha = r_p < r_d$
(3) $r_\alpha > r_d > r_p$
(4) $r_\alpha = r_d > r_p$ **(AIEEE 2012)**

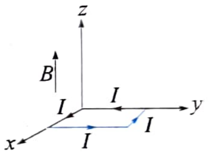
2. A conductor lies along the z-axis at $-1.5 \le z < 1.5$ m and carries a fixed current of 10.0 A in $-\hat{a}_z$ direction (see figure). For a field $\vec{B} = 3.0 \times 10^{-4} e^{-0.2x}\, \hat{a}_y$ T, find the power required to move the conductor at constant speed to $x = 2.0$ m, $y = 0$ m in 5×10^{-3} s. Assume parallel motion along the x-axis

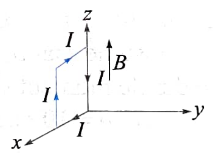


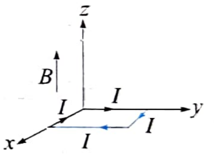
(1) 14.85 W
(2) 29.7 W
(3) 1.57 W
(4) 2.97 W **(JEE Main 2014)**

3. A rectangular loop of sides 10 cm and 5 cm carrying a current I of 12 A is placed in different orientations as shown in the figures below:

(a)


(b)


(c)


(d)


If there is a uniform magnetic field of 0.3 T in the positive z-direction, in which orientations the loop would be in (a) stable equilibrium and (b) unstable equilibrium ?

(1) (a) and (b), respectively
(2) (a) and (c), respectively
(3) (b) and (d), respectively
(4) (b) and (c), respectively **(JEE Main 2015)**

4. An electron, a proton and an alpha particle having the same kinetic energy are moving in circular orbits of radii r_e, r_p, r_α respectively in a uniform magnetic field B. The relation between r_e, r_p, r_α is

(1) $r_e < r_\alpha < r_p$
(2) $r_e > r_p = r_\alpha$
(3) $r_e < r_p = r_\alpha$
(4) $r_e < r_p < r_\alpha$

(JEE Main 2018)

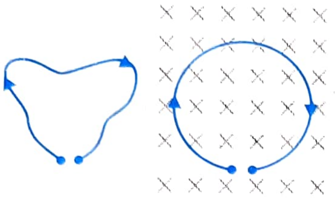
5. The dipole moment of a circular loop carrying a current I is m and the magnetic field at the centre of the loop is B_1. When the dipole moment is doubled by keeping the current constant, the magnetic field at the centre of the loop is B_2. The ratio B_1/B_2 is

(1) $\dfrac{1}{\sqrt{2}}$
(2) 2
(3) $\sqrt{3}$
(4) $\sqrt{2}$ **(JEE Main 2018)**

JEE ADVANCED

Single Correct Answer Type

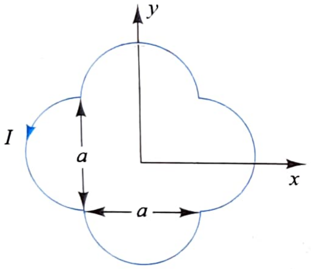
1. A thin flexible wire of length L is connected to two adjacent fixed points and carries a current I in the clockwise direction, as shown in figure. When the system is put in a uniform magnetic field of strength B going into the plane of the paper, the wire takes the shape of a circle. The tension in the wire is



(1) IBL (2) $\frac{IBL}{\pi}$

(3) $\frac{IBL}{2\pi}$ (4) $\frac{IBL}{4\pi}$ **(IIT-JEE 2010)**

2. A loop carrying current I lies in the x–y plane as shown in figure. The unit vector $\hat{k}$ is coming out of the plane of the paper. The magnetic moment of the current loop is



(1) $a^2 I\hat{k}$ (2) $\left(\frac{\pi}{2}+1\right)a^2 I\hat{k}$

(3) $-\left(\frac{\pi}{2}+1\right)a^2 I\hat{k}$ (4) $(2\pi+1)a^2 I\hat{k}$

(IIT-JEE 2012)

Multiple Correct Answers Type

1. An electron and a proton are moving on straight parallel paths with same velocity. They enter a semi-infinite region of uniform magnetic field perpendicular to the velocity. Which of the following statement(s) is/are true?
(1) They will never come out of the magnetic field region.
(2) They will come out travelling along parallel paths.
(3) They will come out at the same time.
(4) They will come out at different times.

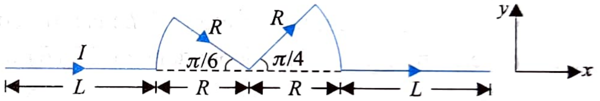
(IIT-JEE 2011)

2. Consider the motion of a positive point charge in a region where are simultaneous uniform electric and magnetic field $\vec{E} = E_0\hat{j}$ and $\vec{B} = B_0\hat{j}$. At time $t = 0$, this charge has velocity $\vec{v}$ in the x-y plane, making an angle θ with the x-axis. Which of the following option(s) is(are) correct for time $t > 0$?
(1) If $\theta = 0°$, the charge moves in a circular path in the x-z plane
(2) If $\theta = 0°$, the charge undergoes helical motion with constant pitch along the y-axis
(3) If $\theta = 10°$, the charge undergoes helical motion with its pitch increasing with time, along the y-axis
(4) If $\theta = 90°$, the charge undergoes linear but accelerated motion along the y-axis **(JEE Advanced 2013)**

3. A particle of mass M and positive charge Q, moving with a constant velocity $u_1 = 4\hat{i}\,\mathrm{m\,s^{-1}}$, enters a region of uniform static magnetic field normal to the x-y plane. The region of the magnetic field extends from $x = 0$ to $x = L$ for all values of y. After passing through this region, the particle emerges on the other side after 10 milliseconds with a velocity $\vec{u}_2 = 2(\sqrt{3}\hat{i}+\hat{j})\,\mathrm{m\,s^{-1}}$. The correct statement(s) is (are)
(1) The direction of the magnetic field is $-z$ direction.
(2) The direction of the magnetic field is $+z$ direction.
(3) The magnitude of the magnetic field $\frac{50\pi M}{3Q}$ units
(4) The magnitude of the magnetic field is $\frac{100\pi M}{3Q}$ units

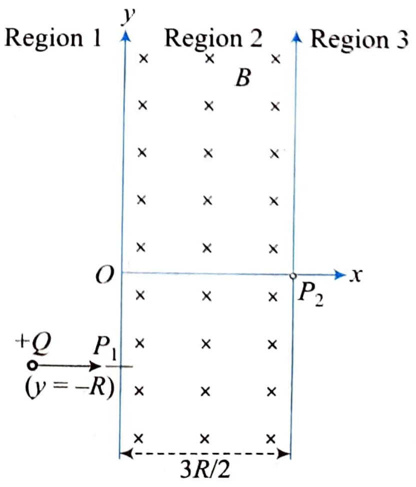
(JEE Advanced 2013)

4. A conductor (shown in the figure) carrying constant current I is kept in the x-y plane in a uniform magnetic field $\vec{B}$. If F is the magnitude of the total magnetic force acting on the conductor, then the correct statement(s) is (are):



(1) If $\vec{B}$ is along $\hat{z}$, $F \propto (L+R)$
(2) If $\vec{B}$ is along $\hat{x}$, $F = 0$
(3) If $\vec{B}$ is along $\hat{y}$, $F \propto (L+R)$
(4) If $\vec{B}$ is along $\hat{z}$, $F = 0$ **(JEE Advanced 2015)**

5. A uniform magnetic field B exists in the region between $x = 0$ and $x = \frac{3R}{2}$ (region 2 in the figure) pointing normally into the plane of the paper. A particle with charge $+Q$ and momentum p directed along x-axis enters region 2 from region 1 at point $P_1(y = -R)$. Which of the following options(s) is/are correct?



(1) For $B = \frac{8}{13}\frac{p}{QR}$, the particle will enter region 3 through the point P_2 on x-axis

(2) For $B > \frac{2}{3}\frac{p}{QR}$, the particle will re-enter region 1

(3) For a fixed B, particle of same charge Q and same velocity v, the distance between the point P_1 and the point of re-entry into region 1 is inversely proportional to the mass of the particle

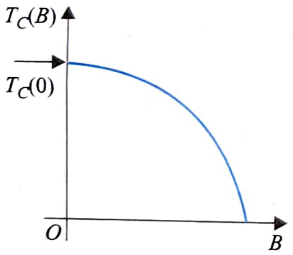
(4) When the particle re-enters region 1 through the longest possible path in region 2, the magnitude of the change in its linear momentum between point P_1 and the farthest point from y-axis is $\frac{p}{\sqrt{2}}$

(JEE Advanced 2017)

Linked Comprehension Type

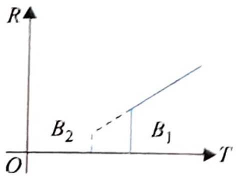
For Problems 1–2

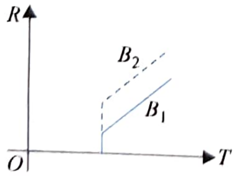
Electrical resistance of certain materials, known as superconductors, changes abruptly from a nonzero value to zero as their temperature is lowered below a critical temperature $T_c(0)$. An interesting property of superconductors is that their critical temperature becomes smaller than $T_c(0)$ if they are placed in a magnetic field, i.e., the critical temperature $T_c(B)$ is a function of the magnetic field strength B. The dependence of $T_c(B)$ on B is shown in figure.

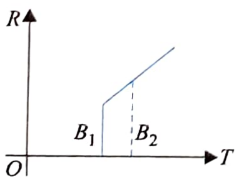


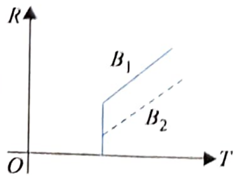
(IIT-JEE 2010)

1. In the graphs below, the resistance R of a superconductor is shown as a function of its temperature T for two different magnetic fields B_1 (solid line) and B_2 (dashed line). If B_2 is larger than B_1, which of the following graphs shows the correct variation of R with T in these fields?

(1)


(2)


(3)


(4)


2. A superconductor has $T_c(0) = 100$ K. When a magnetic field of 7.5 T is applied, its T_c decreases to 75 K. For this material one can definitely say that when

(1) $B = 5$ T, $T_c(B) = 80$ K

(2) $B = 5$ T, $75 \text{ K} < T_c(B) < 100$ K

(3) $B = 10$ T, $75\text{K} < T_c(B) < 100$ K

(4) $B = 10$ T, $T_c = 70$K

Matrix Match Type

Answer Q.1, Q.2 and Q.3 by appropriately matching the information given in the three columns of the following table. A charged particle (electron or proton) is introduced at the origin ($x = 0$, $y = 0$, $z = 0$) with a given initial velocity $\vec{v}$. A uniform electric field $\vec{E}$ and a uniform magnetic field $\vec{B}$ exis everywhere. The velocity $\vec{v}$, electric field $\vec{E}$ and magnetic fiel $\vec{B}$ are given in columns I, II and III, respectively. The quantitie E_0 and B_0 are positive in magnitude.

	Column I		Column II		Column III
(I)	Electron with $\vec{v} = 2\frac{E_0}{B_0}\hat{x}$	(i)	$\vec{E} = E_0\hat{z}$	(P)	$\vec{B} = -B_0\hat{x}$
(II)	Electron with $\vec{v} = \frac{E_0}{B_0}\hat{y}$	(ii)	$\vec{E} = -E_0\hat{y}$	(Q)	$\vec{B} = B_0\hat{x}$
(III)	Proton with $\vec{v} = 0$	(iii)	$\vec{E} = -E_0\hat{x}$	(R)	$\vec{B} = B_0\hat{y}$
(IV)	Proton with $\vec{v} = 2\frac{E_0}{B_0}\hat{x}$	(iv)	$\vec{E} = E_0\hat{x}$	(S)	$\vec{B} = B_0\hat{z}$

1. In which case will the particle move in a straight line wit constant velocity?

(1) (IV) (i) (S)
(2) (III) (ii) (R)
(3) (III) (iii) (P)
(4) (II) (iii) (S)

2. In which case would the particle move in a straight line alon the negative direction of y-axis (i.e., move along $-\hat{y}$)?

(1) (III) (ii) (P)
(2) (III) (ii) (R)
(3) (IV) (ii) (S)
(4) (II) (iii) (Q)

3. In which case will the particle describe a helical path wit axis along the positive z-direction?

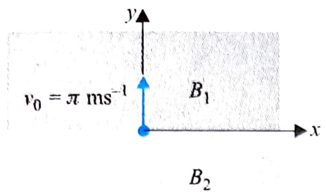
(1) (III) (iii) (P)
(2) (II) (ii) (R)
(3) (IV) (ii) (R)
(4) (IV) (i) (S)

Numerical Value Type

1. Two parallel wires in the plane of the paper are distance apart. A point charge is moving with speed u between th wires in the same plane at a distance X_1 from one of th wires. When the wires carry current of magnitude I in th same direction, the radius of curvature of the path of th point charge is R_1 In contrast, if the currents I in the tw wires have directions opposite to each other, the radius curvature of the path is R_2. If $\frac{X_0}{X_1} = 3$, the value of $\frac{R_1}{R_2}$ i

(JEE Advanced 2014)

2. In the x-y plane, the region $y > 0$ has a uniform magnetic field $B_1\hat{k}$ and the region $y < 0$ has another uniform magnetic field $B_2\hat{k}$. A positively charged particle is projected from the origin along the positive y-axis with speed $v_0 = \pi \text{ms}^{-1}$ at $t = 0$, as shown in the figure.



Neglect gravity in this problem. Let $t = T$ be the time when the particle crosses the x-axis from below for the first time. If $B_2 = 4B_1$, the average speed of the particle, in ms^{-1}, along the x-axis in the time interval T is ________.

(JEE Advanced 2018)

3. A moving coil galvanometer has 50 turns and each turn has an area 2×10^{-4} m^2. The magnetic field produced by the magnet inside the galvanometer is 0.02 T. The torsional constant of the suspension wire is 10^{-4} Nm rad^{-1}. When a current flows through the galvanometer, a full scale deflection occurs if the coil rotates by 0.2 rad. The resistance of the coil of the galvanometer is 50 Ω. This galvanometer is to be converted into an ammeter capable of measuring current in the range 0–1.0 A. For this purpose, a shunt resistance is to be added in parallel to the galvanometer. The value of this shunt resistance, in ohms, is........... .

(JEE Advanced 2018)

Answers Key

EXERCISES

Single Correct Answer Type

1. (1) **2.** (1) **3.** (3) **4.** (2) **5.** (3)
6. (2) **7.** (4) **8.** (1) **9.** (1) **10.** (1)
11. (2) **12.** (3) **13.** (3) **14.** (2) **15.** (2)
16. (1) **17.** (4) **18.** (3) **19.** (2) **20.** (2)
21. (2) **22.** (1) **23.** (4) **24.** (2) **25.** (2)
26. (2) **27.** (2) **28.** (2) **29.** (4) **30.** (1)
31. (4) **32.** (2) **33.** (4) **34.** (2) **35.** (2)
36. (3) **37.** (1) **38.** (4) **39.** (3) **40.** (1)
41. (3) **42.** (4) **43.** (3) **44.** (1) **45.** (2)
46. (1) **47.** (3) **48.** (1) **49.** (3) **50.** (2)
51. (3) **52.** (1) **53.** (1) **54.** (3) **55.** (1)
56. (1) **57.** (1) **58.** (3) **59.** (1) **60.** (1)
61. (3) **62.** (4) **63.** (2) **64.** (3) **65.** (2)
66. (2) **67.** (1) **68.** (1) **69.** (3) **70.** (2)
71. (4) **72.** (1) **73.** (2) **74.** (2) **75.** (4)
76. (3) **77.** (4) **78.** (3) **79.** (1) **80.** (1)
81. (1) **82.** (2) **83.** (2) **84.** (3) **85.** (2)
86. (3) **87.** (2) **88.** (1)

Multiple Correct Answers Type

1. (1),(2) **2.** (1),(2) **3.** (1),(2)
4. (1),(2),(3),(4) **5.** (1),(2) **6.** (1),(4)
7. (1),(2),(3) **8.** (2),(4) **9.** (1),(2),(3)
10. (1),(2),(3) **11.** (2),(3),(4) **12.** (1),(2)
13. (1),(3) **14.** (1),(2),(3) **15.** (1),(4)
16. (1),(3) **17.** (1),(2),(4) **18.** (2),(3)
19. (1),(2),(3),(4) **20.** (2),(4) **21.** (1),(2),(3)
22. (1),(2) **23.** (3),(4) **24.** (1),(2),(3),(4)
25. (2),(4)

Linked Comprehension Type

1. (2) **2.** (1) **3.** (3) **4.** (3) **5.** (1)
6. (1) **7.** (4) **8.** (2) **9.** (3) **10.** (1)
11. (1) **12.** (1) **13.** (4) **14.** (1) **15.** (1)
16. (1) **17.** (2) **18.** (3) **19.** (2) **20.** (4)
21. (1),(2) **22.** (2) **23.** (1) **24.** (2) **25.** (3)
26. (1) **27.** (2) **28.** (2) **29.** (1) **30.** (2)
31. (4) **32.** (4) **33.** (3) **34.** (1) **35.** (1)
36. (2) **37.** (3) **38.** (2) **39.** (4) **40.** (1)
41. (2) **42.** (2) **43.** (1)

Matrix Match Type

1. i. → a., c., d.; ii. → a., c., d.; iii. → a., b.; iv. → a., b.
2. i.→ a., b., d.; ii. → a., b., c., d.; iii. → a., b., d.; iv. → b.
3. i. → d.; ii. → a.; iii. → b.; iv. → c.
4. i. → a., c.; ii. → a., b.; iii → d.; iv. → a.
5. i. → b.; ii. → a., c.; iii. → a., c.; iv. → a., d.
6. i. → c., d.; ii. → c., d.; iii. → b., c.; iv. → a., c.
7. i. → a., b., c., d.; ii. → a., b., d.; iii. → d.; iv. → c.
8. i. → b., d.; ii. → a., b., c.; iii. → b., d.; iv. → b., c.
9. i. → c., d.; ii. → c., d.; iii. → a.; iv. → b.

Numerical Value Type

1. (3) **2.** (1) **3.** (2) **4.** (2) **5.** (5)
6. (4) **7.** (2) **8.** (4) **9.** (5) **10.** (2)
11. (60) **12.** (0.01) **13.** (1.6) **14.** (2) **15.** (3.46)

ARCHIVES

JEE Main

Single Correct Answer Type

1. (2) **2.** (4) **3.** (3) **4.** (3) **5.** (4)

JEE Advanced

Single Correct Answer Type

1. (3) **2.** (2)

Multiple Correct Answers Type

1. (2),(4) **2.** (3),(4) **3.** (1),(3)
4. (1),(2),(3) **5.** (1),(2)

Linked Comprehension Type

1. (1) **2.** (2)

Matrix Match Type

1. (4) **2.** (2) **3.** (4)

Numerical Value Type

1. (3) **2.** (2) **3.** (5.55)

2 Sources of Magnetic Field

INTRODUCTION

In electrostatics we learned that a static charge produces electric field in its surroundings. A moving charge can produce both electric and magnetic field. Thus, a current of a moving charged particles produces a magnetic field around the current.

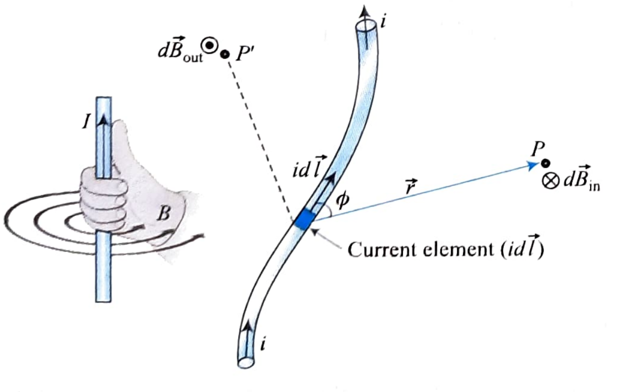
This feature of electromagnetism, which is the combined study of electric and magnetic effects, has become enormously important in everyday life because it is the basis of countless electromagnetic devices.

In this chapter, first we shall learn how to find the magnetic field due to the current in a very small section of current carrying wire, called current element. Then we shall find the magnetic field due to entire wire for several different arrangements of the wire.

MAGNETIC FIELD DUE TO CURRENT ELEMENT: BIOT AND SAVART LAW

We have seen that a current-carrying wire can experience a magnetic force when placed in a magnetic field that is produced by an external source, such as a permanent magnet. A current-carrying wire also produces a magnetic field of its own, as we will see in this section.

Jean-Baptiste **Biot** and Félix **Savart** performed quantitative experiments on the force exerted by an electric current on a nearby magnet. From their experimental results, Biot and Savart arrived at a mathematical expression that gives the magnetic field at some point in space in terms of the current that produces the field. That expression is based on the experimental observations for the magnetic field $d\vec{B}$ at a point P in its neighborhood, associated with a length element $d\vec{l}$ of a wire carrying a steady current i, depends upon four factors,



(i) dB is directly proportional to the current in the element.

$$dB \propto i \qquad \text{...(i)}$$

(ii) dB is directly proportional to the length of the element

$$dB \propto dl \qquad \text{...(ii)}$$

(iii) dB is inversely proportional to the square of the distance r of the point P from the element

$$dB \propto \frac{1}{r^2} \qquad \text{...(iii)}$$

(iv) dB is directly proportional to the sine of the angle ϕ between the direction of the current flow and the line joining the element to the point P

$$dB \propto \sin\theta \qquad \text{...(iv)}$$

Combining all the four factors, we get

$$dB \propto \frac{idl\sin\phi}{r^2} \Rightarrow dB = K\frac{idl\sin\phi}{r^2} \qquad \text{...(v)}$$

Where K is a proportionality constant and its value depends upon the nature of the medium surrounding the current carrying wire. If the current carrying wire is placed in vacuum, then in SI units its value is given as

$$K = \frac{\mu_0}{4\pi} = 10^{-7}\,\text{Tm/A}$$

Here μ_0 is called permeability of vacuum or free space. It is a physical quantity for a given medium which is a measure of a medium's ability on the extent to which it allows external magnetic field to polarize the material inside the volume of the body. Now we can write Biot & Savart's Law as

$$dB = \frac{\mu_0}{4\pi}\frac{idl\sin\phi}{r^2} \qquad \text{...(vi)}$$

The constant $\mu_0$is similar in nature to the constant $\in_0$ we discussed in the chapter electrostatics. For a given medium, its magnetic permeability is defined as

$$\mu = \mu_0\mu_r$$

Where μ_r is called 'Relative Permeability of Medium' which is casually also referred as diamagnetic constant of the medium. Unlike to the case of electrostatics where dielectric constant of any medium is always greater than unity, in case of magnetism, depending upon the types of mediums' value of μ_r can be greater or smaller than unity. If the current carrying wire is placed in a material medium then Eq. (vi) can be rewritten as

$$dB = \frac{\mu}{4\pi}\frac{idl\sin\phi}{r^2} = \frac{\mu_0\mu_r}{4\pi}\frac{idl\sin\phi}{r^2} \qquad \text{...(vii)}$$

Above equation gives the magnitude of magnetic induction produced due to a small current element.

Equation (vi) can be written in vector form as,

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,d\vec{l}\times\hat{r}}{r^2}$$

$$\text{or}\quad d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,d\vec{l}\times\vec{r}}{r^3} \qquad \text{...(viii)}$$

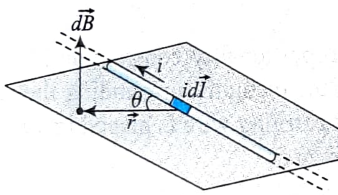
The magnetic field at point P due to whole wire can be obtained by integrating the above expression for the entire length of the wire within proper limits according to the shape of the wire and it is given as

$$\vec{B} = \int d\vec{B} = \int\left(\frac{\mu_0}{4\pi}\right)\frac{I\,d\vec{l}\times\vec{r}}{r^3} \qquad \text{...(ix)}$$

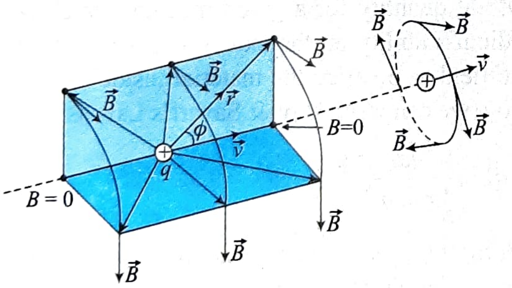
The expression '$i\,d\vec{l}$' in above equations which is the product of current and length of element is referred as a vector quantity called 'Current Element' which is measured in units of 'Ampere-meter' or 'A-m', the direction of this vector is in the direction of current.

Important Points:

- The direction of the magnetic field produced by the current element is perpendicular to both the position vector $\vec{r}$ and the current element, $i\,d\vec{l}$.



- If a charged particle having charge q moving with velocity v is shown in figure.

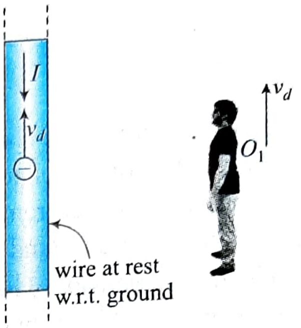


Based on experiments, the magnetic field produced by the particle is given by, $B = \frac{\mu_0}{4\pi}\frac{|q|v\sin\phi}{r^2}$

In vector form: $\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$

- **Frame Dependence of $\vec{B}$:**
  (a) The motion of anything is a relative term. A charge may appear at rest by an observer (say O_1) and moving at some velocity $\vec{v}_1$ with respect to observer O_2 and at velocity $\vec{v}_2$ with respect to observer O_3. Then, $\vec{B}$ due to that charge w.r.t. O_1 will be zero and w.r.t. O_2 and O_3 it will be $\vec{B}_1$ and $\vec{B}_2$ (that means different).

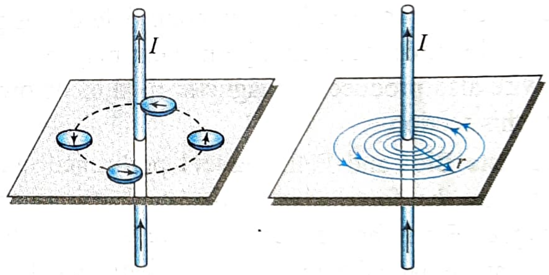
  (b) In a current carrying wire electrons move in the opposite direction to that of the current and +ve ions (of the metal) are static w.r.t. the wire. Now, if some observer (O_1) moves with velocity v_d in the direction of motion of the electrons, then electrons will have zero velocity and +ve ions will have velocity v_d in the downward direction w.r.t. O_1. The density (n) of +ve ions is same as the density of free electrons and their charges are of the same magnitudes.



So, w.r.t. O_1 electrons will produce zero magnetic field but +ve ions will produce the magnetic field.

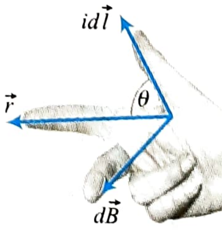
DIRECTION OF MAGNETIC FIELD DUE TO A STRAIGHT CURRENT CARRYING WIRE

When a current is present, the compass needles point in a circular pattern about the wire. The pattern indicates that the magnetic field lines produced by the current are circles centered on the wire. If the direction of the current is reversed, the needles also reverse their directions, indicating that the direction of the magnetic field has reversed.

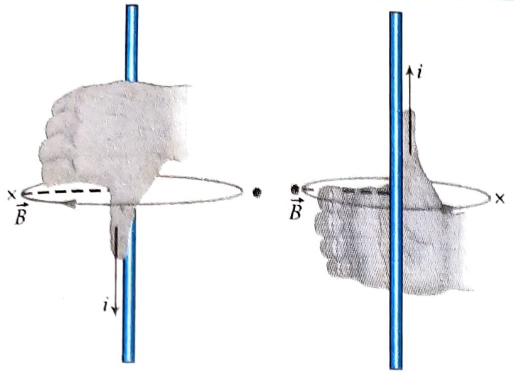


The direction of the field can be obtained by using *Right-Hand Rule 1 or 2.*

Right-hand rule 1: To determine the direction of the magnetic field using your right hand, point your thumb in the direction of the differential current element and your index finger in the direction of the position vector, and your middle finger will point in the direction of the differential magnetic field.



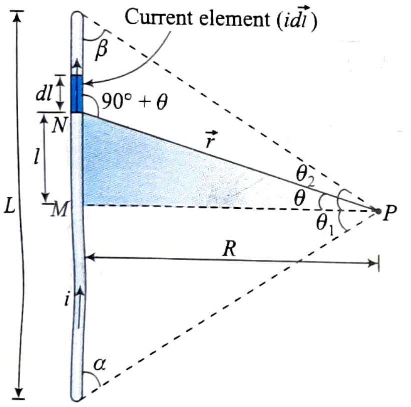
Right-hand rule 2: According to this rule, if we grasp the conductor in the palm of our right hand so that the thumb points in the direction of the flow of current, then the direction in which the fingers curl gives the direction of magnetic lines of force.



MAGNETIC FIELD DUE TO A STRAIGHT CURRENT CARRYING WIRE

Figure shows a straight wire of length L carrying a current I and P is a point at a perpendicular distance R from the wire. Using right hand thumb rule here we can state that the magnetic field at point P is in the direction away from the reader (into the page). Let us take a current element '$i\,d\vec{l}$', the magnetic field at point P due to this current element,

$$dB = \frac{\mu_0}{4\pi}\frac{idl\,\sin\phi}{r^2} = \frac{\mu_0}{4\pi}\frac{idl\,\sin(90°+\theta)}{r^2} \qquad \text{...(i)}$$



In ΔMNP, $r = R\sec\theta$

Now, $l = R\tan\theta$, $dl = R\sec^2\theta \cdot d\theta$ ...(ii)

Also, $r = R\sec\theta$

$$dB = \frac{\mu_0}{4\pi}\frac{i(R\sec^2\theta d\theta)(\cos\theta)}{R^2\sec^2\theta}$$

$$\Rightarrow \quad dB = \frac{\mu_0 i}{4\pi R}\cos\theta\; d\theta$$

$$B = \int dB = \frac{\mu_0 i}{4\pi R}\int_{-\theta_1}^{\theta_2}\cos\theta d\theta \qquad \text{...(iii)}$$

$$B = \int dB = \frac{\mu_0 i}{4\pi R}[\sin\theta]_{-\theta_1}^{\theta_2} = \frac{\mu_0 i}{4\pi R}[\sin\theta_2 - \sin(-\theta_1)]$$

$$B = \frac{\mu_0 i}{4\pi R}[\sin\theta_1 + \sin\theta_2] = \frac{\mu_0 i}{4\pi R}[\cos\alpha + \cos\beta] \qquad \text{...(iv)}$$

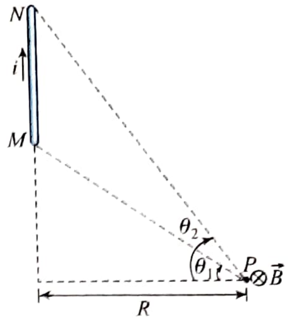
In the vector form:

$$\vec{B} = \frac{\mu_0 i}{4\pi R}[\sin\theta_1 + \sin\theta_2](-\hat{k}) \qquad \text{...(v)}$$

- If point 'P' lies outside the line of wire, as shown in figure then magnetic field at point 'P'.

  In this case, the limits of integration of θ in Eq. (iii) should be from θ_1 to θ_2

  Hence magnetic field at P, should be

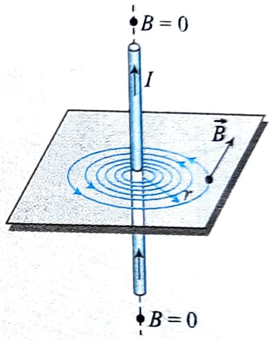


$$B = \int dB = \frac{\mu_0 i}{4\pi R}[\sin\theta]_{\theta_1}^{\theta_2}$$

$$= \frac{\mu_0 i}{4\pi R}[\sin\theta_2 - \sin\theta_1]$$

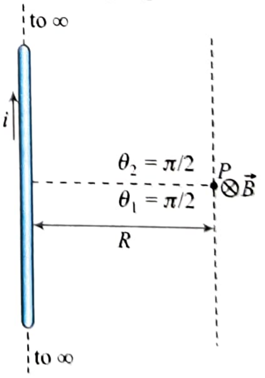
Important Points:

- For points along the length of the wire, the field is always zero.
- In a plane perpendicular to the wire and containing the point, the lines of force are concentric circles encircling the wire.



MAGNETIC FIELD DUE TO A STRAIGHT CURRENT CARRYING WIRE OF INFINITE LENGTH

The magnitude of the magnetic induction at P can be calculated by using Eq. (v). If we look at figure, we can observe that $\theta_1 = \theta_2 = 90°$. We know the general expression for magnetic field at point P due to a current carrying wire is



$$B = \frac{\mu_0 i}{4\pi R}[\sin\theta_2 + \sin\theta_1]$$

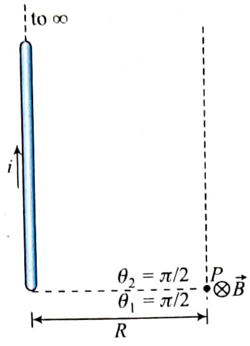
At point P, $\theta_1 = \theta_2 = \frac{\pi}{2}$; then

$$B = \frac{\mu_0 i}{4\pi R}\left[\sin\frac{\pi}{2} + \sin\frac{\pi}{2}\right] = \frac{\mu_0}{4\pi}\frac{i}{R}[1+1] = \frac{\mu_0}{4\pi}\frac{2i}{R}$$

$$B = \frac{\mu_0}{2\pi}\frac{i}{R}$$

MAGNETIC FIELD DUE TO A SEMI-INFINITE STRAIGHT WIRE

If we observe the figure, we can see that at point P, $\theta_1 = 0$ and $\theta_2 = \frac{\pi}{2}$; then



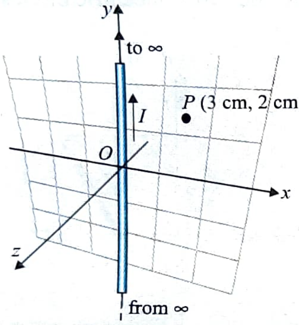
$$B = \frac{\mu_0 i}{4\pi R}\left[\sin\frac{\pi}{2} + \sin 0\right]$$

$$= \frac{\mu_0 i}{4\pi R}[1+0]$$

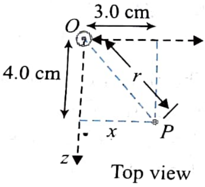
i.e., $$B = \frac{\mu_0}{4\pi}\frac{i}{R}$$

ILLUSTRATION 2.1

A long straight wire carrying current $i = 10$ A lies along y-axis. Find the magnetic field at P (3 cm, 2 cm, 4 cm).



Sol. Perpendicular distance of wire from P is



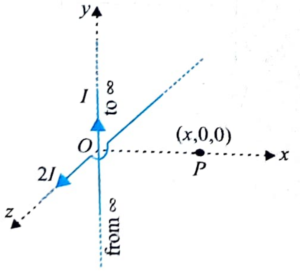
$$r = \sqrt{x^2+z^2} = \sqrt{3^2+4^2} = 5 \text{ cm}$$

$$B_P = \frac{\mu_0}{4\pi}\frac{2I}{a} = \frac{10^{-7}\times 2\times 10}{0.05}$$

$$= 4\times 10^{-5}\text{ T} = 0.04 \text{ mT}$$

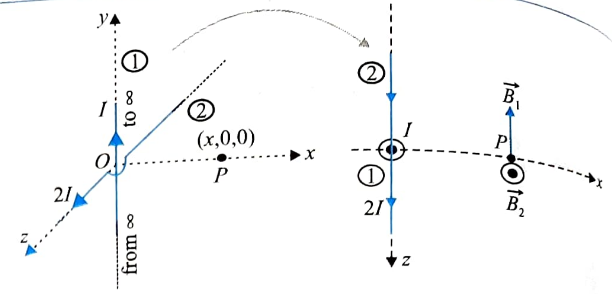
ILLUSTRATION 2.2

Two long straight current carrying wires having currents I and $2I$ lie along y- and z-axes, respectively, as shown in figure. Find $\vec{B}$ at the point P $(x, 0, 0)$.



Sol. Magnetic field at P due to current along y-axis:

$$\vec{B}_1 = \frac{\mu_0}{2\pi}\frac{i}{x}(-\hat{k})$$



Due to current along z-axis: $\vec{B}_2 = \frac{\mu_0}{2\pi}\frac{2i}{x}\hat{j}$

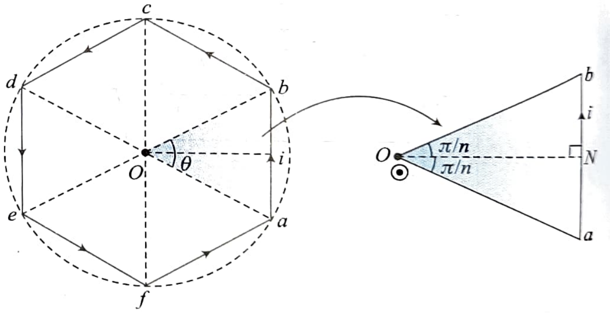
Net field at P: $\vec{B} = \vec{B}_1 + \vec{B}_2 = \frac{\mu_0 i}{2\pi x}\left[2\hat{j} - \hat{k}\right]$

$$B = \sqrt{B_1^2 + B_2^2} = \frac{\sqrt{5}\mu_0 i}{2\pi x}$$

ILLUSTRATION 2.3

A current i flows along a thin wire shaped as a regular polygon with n sides which can be inscribed into a circle of radius R. Find the magnetic induction at the center of the polygon.

Sol. The angle made by one segment of polygon (say ab) the center, $\theta = \left(\frac{2\pi}{n}\right)$



We know the general expression for magnetic field at point due to a current carrying wire,

$$B = \frac{\mu_0 i}{4\pi r}[\sin\theta_2 + \sin\theta_1]$$

Here, $r = ON = R\cos\left(\frac{\pi}{n}\right)$ and $\theta_1 = \theta_2 = \left(\frac{\pi}{n}\right)$

Hence magneticfield at O due to a straight current carrying element ab is given as

$$B_1 = \frac{\mu_0 i}{4\pi}\times\frac{1}{R\cos(\pi/n)}\left[\sin\left(\frac{\pi}{n}\right) + \sin\left(\frac{\pi}{n}\right)\right]$$

$$B_1 = \frac{\mu_0 i}{4\pi}\times\frac{1}{R\cos(\pi/n)}\times 2\sin\left(\frac{\pi}{n}\right)$$

As there are n sides in the polygon, the total magnetic inductio due to polygon is given as

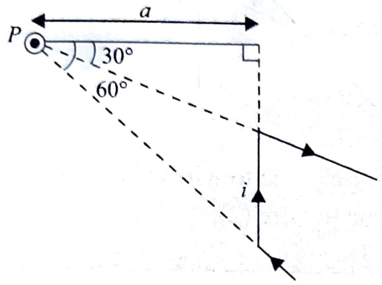
$$B = \frac{\mu_0 n i}{2\pi R}\tan\left(\frac{\pi}{n}\right)$$

Using right hand rule here we can state that the magnetic field at point P is in the direction away towards the reader (away from the page).

$$\vec{B} = \frac{\mu_0 ni}{2\pi R}\tan\left(\frac{\pi}{n}\right)\odot$$

ILLUSTRATION 2.4

Find the magnitude and direction of magnetic field at point P due to the current carrying wire as shown in figure.



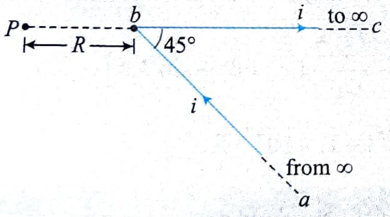
Sol. $B = \dfrac{\mu_0 I}{4\pi R}[\sin\theta_1 + \sin\theta_2]$

Here $\theta_1 = -30°$, $\theta_2 = 60°$. Putting these values, we get

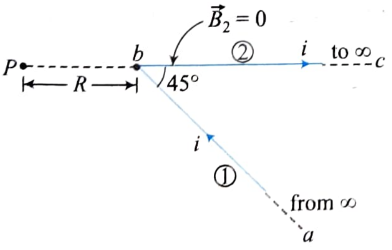
$$B = \frac{\mu_0 I}{4\pi R}\left[-1/2 + \sqrt{3}/2\right] = \frac{\mu_0 I}{8\pi R}(\sqrt{3} - 1)$$

ILLUSTRATION 2.5

A long straight wire, carrying current i, is bent at its midpoint to form an angle of 45°. Find the induction of magnetic field at point P, at a distance R from the point of bending.

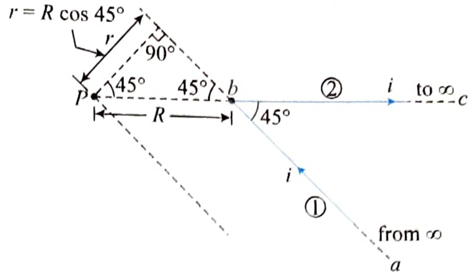


Sol. Since point P lies on axis of straight part bc, therefore, magnetic induction due to this part is equal to zero.



Both the ends a and b are on the same side of base line, it means we should use the expression

$$B = \frac{\mu_0 i}{4\pi r}[\sin\theta_2 - \sin\theta_1]$$



Here, $r = R\cos 45° \Rightarrow r = \dfrac{R}{\sqrt{2}}$

$$B_1 = \frac{\mu_0}{4\pi}\frac{i}{(R/\sqrt{2})}\left[\sin\frac{\pi}{2} - \sin\frac{\pi}{4}\right]$$

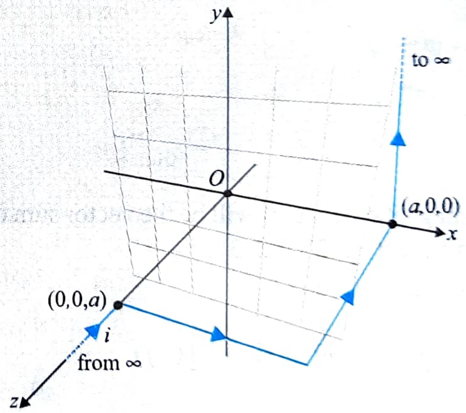
$$= \frac{\mu_0}{4\pi}\frac{i\sqrt{2}}{R}\left[1 - \frac{1}{\sqrt{2}}\right] = \frac{\mu_0}{4\pi}\frac{i}{R}[\sqrt{2} - 1]$$

Using right hand rule, here we can state that the magnetic field at point P is in the direction away towards the reader (away from the page).

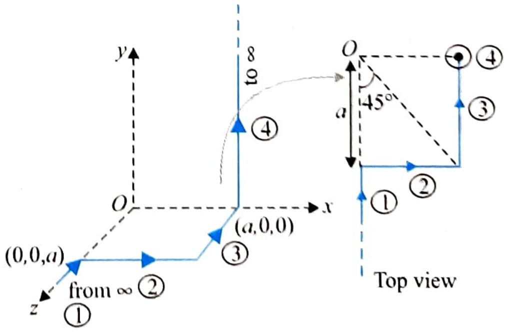
Hence $\vec{B} = \vec{B}_1 + \vec{B}_2 = \dfrac{\mu_0}{4\pi}\dfrac{i}{R}[\sqrt{2} - 1]\odot$ (away from the page).

ILLUSTRATION 2.6

Find $\vec{B}$ at the origin due to the long wire carrying current i (as shown in figure).



Sol.



Net magnetic field at origin is the vector sum of magnetic fields due to wires (1), (2), (3) and (4).

$$\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 + \vec{B}_4$$

where $B_1 = 0$, as direction of current wire passes through origin.

Magnetic field due to wire (2),

$$\vec{B}_2 = \frac{\mu_0}{4\pi}\frac{I}{a}[\sin 0° + \sin 45°]\hat{j}$$

Magnetic field due to wire (2) and wire (3) is equal to $\vec{B}_2 = \vec{B}_3$

$$\Rightarrow \quad \vec{B}_2 = \vec{B}_3 = \frac{\mu_0 I}{4\sqrt{2}\pi a}\hat{j}$$

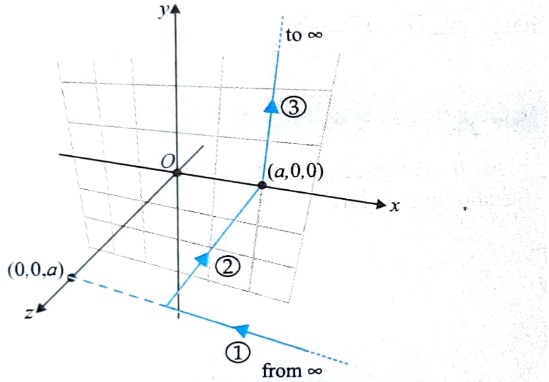
Magnetic field due to wire (4),

$$\vec{B}_4 = \frac{\mu_0}{4\pi}\frac{I}{a}[\sin 0° + \sin 90°]\hat{k} = \frac{\mu_0 I}{4\pi a}\hat{k}$$

So $\quad \vec{B} = 2\vec{B}_2 + \vec{B}_4 = \frac{\mu_0 I}{4\pi a}[\sqrt{2}\,\hat{j} + \hat{k}]$

ILLUSTRATION 2.7

An infinite current carrying conductor is bent into three segments (1), (2) and (3) as shown in figure. If it carries a current i, find the magnetic induction at the origin.

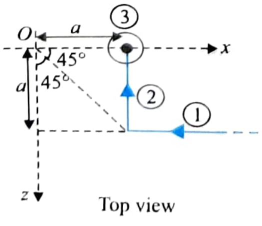


Sol. Net magnetic field at O will be the vector sum of magnetic fields due to wires (1), (2) and (3).

Magnetic field due to wire (1),

$$\vec{B}_1 = \frac{\mu_0}{4\pi}\frac{i}{a}[\sin 90° - \sin 45°](-\hat{j})$$

$$= \frac{\mu_0}{4\pi}\frac{i}{a}\left[1 - \frac{1}{\sqrt{2}}\right](-\hat{j})$$



Magnetic field due to wire (2),

$$\vec{B}_2 = \frac{\mu_0}{4\pi}\frac{i}{a}[\sin 45° + \sin 0°](\hat{j})$$

$$= \frac{\mu_0}{4\pi}\frac{i}{a}\left[\frac{1}{\sqrt{2}}\right](\hat{j})$$

Magnetic field due to wire (3),

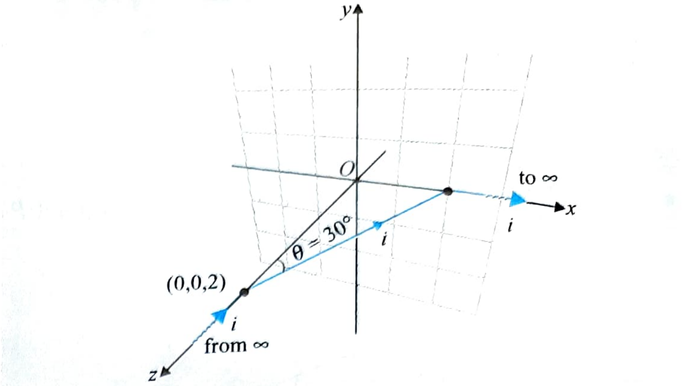
$$\vec{B}_3 = \frac{\mu_0}{2\pi}\frac{i}{a}(\hat{k})$$

Hence, net magnetic field $\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3$

$$\Rightarrow \quad \vec{B} = \frac{\mu_0}{4\pi}\frac{i}{a}\left[(\sqrt{2}-1)\hat{j} + \hat{k}\right]$$

ILLUSTRATION 2.8

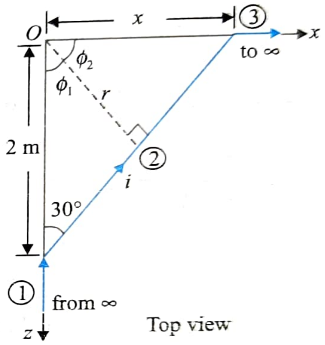
Find the field at the origin O due to the current $i = 2$ A.



Sol. Magnetic field due to wires (1) and (3) will be zero.

Magnetic field due to wire (2),

$$B_2 = \frac{\mu_0}{4\pi}\frac{i}{r}[\sin\phi_1 + \sin\phi_2]$$



Here, $r = 2\sin 30° = 1$,

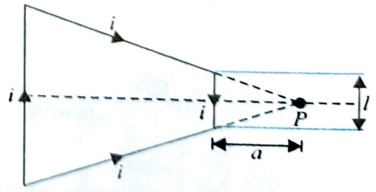
$$\phi_1 = 60°,\ \phi_2 = 30°$$

$$\vec{B}_2 = \frac{10^{-7}\times 2}{1}[\sin 60° + \sin 30°]$$

$$= (\sqrt{3}+1)\times 10^{-7}\ \text{T}$$

ILLUSTRATION 2.9

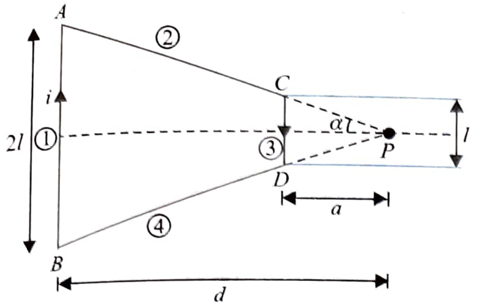
A current I flows in a circuit shaped like an isosceles trapezium. The ratio of the bases is 2. The length of the smaller base is l. Calculate the magnetic induction at point P located in the plane of the trapezium, but at a distance a from the midpoint of the smaller base.



Sol. From similar triangles APF and CPE, we find $d = 2a$ and $\tan\alpha = \frac{1}{2a}$. Magnetic field at P due to wire 1 is

$$B_1 = \frac{\mu_0 I}{4\pi d}[\sin\alpha - \sin(-\alpha)] = \frac{\mu_0 I\sin\alpha}{2\pi d} = \frac{\mu_0 I\sin\alpha}{4\pi a}$$

The direction of the magnetic field is into the plane of the paper. α is the half-angle subtended by the wire 1 at point P and d is the perpendicular distance between the wire and point P.



Magnetic field at P due to wires (2) and (4) is zero.

Magnetic field at P due to wire 3 is

$$B_3 = \frac{\mu_0 I}{4\pi a}[\sin\alpha - \sin(-\alpha)] = \frac{\mu_0 I \sin\alpha}{2\pi a}$$

And the field is directed out of the plane of the paper.

The resultant magnetic field at P is the vector sum of the four fields and is given by

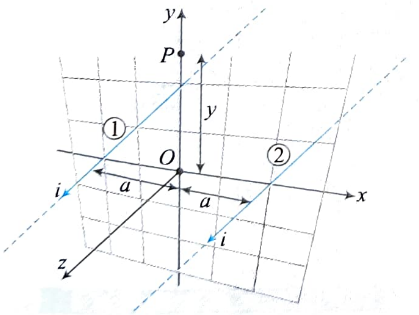
$$B = B_3 - B_1 = \frac{\mu_0 I \sin\alpha}{2\pi a} - \frac{\mu_0 I \sin\alpha}{4\pi a} = \frac{\mu_0 I \sin\alpha}{4\pi a}$$

or $$B = \frac{\mu_0 I}{4\pi a}\left[\frac{l}{\sqrt{l^2+4a^2}}\right]$$

Out of the plane of the paper.

ILLUSTRATION 2.10

In given figure two long wires W_1 and W_2, each carrying current I, both are placed parallel to z-axis. The direction of current in both the wires are same (outward) as shown in figure. Find $\vec{B}$ at P.



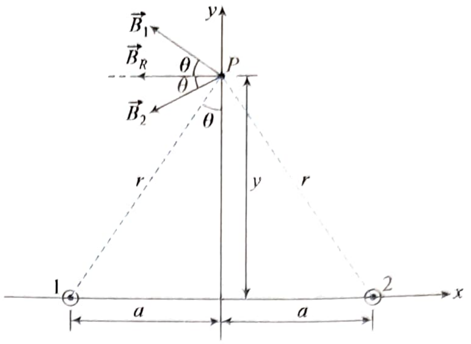
Sol. Point P is situated equidistant from wires 1 and 2. Hence, the magnitude of the magnetic field at P due to both the wires will be the same.

$$|\vec{B}_1| = |\vec{B}_2| = B = \frac{\mu_0 i}{2\pi r} = \frac{\mu_0 i}{2\pi\sqrt{a^2+y^2}} \qquad \text{...(i)}$$

Magnetic field at P: $B_p = 2B\cos\theta$ ...(ii)

The direction of net magnetic field will be along negative x-direction.

From figure, $\cos\theta = \dfrac{y}{r} = \dfrac{y}{\sqrt{a^2+y^2}}$ ...(iii)



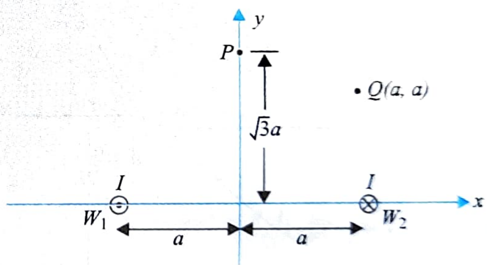
From (i), (ii) and (iii), we get

$$B_p = 2\left(\frac{\mu_0 i}{2\pi\sqrt{a^2+y^2}}\right)\left(\frac{y}{\sqrt{a^2+y^2}}\right) = \frac{\mu_0 iy}{\pi(a^2+y^2)}$$

$$\therefore \quad \vec{B}_P = \frac{\mu_0 iy}{\pi(a^2+y^2)}(-\hat{i})$$

ILLUSTRATION 2.11

In figure, two long wires W_1 and W_2, each carrying current I, are placed parallel to each other and parallel to z-axis. The direction of current in W_1 is outward and in W_2 it is inward. Find $\vec{B}$ at P and Q.



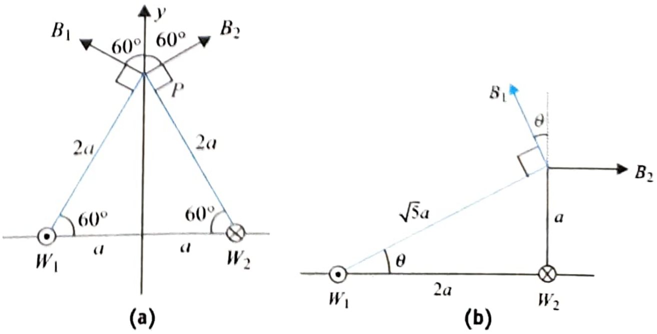
Sol. Magnetic field at P:

Let $\vec{B}$ due to W_1 be $\vec{B}_1$ and due to W_2 be $\vec{B}_2$. By symmetry,

$$|\vec{B}_1| = |\vec{B}_2| = B \text{ [Fig. (a)]}$$

$$\Rightarrow \quad B_p = 2B\cos 60° = B = \frac{\mu_0 I}{2\pi 2a} = \frac{\mu_0 I}{4\pi a}$$

$$\therefore \quad \vec{B}_P = \frac{\mu_0 I}{4\pi a}\hat{j}$$



(a) (b)

For magnetic field at Q [Fig. (b)],

Magnetic field due to W'_1, $B_1 = \frac{\mu_0 I}{2\pi\sqrt{5}a}$

Magnetic field due to W'_2, $B_2 = \frac{\mu_0 I}{2\pi a}$

$$\vec{B}_Q = (B_1\cos\theta\,\hat{j}) + (B_2 - B_1\sin\theta)\,\hat{i}$$

$$\therefore \quad \vec{B}_Q = \frac{\mu_0 I}{5\pi a}\hat{j} + \left(\frac{\mu_0 I}{2\pi a} - \frac{\mu_0 I}{10\pi a}\right)\hat{i}$$

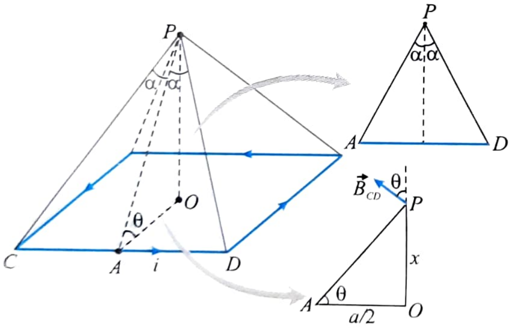
ILLUSTRATION 2.12

A square loop of wire, edge length a, carries a current i. Compute the magnitude of the magnetic field produced at a point on the axis of the loop at a distance x from the center.

Sol. Let us consider magnetic field at point 'P' due to wire CD.

$$AP = \sqrt{x^2 + \frac{a^2}{4}} = \frac{\sqrt{4x^2 + a^2}}{2}$$

$$CP = \sqrt{AC^2 + AP^2} = \sqrt{x^2 + \frac{a^2}{2}} = \frac{\sqrt{4x^2 + 2a^2}}{2}$$



$$\sin\alpha = \frac{AC}{CP} = \frac{a}{\sqrt{4x^2 + 2a^2}}$$

Now $B = \frac{\mu_0 i}{4\pi(AP)} 2\sin\alpha = \frac{\mu_0 i a}{\pi\sqrt{4x^2 + a^2}\sqrt{4x^2 + 2a^2}}$

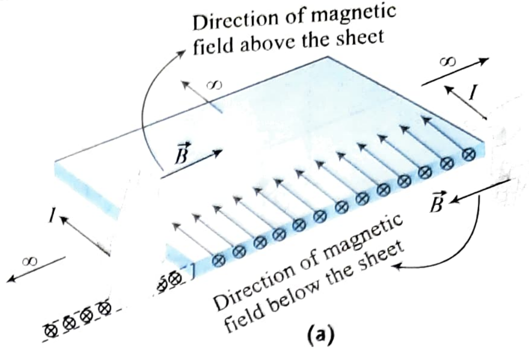
When the fields of all the four sides are considered, the horizontal components add to zero. So, the total field is given by

$$B_p = 4B\cos\theta = \frac{4\mu_0 i a^2}{\pi(4x^2 + a^2)\sqrt{4x^2 + 2a^2}}$$

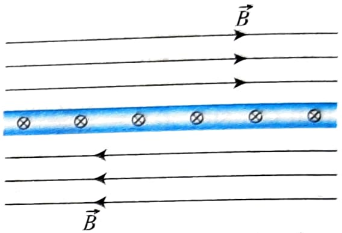
For $x = 0$, the expression reduces to $B_R = \frac{4\mu_0 i a^2}{\pi a^2\sqrt{2a^2}} = \frac{2\sqrt{2}\mu_0 i}{\pi a}$

MAGNETIC INDUCTION DUE TO A LARGE CURRENT CARRYING SHEET

A large metal sheet (infinite sheet of current) can be considered as a group of long current carrying wires placed side by side parallel to each other. The current is distributed along a side with linear current density j ampere per metre. The direction of magnetic field due to this sheet above and below can be find by right hand rule as shown in Figs. (a) and (b).


Direction of magnetic field above the sheet
∞
∞
I
$\vec{B}$
$\vec{B}$
I
∞
Direction of magnetic field below the sheet

(a)



Front view of current carrying sheet

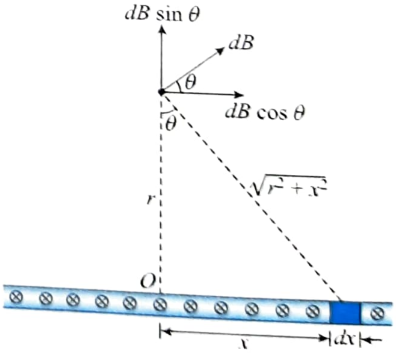
(b)

Let us subdivide the sheet into long, infinitesimal wires of width dx, each of which may be treated as a wire carrying a current element, given by $di = j dx$

To determine the magnitude of magnetic field at a point P located a distance r above the sheet, we consider an elemental wire in the sheet of width dx at a distance x from the base of perpendicular drawn from P on the sheet.

Magnetic field dB due to the elemental wire on sheet at point P is given as

$$dB = \frac{\mu_0 di}{2\pi\sqrt{x^2 + r^2}}$$



The direction of dB is shown in above figure can be verified by right hand rule. From figure we can see that the component of dB normal to the sheet '$dB\sin\theta$' will get cancelled out by the opposite direction component at P due to another elemental wire on sheet on the other side of point O at the same distance x. Only the component of $d\vec{B}$ — namely, '$dB\cos\theta$' along the surface of sheet is effective. This component will be added up due to all the elemental wires on the sheet, thus net magnetic induction at point P due to whole sheet will be given by integrating $dB\cos\theta$ for the whole length of sheet from $-\infty$ to $+\infty$.

Hence, $B = \int dB\cos\theta$

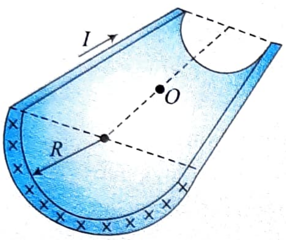
$$\Rightarrow \quad B=\int_{-\infty}^{+\infty}\frac{\mu_0(idx)}{2\pi\sqrt{x^2+r^2}}\cdot\frac{r}{\sqrt{x^2+r^2}}=\frac{\mu_0 ir}{2\pi}\int_{-\infty}^{+\infty}\frac{dx}{(x^2+r^2)}$$

$$\Rightarrow \quad B=\frac{\mu_0 ir}{2\pi}\left[\frac{1}{r}\tan^{-1}\left(\frac{x}{r}\right)\right]_{-\infty}^{+\infty}=\frac{\mu_0 i}{2\pi}\left[\left(\frac{\pi}{2}\right)-\left(\frac{\pi}{2}\right)\right]$$

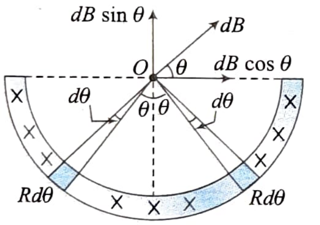
$$\Rightarrow \quad B=\frac{1}{2}\mu_0 i$$

ILLUSTRATION 2.13

A current i flows in a long straight wire with cross-section having the form of a thin semicircular-ring of radius R as shown in figure. Find the magnetic field at the point O, situated on the axis of the wire.



Sol. Here we can subdivide the semicircular cross-section into infinitesimal filaments of width dx. Let us consider an elemental wire of thickness dx at angular position θ as shown in the figure. The current flowing through the elemental wire along the length of the wire,



Current in the elemental wire is given as

$$di=\frac{i}{\pi R}dx=\frac{i}{\pi R}Rd\theta=\frac{i}{\pi}d\theta$$

The magnetic induction due to a long wire can be used to calculate the magnetic induction due to this elemental wire at point O which is given as

$$dB=\frac{\mu_0 di}{2\pi R}=\frac{\mu_0 id\theta}{2\pi^2 R} \quad \text{...(i)}$$

All the components $dB\sin\theta$ will be cancelled out due to the corresponding elemental wires on the other half of the cross section as these are in opposite directions and the components $dB\cos\theta$ due to all the elemental wires are in same directions so will be added up. Thus net magnetic induction due to the total current in the complete wire with the cross section shown is given by integrating $dB\cos\theta$ which is given as

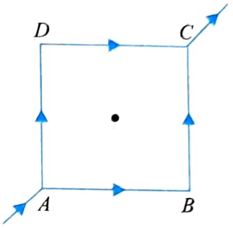
$$B_O=\int dB\cos\theta$$

$$\Rightarrow \quad B_O=2\int_0^{\pi/2}\left(\frac{\mu_0 id\theta}{2\pi^2 R}\right)\cos\theta=\frac{\mu_0 i}{\pi^2 R}[\sin\theta]_0^{\pi/2}$$

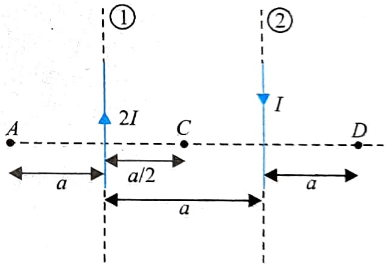
$$\Rightarrow \quad B_O=\frac{\mu_0 i}{\pi^2 R}[1-(0)]=\frac{\mu_0 i}{\pi^2 R}$$

CONCEPT APPLICATION EXERCISE 2.1

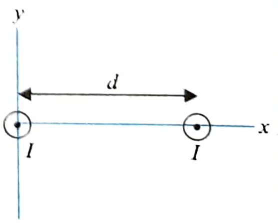
1. A +6 μC point charge is moving at a constant velocity of $8\times10^6\ \text{ms}^{-1}$ in the $+y$ direction, relative to a reference frame. At the instant when the point charge is at the origin of this reference frame, what is the magnetic field vector it produces at the following points?
   (a) $x=0.500$ m, $y=0$, $z=0$, and
   (b) $x=0$, $y=-0.500$ m, $z=0$.
2. Figure shows a square loop made from a uniform wire. Find the magnetic field at the center of the square if a battery is connected between points A and C.



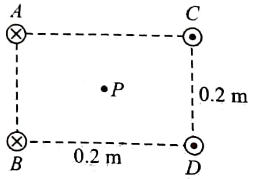
3. In figure, there are two parallel long wires (placed in the plane of paper) carrying currents $2I$ and I. Consider points A, C, and D on the line perpendicular to both the wires and also in the plane of the paper. The distances are mentioned. Find **(a)** $\vec{B}$ at A, C and D **(b)** position of points on line ACD where $\vec{B}$ is 0.



4. An infinitely long straight wire with current I flowing along the positive z-axis is located at coordinates $P(-12\text{ m}, 5\text{ m})$. Determine the unit vector showing the direction of magnetic field at the origin.
5. A long, vertical wire carrying a current of 10 A in the upward direction is placed in a region where a horizontal magnetic field of magnitude 2.0×10^{-3} T exists from south to north. Find the point where the resultant magnetic field is zero.
6. Let two long parallel wires, a distance d apart, carry equal currents I in the same direction. One wire is at $x=0$, the other at $x=d$ (as shown in figure). Determine magnetic field at any point on x-axis between the wires as a function of x.



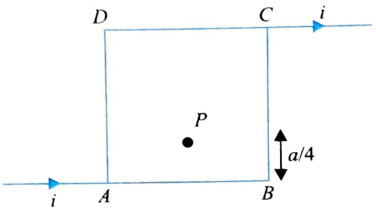
7. Four long, parallel conductors carry equal currents of 5.0 A. The direction of the currents is into the page at points A and B and out of the page at



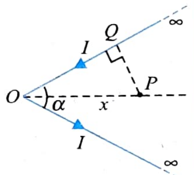
C and D. Calculate the magnitude and direction of the magnetic field at point P, located at the center of the square.

8. An infinitely long, non-conducting cylinder of radius R lies along the z-axis. Five long, conducting wires are parallel to the cylinder and spaced equally on the upper half of its surface. Each wire carries a current I in the opposite z-direction. Find the magnetic field on the z-axis.

9. Figure shows a square loop of side a made of a uniform wire. A current i enters the loop at the point A and leaves it at point C. Find the magnetic field at point P which is on the perpendicular bisector of AB at a distance $a/4$ from it.

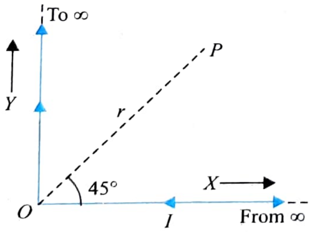


10. Find magnetic field at point P shown in figure, point P is on the bisector of angle between the wires.

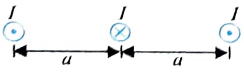


11. Two long, straight wires, one above the other, are separated by a distance $2a$ and are parallel to the x-axis. Let the y-axis be in the plane of the wires in the direction from the lower wire to the upper wire. Each wire carries current I in the $+x$-direction. What are the magnitude and direction of the net magnetic field of the two wires at a point in the plane of wires:
   (a) midway between them?
   (b) at a distance a above the upper wire?
   (c) at a distance a below the lower wire?

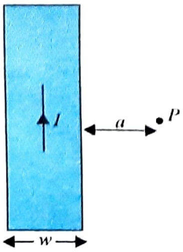
12. Current I flows through a long conducting wire bent at right angle as shown in figure. Find the magnetic field at point P on the right bisector of the angle XOY at distance r from O.



13. Shown in figure, is an end-on view of three long, straight, parallel conductors spaced equal distances a apart. The outer conductors carrry current I out of the page; the middle conductor carries current I into the page. Where in the plane of the page is the magnetic field zero?



14. In figure, a large metal sheet of width w carries a current I (uniformly distributed in its width w). Find the magnetic field at point P which lies on the plane of the sheet.



ANSWERS

1. (a) $-(1.92\times10^{-5}\ \mathrm{T})\hat{k}$ (b) 0 **2.** 0

3. (a) $\vec{B}$ at $A=\frac{3}{4}\cdot\frac{\mu_0 I}{\pi a}$; $\vec{B}$ at $C=\frac{3\mu_0 I}{\pi a}$; $\vec{B}$ at $D=0$

(b) at Point D

4. $\frac{1}{13}(5\hat{i}+12\hat{j})$

5. At $r=1$ mm to the left of the wire in the indicated yz-plane.

6. $\frac{\mu_0 i(d-2x)}{2\pi x(d-x)}\hat{j}$ **7.** $2\times10^{-5}T(-\hat{j})$, into the page

8. $\frac{\mu_0 I}{2\pi r}(1+\sqrt{2})\hat{i}$ **9.** $\frac{2\mu_0 i}{\pi a}\left(\frac{1}{\sqrt{5}}-\frac{1}{3\sqrt{13}}\right)$

10. $\frac{\mu_0 I}{2\pi x\sin\left(\frac{\alpha}{2}\right)}\left[1+\cos\left(\frac{\alpha}{2}\right)\right]$

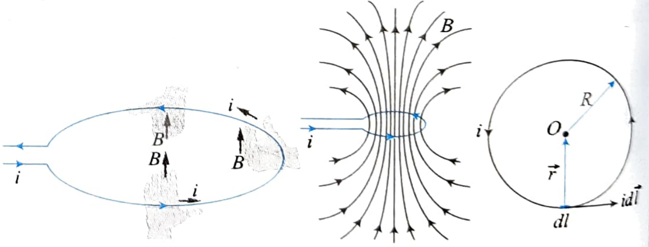
11. (a) 0, opposite direction (b) $\frac{2\mu_0 I}{3\pi a}\hat{k}$, same direction

(c) $-\frac{2\mu_0 I}{2\pi a}\hat{k}$, opposite direction

12. $\frac{\mu_0}{2\pi}\frac{I}{r}(\sqrt{2}+1)$ **13.** $y=\pm a$ **14.** $\frac{\mu_0 I}{2\pi w}\ln\left(\frac{a+w}{a}\right)$

MAGNETIC FIELD AT THE CENTER OF A CIRCULAR COIL

Figure (a) shows a circular loop of radius R carrying a current I. We can apply Biot and Savart law for finding the magnetic field at the center of the loop. Let us consider an element of length dl on its circumference, we can find the direction of magnetic induction dB at the center of coildue to this element by using the right-hand rule.We can observe that the direction of magnetic induction dB at the center of coil is in outward direction as shown in Fig. (b).



(a) (b) (c)

The magnetic field due to the element dl, $d\vec{B}=\frac{\mu_0 i\vec{dl}\times\vec{r}}{4\pi r^3}$

Here, $dB=\frac{\mu_0}{4\pi}\frac{idlR\sin 90°}{R^3}$ (as $i\vec{dl}\perp\vec{r}$)

or, $dB=\frac{\mu_0}{4\pi}\frac{idl\sin(90°)}{R^2}\Rightarrow dB=\frac{\mu_0 idl}{4\pi R^2}$ ...(i)

We can observe in Fig. (a) the direction of magnetic field due to all the current elements of the loop are in same direction at every interior point of the coil and the magnetic lines are making closed loops from outside of the loop as shown. It means net magnetic field at center of loop can be given by integrating Eq. (i) for the complete length of the loop which is given as

$$B=\int dB=\frac{\mu_0 i}{4\pi R^2}\int_0^{2\pi R} dl$$

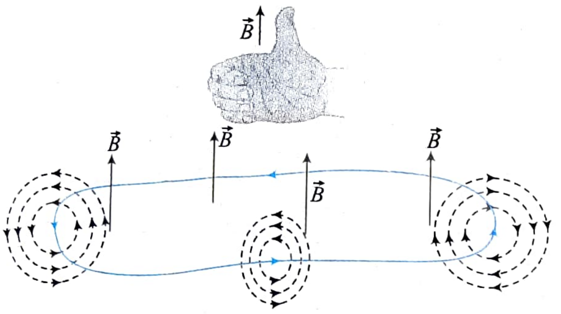
$$\Rightarrow \quad B=\frac{\mu_0 i}{4\pi R^2}[2\pi R-0] \Rightarrow B=\frac{\mu_0 i}{2R} \qquad \text{...(ii)}$$

If there are N turns of wire in the loop, then the magnetic induction at the center of coil is given as

$$B=\frac{\mu_0 iN}{2R} \qquad \text{...(iii)}$$

Important Points:

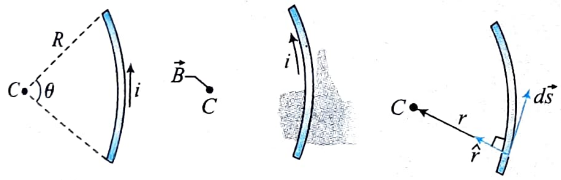
- We can find the direction of magnetic field for any closed current carrying loop of any shape in a plane of the loop by using right hand thumb rule in a different way. For this we circulate our right-hand fingers along the direction of current in the loop as shown in figure then the direction of our right hand thumb gives the direction of magnetic induction at interior points in the plane of the loop.



- The magnetic field at the centre of a circular arc can be written as

$$B=\frac{\mu_0 i}{2R}\left(\frac{\theta}{2\pi}\right)=\frac{\mu_0 i}{4\pi R}\theta$$

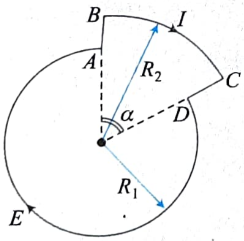
And for direction of the magnetic field we use right hand rule.



(a) (b) (c)

ILLUSTRATION 2.14

In the given network of wires shown in figure, a current i is allowed to flow. Calculate the magnetic field at the centre O. If angle α is equal to 90°, then what will be the value of magnetic induction at O?



Sol. Magnetic field at O due to the circular segment BC is

$$B_1=\frac{\mu_0 i}{4\pi R_2}\alpha\otimes$$

Similarly, the magnetic field at O due to circular segment AED is

$$B_2=\frac{\mu_0 i}{4\pi R_1}(2\pi-\alpha)\otimes$$

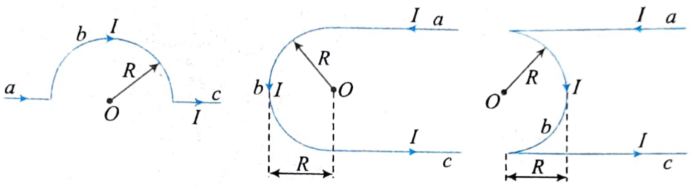
Magnetic field due to segments AB and CD is zero, because point 'O' lies on the direction of these parts.

Hence resultant magnetic induction at O is

$$\vec{B}=\vec{B}_1+\vec{B}_2=\frac{\mu_0 i}{4\pi}\left(\frac{\alpha}{R_2}+\frac{2\pi-\alpha}{R_1}\right),\otimes$$

ILLUSTRATION 2.15

Calculate the field at the centre of a semi-circular wire of radius R in situations given in Figs. (i), (ii) and (iii) if the straight wire is of infinite length.



(i) (ii) (iii)

Sol. We know

- The magnetic field due to a straight current carrying wire of semi-infinite wire length, for a point at the end of wire at a distance R perpendicular to its length is $\frac{\mu_0 I}{4\pi R}$.
- The magnetic field is zero if the point under consideration is lying along its length.
- The magnetic field at the center of semicircular wire is $\frac{\mu_0 I}{4\pi R}(\pi)=\frac{\mu_0 I}{4R}$

So net magnetic field at O in given cases should be,

$$\vec{B}_R=\vec{B}_a+\vec{B}_b+\vec{B}_c$$

(i) In case (i), the magnetic field due to straight parts should be zero $\vec{B}_R=0+\frac{\mu_0}{4}\frac{I}{R}\otimes+0=\frac{\mu_0 I}{4R}\otimes$ (into the page)

(ii) In case (ii), the magnetic field due to upper and lower semi-in finite segment will be $\vec{B}_a=\vec{B}_c=\frac{\mu_0}{4\pi}\frac{I}{R}\odot$ (Direction can be found out by right hand rule)

The magnetic field at the center of semicircular wire is $\vec{B}_b=\frac{\mu_0 I}{4R}\odot$

Hence,

$$\vec{B}_R=\frac{\mu_0}{4\pi}\frac{I}{R}\odot+\frac{\mu_0}{4}\frac{I}{R}\odot+\frac{\mu_0}{4\pi}\frac{I}{R}\odot=\frac{\mu_0}{4\pi}\frac{I}{R}[\pi+2]\odot \text{ (out of the page)}$$

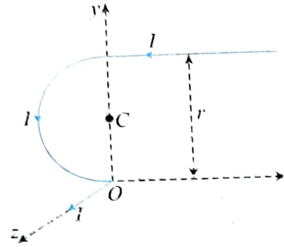
(iii) Similarly the magnetic field in case (iii) should be

$$\vec{B}_R=\frac{\mu_0}{4\pi}\frac{I}{R}\odot+\frac{\mu_0}{4}\frac{I}{R}\otimes+\frac{\mu_0}{4\pi}\frac{I}{R}\odot=\frac{\mu_0}{4\pi}\frac{I}{R}[\pi-2]\otimes$$

(into the page)

ILLUSTRATION 2.16

An infinitely long conductor as shown in figure carrying a current I with a semicircular loop on X-Y plane and two straight parts, one parallel to x-axis and another coinciding with Z-axis. What is the magnetic field at the centre C of the semi-circular loop?



Sol. The magnetic field at C due to semi-infinite wire parallel to X-axis is

$$\vec{B}_1 = \frac{\mu_0}{4\pi}\frac{I}{(r/2)}(\hat{k}) = \frac{\mu_0}{2\pi}\frac{I}{r}(\hat{k})$$

For finding the direction, we use right hand rule.

The magnetic field induction at C due to current through the semi-circular loop in X-Y plane is

$$B_2 = \frac{\mu_0}{4\pi}\frac{I}{(r/2)}(\pi) = \frac{\mu_0}{2}\frac{I}{r} \text{ acting along } +Z\text{-direction, i.e.}$$

$$\vec{B}_2 = \frac{\mu_0 I}{2r}\hat{k}$$

The magnetic field induction at C due to current through the straight part of the conductor coinciding with Z-axis is

$$B_3 = \frac{\mu_0}{4\pi}\frac{I}{r/2}\left[\sin\frac{\pi}{2} + \sin 0\right] = \frac{\mu_0 I}{2\pi r} \text{ acting along } (-X)\text{-axis}$$

i.e. $\vec{B}_3 = \frac{\mu_0}{2\pi}\frac{I}{r}(-\hat{i})$

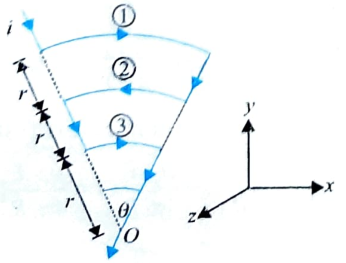
Total magnetic field induction at C is

$$\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 = \frac{\mu_0 I}{2\pi r}\hat{k} + \frac{\mu_0 I}{2\pi}\hat{k} - \frac{\mu_0 I}{2\pi r}\hat{i}$$

$$\Rightarrow \quad \vec{B} = \frac{\mu_0 I}{2\pi r}[(1+\pi)\hat{k} - \hat{i}]$$

ILLUSTRATION 2.17

Shown in figure is a conductor carrying current I. Find the magnetic field intensity at point O.



Sol. The magnetic field at the center of an arc is equal to

$$B = \frac{\mu_0 I}{4\pi r}\theta$$

Magnetic field due to arc (1), $B_1 = \frac{\mu_0 I\theta}{4\pi \times 3r}(-\hat{k})$

Magnetic field due to arc (2), $B_2 = \frac{\mu_0 I\theta}{4\pi \times 2r}(\hat{k})$

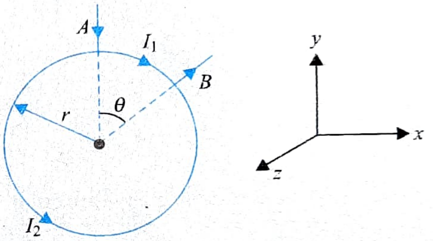
Magnetic field due to arc (3), $B_3 = \frac{\mu_0 I\theta}{4\pi r}(-\hat{k})$

Net magnetic field, $B = B_1 + B_2 + B_3$

Hence, net $B = \frac{\mu_0 I}{4\pi}\left[-\frac{1}{r} + \frac{1}{2r} - \frac{1}{3r}\right]\theta(\hat{k}) = -\frac{5\mu_0 I\theta}{24\pi r}\hat{k}$

ILLUSTRATION 2.18

A battery is connected between two points A and B on the circumference of a uniform conducting ring of radius r and resistance R as shown in figure. One of the arcs AB of the ring subtends angle θ at the center. Show that the magnetic field at the center of the coil is zero and independent of θ.



Sol. Magnetic field at the center of an arc is given by

$$B = \frac{\mu_0 I}{2R} \times \frac{\theta}{2\pi}$$

Magnetic field due to a smaller arc, $\vec{B}_1 = \frac{\mu_0 I_1}{2r} \times \frac{\theta}{2\pi}(-\hat{k})$

Magnetic field due to larger arc, $\vec{B}_2 = \frac{\mu_0 I_2}{2r} \times \frac{(2\pi-\theta)}{2\pi}(+\hat{k})$

$$\text{Resultant magnetic field} = \left[-\frac{\mu_0 I_1\theta}{4\pi r} + \frac{\mu_0 I_2(2\pi-\theta)}{4\pi r}\right](\hat{k}) \quad \text{...(i)}$$

Two arcs form a parallel combination of resistors.

$$\text{Here } I_1 R_1 = I_2 R_2 \Rightarrow \frac{I_1}{I_2} = \frac{R_2}{R_1} \quad \text{...(ii)}$$

Here R_1 and R_2 are the resistances of smaller and bigger arcs respectively.

$$\text{But } \frac{R_2}{R_1} = \frac{l_2}{l_1} = \frac{(2\pi-\theta)r}{\theta r} = \frac{2\pi-\theta}{\theta} \quad \text{...(iii)}$$

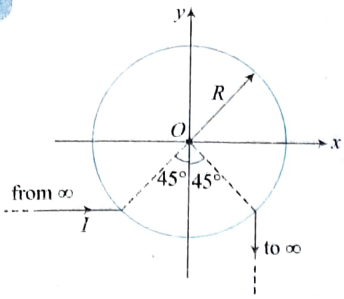
From Eqs. (ii) and (iii), we get

$$I_1\theta = I_2(2\pi - \theta) \quad \text{...(iv)}$$

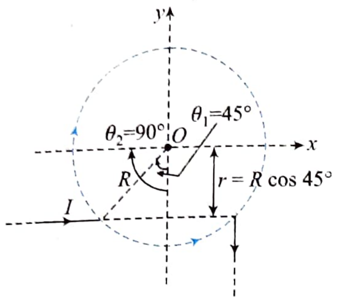
Hence, from Eqs. (i) and (iv), the net magnetic field at the center of ring will be zero.

ILLUSTRATION 2.19

What is the magnitude of magnetic field at the center O of loop of radius R made of uniform wire when a current of I enters in the loop and taken out of it by two long wires as shown in the figure?



Sol. Let us consider a wire parallel to x-axis.



Magnetic field due to wire at O,

$$B_1 = \frac{\mu_0 I}{4\pi r}[\sin\theta_2 - \sin\theta_1]$$

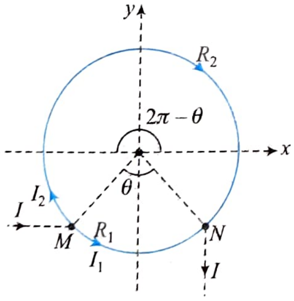
$$= \frac{\mu_0 I}{4\pi(R\cos 45°)}[\sin 90° - \sin 45°] \quad \text{...(i)}$$

$$= \frac{\mu_0 I}{4\pi(R/\sqrt{2})}\left[1 - \frac{1}{\sqrt{2}}\right] = \frac{\mu_0 I}{4\pi R}[\sqrt{2}-1]$$

From right hand rule, the direction of magnetic field is away from reader or along $+z$ direction.

$$\vec{B}_1 = \frac{\mu_0 I}{4\pi R}(\sqrt{2}-1)(\hat{k}) \quad \text{...(ii)}$$

Now consider circular loop: Here current enters at M and leaves at N. The current entering at M divides into two parts.



We can observe both the circular sections are connected in parallel combination. Hence the currents in these reactions

$$\frac{I_1}{I_2} = \frac{R_2}{R_1} = \frac{\rho \cdot R(2\pi - \theta)}{\rho R\theta} = \frac{2\pi - \theta}{\theta}$$

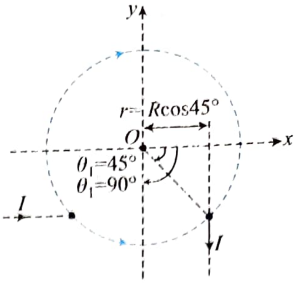
or $\quad I_1\theta = I_2(2\pi - \theta)$

The magnetic field at the centre of the circular loop

$$\vec{B}_2 = \frac{\mu_0 I_1 \theta}{4\pi R}(\hat{k}) + \frac{\mu_0 I_2(2\pi - \theta)}{4\pi R}(-\hat{k}) = 0$$

$$[\text{as } I_1\theta = I_2(2\pi - \theta)]$$

Now considering the wire parallel to y-axis



$$\vec{B}_3 = \frac{\mu_0 I}{4\pi(R\cos 45°)}[\sin 90° - \sin 45°](-\hat{k})$$

$$= \frac{\mu_0 I}{4\pi R}[\sqrt{2}-1](-\hat{k})$$

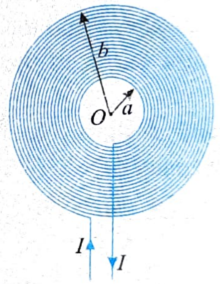
Hence net magnetic field at O

$$\vec{B}_{\text{net}} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3$$

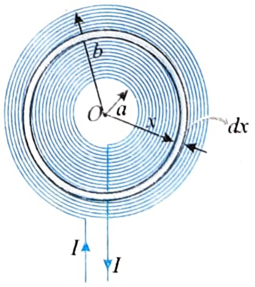
$$= \frac{\mu_0 I}{4\pi R}[\sqrt{2}-1](\hat{k}) + 0 + \frac{\mu_0 I}{4\pi R}[\sqrt{2}-1](-\hat{k}) = 0$$

ILLUSTRATION 2.20

A thin insulated wire forms a plane spiral of $N = 100$ turns carrying a current $i = 8$ mA. The inner and outer radii are equal to $a = 5$ cm and $b = 10$ cm. Find the magnetic induction at the center of the spiral.



Sol. Let n be the number of turns per unit length along the radii of the spiral. Consider a ring of radii x and $x + dx$.



Number of turns in the ring $= n\,dx$

$\therefore$ Magnetic field at the center due to this ring $= \dfrac{\mu_0(n\,dx)i}{2x}$

$$\therefore B \text{ (total field)} = \int_a^b \frac{\mu_0 ni\,dx}{2x} = \frac{1}{2}\mu_0\, ni \ln\frac{b}{a}$$

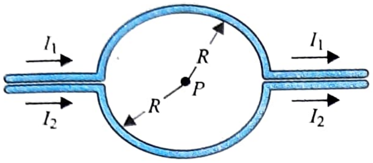
But $\quad n = \dfrac{N}{b-a}$

$$B = \frac{\mu_0 N i}{2(b-a)} \ln\frac{b}{a} = \frac{4\pi\times10^{-7}\times100\times8\times10^{-3}}{2(10-5)\times10^{-2}} \ln\frac{10}{5}$$

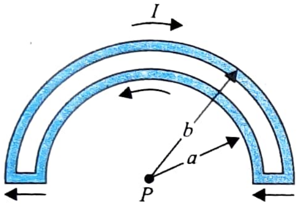
$$= 6.9\times10^{-6}\text{ T} = 6.9\ \mu\text{T}$$

CONCEPT APPLICATION EXERCISE 2.2

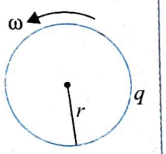
1. Calculate the magnitude of the magnetic field at point P as shown in figure, in terms of R, I_1, and I_2.



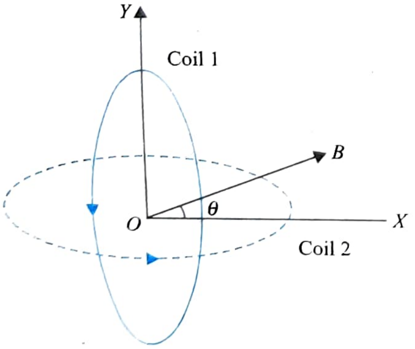
2. Two semicircles shown in figure, have radii a and b. Calculate the net magnetic field (magnitude and direction) that the current in the wires produces at point P.



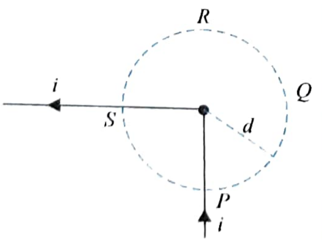
3. A conducting ring of radius r having charge q is rotating with angular velocity ω about its axes. Find the magnetic field at the center of the ring.



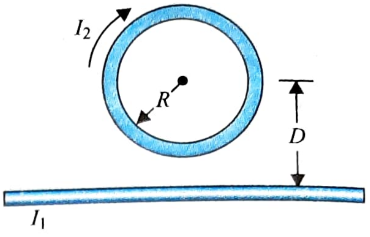
4. For the arrangement shown in figure, determine the magnetic field at center O.



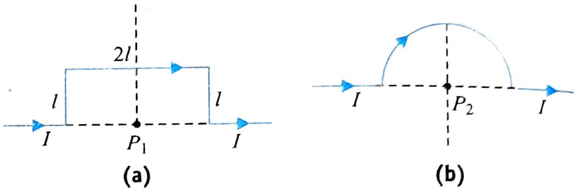
5. Figure shows a long wire bent at the middle to form a right angle. Show that the magnitudes of the magnetic field at points P, Q, R, and S are equal and find this magnitude.



6. A circular loop has radius R and carries current I_2 in a clockwise direction (as shown in figure). The center of the loop is at a distance D above a long, straight wire. What are the magnitude and direction of the current I_l in the wire if the magnetic field at the center of the loop is zero?

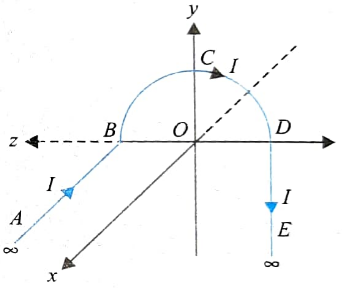


7. A wire is bent into the shape shown in Fig. (a), and the magnetic field is measured at P_1 when the current in the wire is I. The same wire is then formed into the shape shown in Fig. (b), and the magnetic field is measured at point P_2 when the current is again I. If the total length of wire is the same in each case, what is the ratio of B_1/B_2?

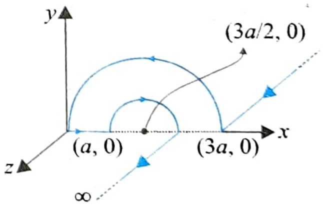


(a) (b)

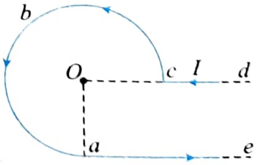
8. Find magnetic field at O by the system of current carrying wire.



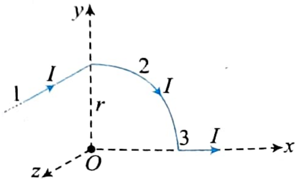
9. In figure, find the magnetic field at common centre.



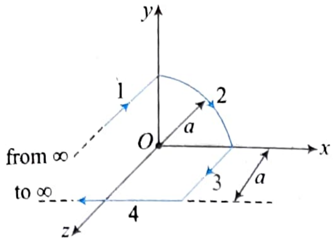
10. Calculate the magnetic induction at the point O, if the current carrying wire is in the shape shown in figure. The radius of the curved part of the wire is a and linear parts are assumed to be very long and parallel.



11. A conductor carrying a current i is bent as shown in figure. Find the magnitude of magnetic field at the origin.



12. A long current carrying wire having a current i is bent into four parts 1, 2, 3 and 4 as shown in the figure. Find the induction at O.



ANSWERS

1. $\frac{1}{2}\left(\frac{\mu_0}{2R}\right)|I_1 - I_2|$ **2.** $\frac{\mu_0 I}{4a}\left(1-\frac{a}{b}\right)$, out of the page

3. $\frac{\mu_0 \omega q}{4\pi r}$ **4.** $\frac{\mu_0}{2R}\sqrt{(i_1^2+i_2^2)}$ **5.** $\frac{\mu_0 i}{4\pi d}$

6. $\frac{\pi D}{R}I_2$, point to the right **7.** $\frac{4\sqrt{2}}{\pi^2}$

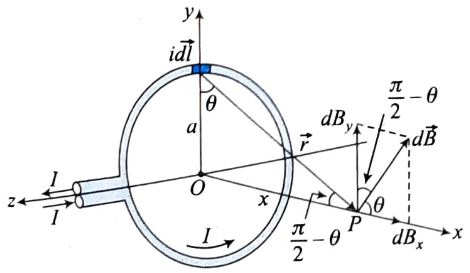
8. $\vec{B}_0 = \left(\frac{\mu_0 I}{4\pi R}\right)(-\hat{j}) + \left[\frac{\mu_0 I}{4R} + \frac{\mu_0 I}{4\pi R}\right](-\hat{i})$

9. $\frac{\mu_0 i}{3a}\left[\frac{2}{\pi}\hat{j} - \hat{k}\right]$ **10.** $\frac{\mu_0 i}{4\pi a}\left(\frac{3\pi}{2}+1\right)$ **11.** $\frac{-\mu_0 i}{4r}\left[\frac{\hat{i}}{\pi}+\frac{\hat{k}}{2}\right]$

12. $\frac{\mu_0 i}{4\pi a}\left[-\hat{i} - (\sqrt{2}+1)\hat{j} - \left(\frac{\pi}{2}\right)\hat{k}\right]$

MAGNETIC FIELD OF A CIRCULAR CURRENT LOOP

A circular current carrying loop of radius R is placed on y-z plane and we are interested to find magnetic field at a point P located on the axis of loop as shown in figure. Let us consider a current element $id\vec{l}$ thatcauses the field $d\vec{B}$, which lies in the x-y plane. The situation has rotational symmetry about the x-axis, so there cannot be a component of the total field $\vec{B}$ perpendicular to this axis. For every current element $id\vec{l}$, there is a corresponding element on the opposite side of the loop, with opposite direction. These two elements give equal contributions to the x-component of $d\vec{B}$, but opposite components perpendicular to the x-axis. Thus, all the perpendicular components cancel out and only the x-components survive.



We can use the law of Biot and Savart to find the magnetic field at point P on the axis of the loop, at a distance x from the center. As figure shows, $id\vec{l}$ and $\vec{r}$ are perpendicular and the direction of field $d\vec{B}$ caused by this particular current element $id\vec{l}$ lies in the x-y plane. Since $r^2 = x^2 + a^2$, the magnitude dB of the field due to this current element $id\vec{l}$ is

$$dB = \frac{\mu_0 I}{4\pi}\frac{dl}{(x^2+a^2)} \qquad \text{...(i)}$$

x component of the vector $d\vec{B}$ is

$$dB_x = dB\cos\theta = \frac{\mu_0 I}{4\pi}\frac{dl}{(x^2+a^2)}\frac{a}{(x^2+a^2)^{1/2}} \qquad \text{...(ii)}$$

To obtain the x-component of the total field $\vec{B}$ we integrate Eq. (ii). Everything in this expression except dl is constant and can be taken outside the integral. So, we have

$$B_x = \int \frac{\mu_0 I}{4\pi}\frac{a\,dl}{(x^2+a^2)^{3/2}} = \frac{\mu_0 I a}{4\pi(x^2+a^2)^{3/2}}\int dl$$

The integral of dl is just the circumference of the circle, i.e., $\int dl = 2\pi a$. So, we finally get

$$B_x = \frac{\mu_0 I a^2}{2(x^2+a^2)^{3/2}} \text{ (on the axis of a circular loop)} \qquad \text{..(iii)}$$

Now suppose that instead of the single loop, we have a coil consisting of N loops, all with the same radius. The loops are closely spaced so that the plane of each loop is essentially the same distance x from the field point P. Each loop contributes equally to the field, and the total field is N times the field of a single loop.

$$B_x = \frac{\mu_0 N I a^2}{2(x^2+a^2)^{3/2}} \text{ (on the axis of } N \text{ circular loops)}$$

If $x >> a$, then $B = \frac{\mu_0 I a^2}{2x^3} = \frac{\mu_0}{2\pi}\frac{I\pi a^2}{x^3} = \frac{\mu_0}{4\pi}\frac{I.2\pi a^2}{x^3}$

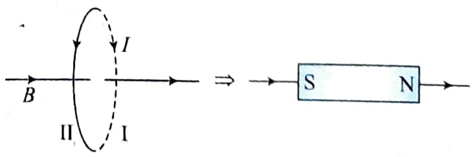
But $\pi a^2 = A =$ Area of cross-section of the coil.

Thus, $B = \frac{\mu_0}{4\pi}\frac{2IA}{x^3} = \frac{\mu_0 2M}{4\pi x^3}$ or, $B = \frac{\mu_0}{4\pi}\frac{2M}{x^3}$

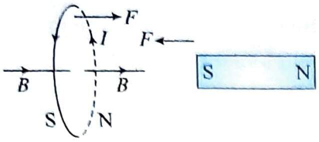
where $\vec{M} = I\vec{A} =$ Magnetic dipole moment of the loop. The direction of $\vec{M}$ is same as the direction of the normal to the area of the loop.

Important Points:

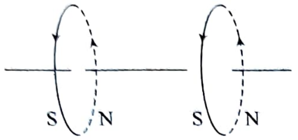
- **A loop as a magnet:** The pattern of the magnetic field is comparable with the magnetic field produced by a bar magnet.



- The side I (the side from which $\vec{B}$ energes out) of the loop acts as "north pole" and side II (the side in which $\vec{B}$ enters) acts as the "south pole". It can be verified by studying force on one loop due to a magnet or a loop.



The loop and the magnet attract each other.

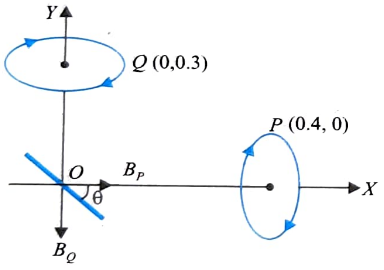


The two loops attract each other.

ILLUSTRATION 2.21

Centers of two similar coils P and Q having same number of turns are located at the coordinates (0.4, 0) and (0, 0.3) such that the plane of coils are perpendicular to X- and Y-axes respectively. The areas of cross section of coils P and Q are in the ratio 4 : 3. Coil P has 16 A current in clockwise direction and coil Q has $9\sqrt{3}$ A current in anticlockwise direction as seen from the origin. A small compass needle is placed at the origin. Find the deflection in the needle, assuming the earth's magnetic field negligible and the radii of the coils very small compared to their distances from the origin.

Sol. We know that the magnetic field on the axis of a current carrying coil of radius r and number of turns N at a distance x from the center of coil is given by



$$B = \frac{\mu_0 N i r^2}{2(r^2+x^2)^{3/2}} = \frac{\mu_0 N i(\pi r^2)}{2\pi(r^2+x^2)^{3/2}} = \frac{\mu_0 N i A}{2\pi(r^2+x^2)^{3/2}}$$

where A = Area of the coil.

When $r << x$, then $B = \dfrac{\mu_0 N i A}{2\pi x^3}$

Now magnetic field at O due to coil P is

$$B_P = \frac{\mu_0 N i_P A_P}{2\pi x^3} \qquad \text{(along } OX\text{)}$$

Similarly, $B_Q = \dfrac{\mu_0 N i_Q A_Q}{2\pi y^3}$ (along YO)

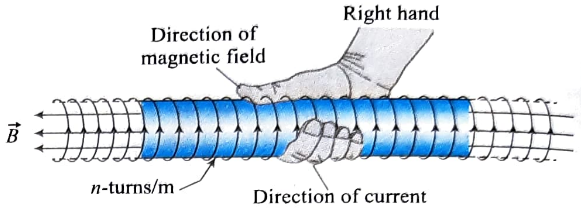
Suppose the compass needle makes an angle θ with x-axis, then

$$\tan\theta = \frac{B_Q}{B_P} = \left(\frac{i_Q}{i_P}\right)\left(\frac{A_Q}{A_P}\right)\left(\frac{x^3}{y^3}\right) = \left(\frac{9\sqrt{3}}{16}\right)\left(\frac{3}{4}\right)\left(\frac{0.4}{0.3}\right)^3 = \sqrt{3}$$

$$\Rightarrow \quad \theta = \tan^{-1}\sqrt{3} = 60°$$

FINDING MAGNETIC FIELD INSIDE A LONG SOLENOID

A solenoid is a long coil of wire wrapped in many turns. When a current pass through it, it creates a nearly uniform magnetic field inside. Let a long tightly wound solenoid with core radius R and number of turns per unit length of solenoid are n, carries a current I. We can consider the solenoid made of a number of circular coils placed coaxially. For finding the direction of magnetic field inside the core we use right hand rule as shown in figure.

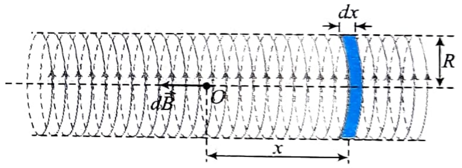


For finding the magnitude of the magnetic field at the axial point of the solenoid, let us consider an elemental ring of width dx on the solenoid core at a distance x from an axial point O as shown in figure.

Number of turns in this elemental ring, $n' = n\,dx$

Due to this elemental ring, magnetic field at point O,

$$dB = \frac{\mu_0 (ndx) IR^2}{2(R^2+x^2)^{3/2}} \qquad \text{...(i)}$$



Net magnetic induction at point O due to complete solenoid can be calculated by integrating above Eq. (i) within limits from $-\infty$ to $+\infty$ which is given as

$$B = \int dB = \int_{-\infty}^{+\infty} \frac{\mu_0 (ndx) IR^2}{2(R^2+x^2)^{3/2}}$$

$$\Rightarrow \quad B = \frac{\mu_0 n I R^2}{2} \int_{-\infty}^{+\infty} \frac{dx}{(R^2+x^2)^{3/2}} \qquad \text{...(ii)}$$

For solving the above integration, we substitute

$x = R\tan\theta$ and $dx = R\sec^2\theta\, d\theta$

The revised limits of integration will be,

when $x = -\infty$ then $\theta = -\dfrac{\pi}{2}$ and when $x = +\infty$ then $\theta = +\dfrac{\pi}{2}$

Now Eq. (ii) will become,

$$B = \frac{\mu_0 n I R^2}{2} \int_{-\pi/2}^{+\pi/2} \frac{R\sec^2\theta\, d\theta}{(R^2 + R^2\tan^2\theta)^{3/2}}$$

$$= \frac{\mu_0 n I R^2}{2} \int_{-\pi/2}^{+\pi/2} \frac{R\sec^2\theta\, d\theta}{R^3\sec^3\theta}$$

$$\text{Or,} \quad B = \frac{\mu_0 n I}{2} \int_{-\pi/2}^{+\pi/2} \cos\theta\, d\theta = \frac{\mu_0 n I}{2} [\sin\theta]_{-\pi/2}^{+\pi/2}$$

$$\Rightarrow \quad B = \frac{1}{2}\mu_0 n I[1-(-1)] = \mu_0 n I \qquad \text{...(iii)}$$

Above expression given in Eq. (iii) is valid only for the case when the solenoid is very long and tightly wound on its core. If there is a gap between turns of solenoid then these turns cannot be considered as circular coils using which above result is derived.

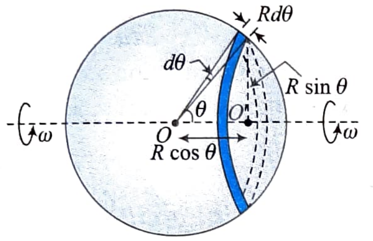
ILLUSTRATION 2.22

A non-conducting sphere of radius R having surface density σ uniformly on the surface, rotates with an angular velocity ω about a diametrical axis passing through its center. Find the magnetic field due to the rotating charge at the center of the sphere.

Sol. We can consider the charged sphere made of a number of coaxial rings, all are rotating with same angular velocity ω. In the given figure, we consider an elemental ring at an angular position θ from the axis of rotation having angular width $d\theta$.

The surface area of the elemental ring , $dA = 2\pi R\sin\theta \times Rd\theta$

The charge on the elemental ring surface, $dq = \sigma dA$



Current due to rotating charge on the elemental ring,

$$di = \frac{dq\omega}{2\pi} \Rightarrow di = \frac{(2\pi\sigma R^2 \sin\theta d\theta)\omega}{2\pi}$$

$$\Rightarrow \quad di = \sigma\omega R^2 \sin\theta\, d\theta$$

Now point O is lying on the axis of this current carrying elemental ring.

Magnetic field at the center O due to the current in elemental ring.

$$dB = \frac{\mu_0 di(R\sin\theta)^2}{2(R^2\cos^2\theta + R^2\sin^2\theta)^{3/2}} \Rightarrow dB = \frac{\mu_0 di \sin^2\theta}{2R} \quad ...(\text{i})$$

Substituting the value of current di in Eq. (i) we get

$$dB = \frac{\mu_0(\sigma\omega R^2 \sin\theta d\theta)\sin^2\theta}{2R}$$

$$\Rightarrow \quad dB = \frac{1}{2}\mu_0\sigma\omega R\sin^3\theta d\theta \quad ...(\text{ii})$$

Net magnetic induction at O is given by integrating Eq. (ii) within limits of θ from 0 to π is given as

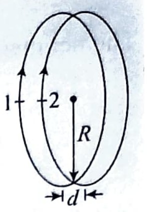
$$B = \int dB = \frac{1}{2}\mu_0\sigma\omega R\int_0^{\pi} \sin^3\theta d\theta$$

$$\Rightarrow \quad B = \frac{1}{2}\mu_0\sigma\omega R\left[\frac{1}{3}\cos^3\theta - \cos\theta\right]_0^{\pi}$$

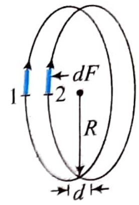
$$\Rightarrow \quad B = \frac{1}{2}\mu_0\sigma\omega R\left[\frac{1}{3}(-1) - (-1) - \frac{1}{3}(1) + (1)\right]_0^{\pi}$$

$$\Rightarrow \quad B = \frac{2}{3}\mu_0\sigma\omega R$$

ILLUSTRATION 2.23



Two identical circular loops each of radius r carrying currents i_1 and i_2 are placed co-axially at a small distance of separation d. Find the force of interaction between them.



Sol. Let consider a current element i_2dl on loop 2.The force on this current element due to loop 2 is

$$dF_2 = i_2 dlB_1 \quad ...(\text{i})$$

B_1 is the magnetic field at the position of this current element due to loop 1. Since, the loops are very close, we can treat the wire of loop 1 near this current element as infinite.

It means $B_1 = \dfrac{\mu_0 i_1}{2\pi d}$

$$\text{Hence, } dF_2 = i_2 dl\left(\frac{\mu_0 i_1}{2\pi d}\right) \quad ...(\text{ii})$$

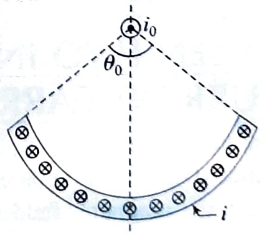
For finding net force on loop 2, integrating Eq. (ii)

$$\text{or} \quad F_2 = \int i_2 dl\left(\frac{\mu_0 i_1}{2\pi d}\right) = \int_0^{2\pi R} i_2 \frac{\mu_0 i_1}{2\pi d} dl$$

$$= \frac{\mu_0 i_1 i_2}{2\pi d}\int_0^{2\pi R} dl = \frac{\mu_0 i_1 i_2}{2\pi d} 2\pi R = \frac{\mu_0 i_1 i_2 R}{d}$$

ILLUSTRATION 2.24

A long straight rigid wire carrying a current is kept co-axially with the current carrying long thin plane which is bent in the form of a circular arc. If the inward current i flows through the plate, find the magnetic force acting on the wire.



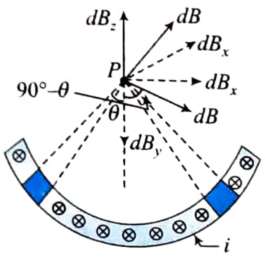
Sol. To find the magnetic field at P let us take the field due to two symmetrical elementary segments of the cross-section of the tube which behave as two long straight current carrying conductors 1 and 2. Since, the vertical components of dB at P due to 1 and 2 will nullify each other, their net field is

$$2dB_x = 2dB\sin(90° - \theta) = 2dB\cos\theta \text{ where } dB = \frac{\mu_0 di}{2\pi R}$$

$$\text{or} \quad 2dB_x = \frac{2\mu_0}{2\pi R}di\cos\theta$$

$$\text{where } di = \frac{i}{R\theta_0}\cdot Rd\theta = \frac{id\theta}{\theta_0}$$

$$\text{or} \quad 2dB_x = \frac{\mu_0 i}{\pi\theta_0}\cos\theta\, d\theta$$



The net magnetic field is $B = \dfrac{\mu_0 i}{\pi\theta_0}\int_0^{\theta_0/2}\cos\theta\, d\theta$

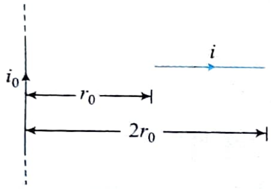
or $$B = \frac{\mu_0 i \sin\dfrac{\theta_0}{2}}{\pi\theta_0}$$

Then, the magnetic (Ampere's) force is

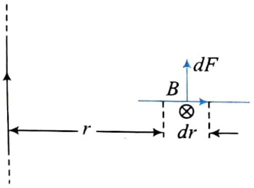
$$F = i_0 lB = \frac{i_0\left(\mu_0 i \sin\dfrac{\theta_0}{2}\right)}{\pi\theta_0} \text{ or } F = \frac{\mu_0 i i_0}{\pi\theta_0}\sin\frac{\theta_0}{2}$$

ILLUSTRATION 2.25

A straight conductor of length r_0 carrying a current i is placed perpendicular to a long straight conductor which carries a current i_0 as shown in the figure. Find the force of interaction between them.



Sol. The force on the current carrying element idl of the conductor is



$$dF = idl\ B\sin 90°$$

$$= idr\,\frac{\mu_0 i_0}{2\pi r} = \frac{\mu_0 i i_0}{2\pi}\frac{dr}{r}$$

The net force is, $F = \int dF$

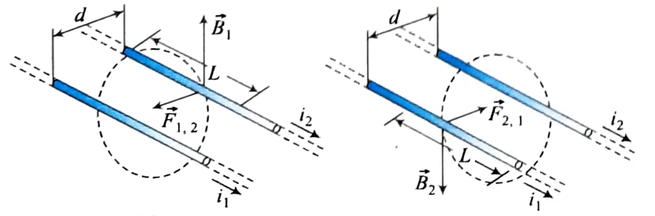
$$= \int_{r_0}^{2r_0}\frac{\mu_0 i i_0}{2\pi r}\,dr = \frac{\mu_0 i i_0}{2\pi}\int_{r_0}^{2r_0}\frac{dr}{r} = \frac{\mu_0 i i_0}{2\pi r}\ln 2$$

FORCE BETWEEN TWO INFINITE PARALLEL CURRENT-CARRYING WIRES

Let two infinite parallel wires carrying currents i_1 and i_2 be separated by a distance d. The two wires exert magnetic forces on each other because the magnetic field of one wire exerts a force on second current carrying wire. Let's first consider wire 1 carrying a current, i_1 towards the right, as shown in Fig. (a). The magnitude of the magnetic field at a perpendicular distance d from wire 1 is

$$B_1 = \frac{\mu_0 i_1}{2\pi d} \quad \text{...(i)}$$

This magnetic field is always perpendicular to the wire with a direction given by right-hand rule.



(a) (b)

Now consider wire 2 carrying a current, i_2, in the same direction as i_1, and placed parallel to wire 1 at a distance d from it [Fig. (b)]. The magnetic field due to wire 1 exerts a magnetic force on the moving charges in the current flowing in wire 2. The magnitude of the magnetic force on a length, L, of wire 2 is then

$$F = iLB\sin\theta = i_2 LB_1 \quad \text{...(ii)}$$

because $\vec{B}_1$ is perpendicular to wire 2 and thus $\theta = 90°$. Substituting for B_1 from Eq. (i) into Eq. (ii), we find the magnitude of the force exerted by wire 1 on a length L of wire 2:

$$F_{1\to2} = i_2 L\left(\frac{\mu_0 i_1}{2\pi d}\right) = \frac{\mu_0 i_1 i_2 L}{2\pi d} \quad \text{...(iii)}$$

According to right-hand rule 1, $\vec{F}_{1\to2}$ points toward wire 1 and is perpendicular to both wires. An analogous calculation allows us to deduce that the force from wire 2 on a length, L, of wire 1 has the same magnitude and opposite direction: $\vec{F}_{2\to1} = \vec{F}_{1\to2}$. This result is a simple consequence of Newton's Third Law.

Note: Force per unit length $= \dfrac{\mu_0 i_1 i_2}{2\pi d}$. We note that the wires carrying current in the same direction attract. The wires carrying current in the opposite direction will repel each other.

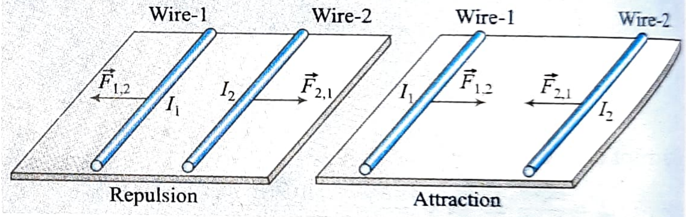
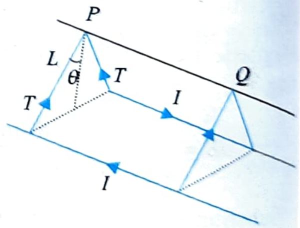


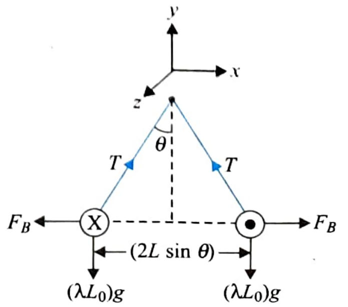
ILLUSTRATION 2.26

Two long parallel wires carry currents of equal magnitude but in opposite directions. These wires are suspended from rod PQ by four chords of same length L as shown in figure. The mass per unit length of the wires is λ. Determine the value of θ assuming it to be small.



Sol. The force per unit length between current carrying parallel wires is $\dfrac{dF}{dL} = \dfrac{\mu_0 I_1 I_2}{2\pi d}$.

If two wires carry current in opposite directions, the magnetic force is repulsive, due to which the parallel wires in figure have moved out so that equilibrium is reached.



The figure shows free body diagram of each wire. In equilibrium,

$$\Sigma F_y = 0,\ 2T\cos\theta = (\lambda L_0)g \quad \text{...(i)}$$

$$\Sigma F_z = 0,\ 2T\sin\theta = F_B \quad \text{...(ii)}$$

Now dividing Eq. (ii) by (i), we get $\tan\theta = \dfrac{F_B}{\lambda L_0 g}$ ...(iii)

The magnetic force $F_B = \left(\dfrac{dF}{dL}\right)\times L_0 = \dfrac{\mu_0 I^2}{4\pi(\sin\theta)L} L_0$ ...(iv)

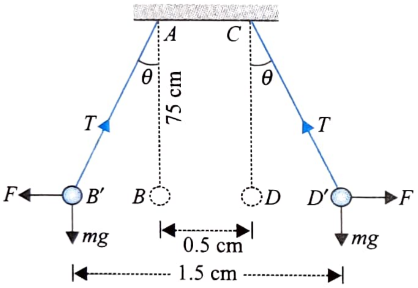
For small θ, $\tan\theta \simeq \sin\theta \simeq \theta$

On substituting Eq. (iv) in (iii), we get $\theta = I\sqrt{\dfrac{\mu_0}{4\pi\lambda g L}}$

ILLUSTRATION 2.27

Two parallel horizontal conductors are suspended by two light vertical threads each 75 cm long. Each conductor has a mass of 40 g m^{-1}, and when there is no current they are 0.5 cm apart. Equal current in the two wires result in a separation of 1.5 cm. Find the values and directions of currents. Take $g = 9.8$ ms^{-2}.

Sol. The situation is shown in figure.



Here, we have $T\cos\theta = mg$ ...(i)

$$T\sin\theta = F = \frac{\mu_0}{4\pi} l \frac{2i_1 i_2}{d}$$

or $\quad T\sin\theta = \dfrac{\mu_0}{4\pi} l \dfrac{2i^2}{d}$ ...(ii)

From Eqs. (i) and (ii), $\tan\theta = \dfrac{\mu_0}{4\pi} l \dfrac{2i^2}{d} \cdot \dfrac{1}{mg}$ ...(iii)

where θ is small, $\tan\theta \approx \sin\theta$

From the figure, $\sin\theta = \dfrac{0.5\times10^{-2}}{75\times10^{-2}}$

and $m = 40.0\times10^{-3} l$ kg

where l = length of conductor in metre

Subsituting in Eq. (iii), we get

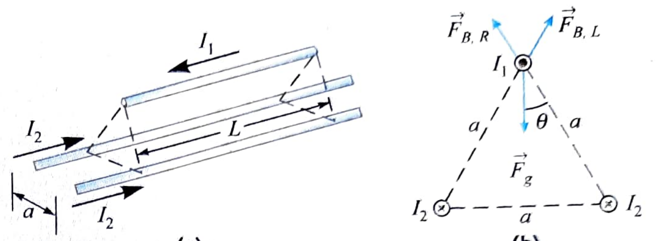
$$\frac{0.5\times10^{-2}}{75\times10^{-2}} = 10^{-7} l \frac{2i^2}{(1.5\times10^{-2})} \times \frac{1}{(40\times10^{-3})l\times9.8}$$

Solving, we get $i = 14$ A

For repulsion, the currents are in opposite directions.

ILLUSTRATION 2.28

Two infinitely long, parallel wires are lying on the ground at distance a = 1 cm apart as shown in Fig. (a). A third wire, of length L = 10 m and mass 400 g, carries a current of I_1 = 100 A and is levitated above the first two wires, at a horizontal position midway between them. The infinitely long wires carry equal currents I_1 in the same direction, but in the direction opposite that in the levitated wire. What current must the infinitely long wires carry so that the three wires form an equilateral triangle?



(a) (b)

Sol. Since the current in the short wire is opposite to those in the long wires, the short wire is repelled from both of the others. Imagine the currents in the long wires in Fig. (a) are increased. The repulsive force becomes stronger, and the levitated wire rises to the point at which the wire is once again levitated in equilibrium at a higher position. Figure (b) shows the desired situation with the three wires forming an equilateral triangle.

Because the levitated wire is subject to forces but does not accelerate, it is modeled as a particle in equilibrium.

The horizontal components of the magnetic forces on the levitated wire cancel. The vertical components are both positive and added together. Choose the z-axis to be upward through the top wire in Fig. (b) and in the plane of the page.

The total magnetic force in the upward direction on the levitated wire:

$$\vec{F}_B = 2\left(\frac{\mu_0 I_1 I_2}{2\pi a}\right)\cos\theta\,\hat{k} = \frac{\mu_0 I_1 I_2}{2\pi a} l\cos\theta\,\hat{k}$$

The gravitational force on the levitated wire:

$$\vec{F}_g = -mg\hat{k}$$

Apply the particle in equilibrium model by adding the forces and setting the net force equal to zero:

$$\sum\vec{F} = \vec{F}_B + F_g = \frac{\mu_0 I_1 I_2}{\pi a} l\cos\theta\,\hat{k} - mg\hat{k} = 0$$

Solve for the current in the wires on the ground:

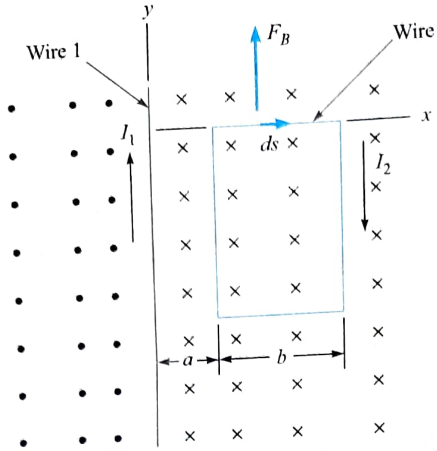
$$I_2 = \frac{mg\pi a}{\mu_0 I_1 l\cos\theta}$$

Substitute numerical values:

$$I_2 = \frac{(0.400\text{ kg})(10\text{ m s}^{-2})\pi(0.0100\text{ m})}{(4\pi\times10^{-7}\text{ T}\cdot\text{m A}^{-2})(100\text{ A})(10\text{ m})\cos30^\circ} = 113\text{ A}$$

ILLUSTRATION 2.29

Wire 1 in figure is oriented along the y-axis and carries a steady current I_1. A rectangular loop located to the right of the wire and in the x-y plane carries a current I_1. Find the magnetic force exerted by wire (1) on the top wire of length b in the loop, labeled “wire (2)” in the figure.



Sol. Consider the force exerted by wire 1 on a small segment ds of wire (2). This force is given by $d\vec{F}_B = I d\vec{s} \times \vec{B}$, where $I = I_2$ and $\vec{B}$ is the magnetic field created by the current in wire (1) at the position of $d\vec{s}$. The field at a distance x from wire (1) is

$$B = \frac{\mu_0 I_1}{2\pi x}(-\hat{k})$$

where the unit vector $-\hat{k}$ is used to indicate that the field due to the current in wire (1) at the position of $d\vec{s}$ points into the page. Because wire (2) is along the x-axis, $d\vec{s} = dx\hat{i}$, we find that

$$dF_B = \frac{\mu_0 I_1 I_2}{2\pi x}[\hat{i} \times (-\hat{k})]dx = \frac{\mu_0 I_1 I_2}{2\pi}\frac{dx}{x}\hat{j}$$

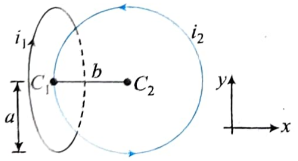
Integrating over the limits $x = a$ to $x = a + b$ gives

$$F_B = \frac{\mu_0 I_1 I_2}{2\pi}\ln x\Big]_a^{a+b}\hat{j} = \frac{\mu_0 I_1 I_2}{2\pi}\ln\left(1+\frac{b}{a}\right)\hat{j}$$

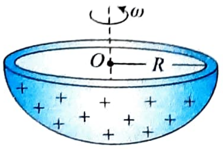
The force on wire (2) points in the positive y-direction, as indicated by the unit vector $\hat{j}$ and as shown in the figure.

CONCEPT APPLICATION EXERCISE 2.3

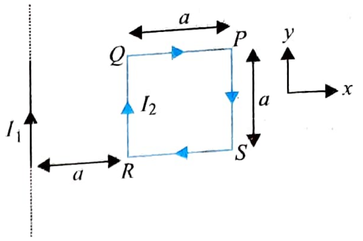
1. Two perpendicular circular loops of radii a and b carry currents i_1 and i_2 respectively. Find the magnetic field at C.



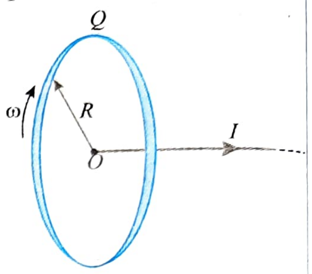
2. A thin hemispherical shell having a uniform surface charge density σ spins with an angular speed ω. Find the magnetic field at O.



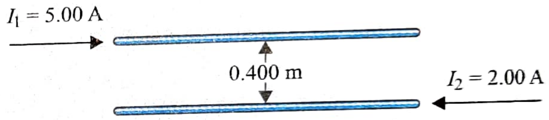
3. Find the magnetic force on loop $PQRS$ due to the wire.



4. A charge Q is uniformly distributed over a ring which is rotating with constant angular velocity ω about an axis passing through its center and perpendicular to the plane. A wire which carries a current I is lying perpendicular to the plane of the ring along its axis having one end at its center. Find resultant magnetic force on the wire by the ring.



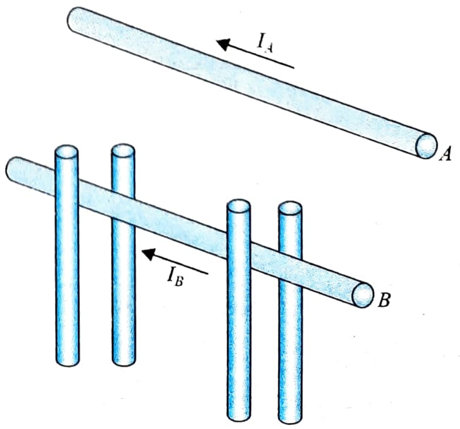
5. Two long, parallel wires are separated by a distance of 0.400 m (as shown in figure). The currents I_1 and I_2 have the directions shown.



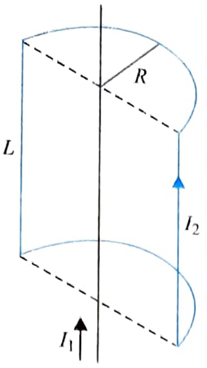
   (a) Calculate the magnitude of the force exerted by each wire on a 1.20 m length of the other. Is the force attractive or repulsive?

   (b) Each current is doubled, so that I_1 becomes 10.0 A and I_2 becomes 4.00 A. Now, what is the magnitude of the force that each wire exerts on a 1.20 m length of the other?

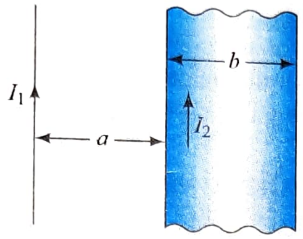
6. Two long, parallel conductors carry currents in the same direction as shown in figure. Conductor A carries a current of 150 A and is held firmly in position. Conductor B carries a current I_B and is allowed to slide freely up and down (parallel to A) between a set of non-conducting guides. If the mass per unit length of conductor B is 0.100 g cm^{-1}, what value of current I_B will result in equilibrium when the distance between the two conductors is 2.50 cm?



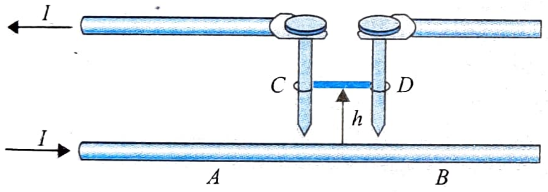
7. An infinitely long straight wire carrying a current I_1 is partially surrounded by a loop as shown in figure. The loop has a length L, radius R, and carries a current I_2. The axis of the loop coincides with the wire. Calculate the force exerted on the loop.



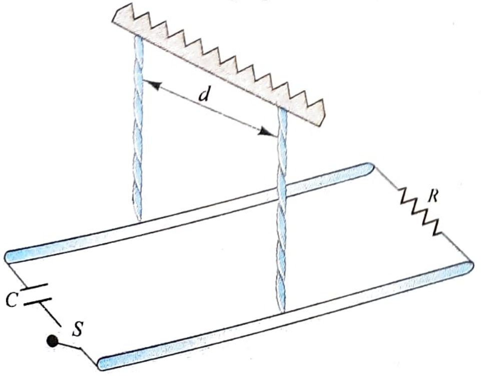
8. A long straight conductor carrying current I_1 is placed in the plane of a ribbon carrying current I_2 parallel to the previous one. The width of the ribbon is b and the straight conductor is at a distance a from the near edge. Find the force of attraction between the two.



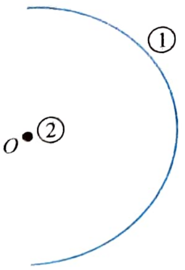
9. A long, horizontal wire AB rests on the surface of a table and carries a current I. Horizontal wire CD is vertically above wire AB, and is free to slide up and down on the two vertical metal guides C and D (as shown in figure). Wire CD is connected through the sliding contacts to another wire that also carries a current I, opposite in direction to the current in wire AB. The mass per unit length of the wire CD is λ. To what equilibrium height h will the wire CD rise, assuming that magnetic force on it is wholly due to the current in wire AB?



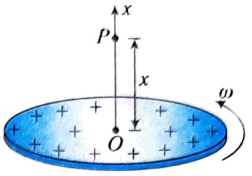
10. Two long conducting rods suspended by means of two insulating threads as shown in figure are connected at one end to a charged capacitor through a switch S, which initially open. At the other end, they are connected by a loose wire. The capacitor has charge Q and mass per unit length of the rod is λ. The effective resistance of the circuit after closing the switch is R. Find the velocity of each rod when the capacitor is discharged after closing the switch. (Assume that the displacement of rods during the discharging time is negligible.)



11. A direct current I flows in a long straight conductor whose cross section has the form of a thin half-ring of radius R. The same current flows in the opposite direction along a thin conductor located on the axis of the first conductor. Find the magnetic induction force between the given conductors reduced to a unit of their length.



12. A thin circular plate of uniform surface charge density σ spins about its axis with an angular speed ω. Find the magnetic field at an axial point P of the spinning plate.



ANSWERS

1. $\frac{\mu_0}{2}\left[-\frac{i_1 a^2}{(a^2+b^2)^{3/2}}\hat{i}+\frac{i_2}{b}\hat{k}\right]$

2. $\frac{2}{3}\mu_0\sigma\omega$

3. $\frac{\mu_0 I_1 I_2}{4\pi}(-\hat{i})$

4. 0

5. (a) 6×10^{-6} N, repulsive (b) 2.4×10^{-5} N

6. $\frac{250}{3}$ A

7. $\frac{\mu_0 I_1 I_2}{\pi R}$

8. $\frac{\mu_0 I_1 I_2}{2\pi b}\ln\frac{a+b}{a}$

9. $h=\frac{\mu_0 I^2}{2\pi\lambda g}$

10. $v=\frac{\mu_0 Q^2}{4\pi dRC\lambda l}$

11. $\frac{\mu_0 I^2}{\pi^2 R}$

12. $\frac{\mu_0\sigma\omega x}{2}\left[\frac{2x^2+R^2}{\sqrt{R^2+x^2}}-2x\right]$

AMPERE'S LAW

Calculating the electric field resulting from a continuous distribution of electric charge may require evaluating a difficult integral. However, if the charge distribution has cylindrical, spherical, or planar symmetry, we can apply Gauss's Law and obtain the electric field in an elegant manner. Similarly, calculating the magnetic field due to an arbitrary distribution of current elements using the Biot-Savart Law may involve the evaluation of a difficult integral. Alternatively, we can avoid using the Biot-Savart Law and instead apply Ampere's Law to calculate the magnetic field from a distribution of current elements when the distribution has cylindrical or other symmetry. Often, problems can be solved with much less effort in this way than by using a direct integration.It relates the line integral of magnetic field along a given closed path and the currents which are enclosed by that closed path. The law is stated as

"Line integral of magnetic induction along a closed path is equal to μ_0 times the total current coming out and enclosed by that closed path."

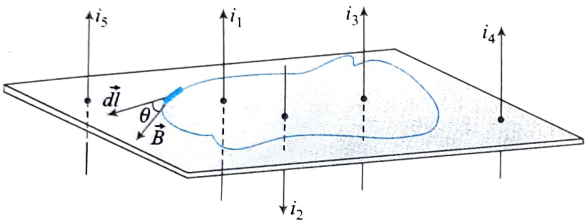
The mathematical statement of Amperes Law is,

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 i_{\text{enc.}} \qquad \text{...(i)}$$

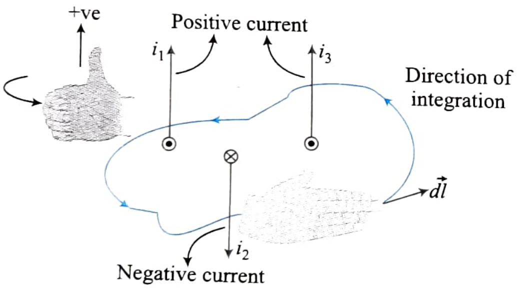
The symbol $\oint$ means that the integrand, $\vec{B} \cdot d\vec{l}$, is integrated over a closed loop, called an Amperian loop. This loop is chosen so that the integral in Eq. (i) is not difficult to evaluate, a procedure similar to that used in applying Gauss's Law. The total current enclosed in this loop is i_{enc}, which is also similar to Gauss's Law, where the chosen closed surface encloses a total net charge.

As an example of how Ampere's Law is used, consider the five currents shown in Fig. (a) below, which are all perpendicular to the plane. An Amperian loop, represented by the blue line, encloses currents i_1, i_2, and $i_3$and excludes currents i_4 and i_5. By Ampere's Law, the closed-loop integral over the magnetic field resulting from these three currents is given by

$$\oint \vec{B} \cdot d\vec{l} = \oint B\cos\theta dl = \mu_0(i_1 - i_2 + i_3) \qquad \text{...(ii)}$$



(a)



Top view of ampere loop

(b)

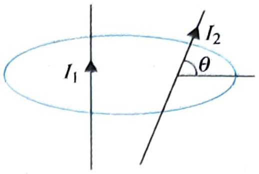
where θ is the angle between the direction of the magnetic field, $\vec{B}$, and the direction of the element of length, $d\vec{l}$ at each point along the Amperian loop. The integration over the Amperian loop can be done in either direction. Figure (a) indicates a direction of integration from the direction of $d\vec{l}$, along with the resulting magnetic field.

In Eq. (ii), the sign of the contributing currents can be determined using a right-hand rule. Here right hand fingers are curled along the direction of line integral on the path and thumb gives the positive direction or the currents in the direction of thumb are taken positive and opposite ones are taken negative as shown in Fig. (b).

Here it is to be noted that in Eq. (ii), the calculation of the quantities on right side is simple, can be calculated adding and subtracting the currents, but the integral $\oint B\cos\theta \cdot dl$ cannot be easily evaluated. However, let's examine some special situations in which Amperian loops contain symmetrical distributions of current that can be exploited to carry out the integral.

ILLUSTRATION 2.30

Figure shows two current carrying wires piercing the plane of an imaginary loop. One of the wires is normal to the plane of the loop while the other is at an angle. Calculate the net current enclosed.



Sol. The current enclosed is simply $I_1 + I_2$. There is no need to take any component of I_2 perpendicular to the plane of the loop. All that matters is just the magnitude of the net charge per unit time that crosses the plane of the loop.

GUIDELINES FOR APPLYING AMPERE'S LAW

Just as in the case of Gauss's law, the selection of the Amperian loop is critical for easy application of the law. The general rules while selecting the loop are:

- Estimate the direction of the magnetic field around the current carrying conductor.
- The loop should include the current carrying element whose magnetic field is to be calculated.
- The magnetic field should be constant over the whole loop or should be easy to calculate on the different portions of the loop.
- The dot product $\oint \vec{B} \cdot d\vec{l}$ should be easy to calculate.

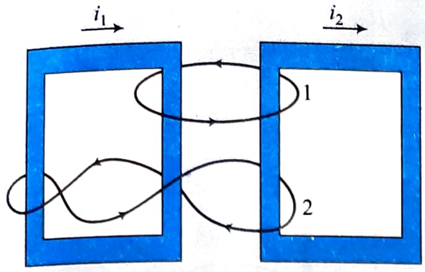
Note:

- If B is everywhere tangent to the integration path and has the same magnitude B at every point on the path, then its line integral is equal to B multiplied by the circumference of the path.
- If B is everywhere perpendicular to the path, for all or some portion of the path, that portion of the path makes no contribution to the line integral.
- In the integral $\oint \vec{B} \cdot d\vec{l}$, $\vec{B}$ is always the total magnetic field at each point on the path. In general, this field is caused partly by currents enclosed by the path and partly by currents outside. Even when no current is enclosed by the path, the field at points on the path need not be zero. In that case, however, $\oint \vec{B} \cdot d\vec{l}$ is always zero.
- Some judgment is required in choosing an integration path. Two useful guiding principles are that the point or points at which the field is to be determined must lie on the path, and that the path must have enough symmetry so that the integral can be evaluated.

ILLUSTRATION 2.31

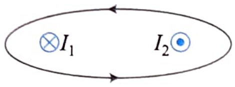
In the figure shown, two closed paths wrapped around two conducting loops carrying currents i_1 =8.0 A and i_2 =5.0 A.

What is the value of the integral $\oint \vec{B}\cdot d\vec{l}$ for (i) path 1 and (ii) path 2?



Sol. We use Ampere's law: $\oint \vec{B}\cdot d\vec{l} = \mu_0 i_{\text{enclosed}}$ where the integral is around a closed loop and i_{enclosed} is the net current through the loop.

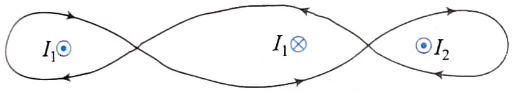
(i) For path 1, if we use right hand rule the current i_1 will be negative while current i_2 will be positive.



Hence, $\oint \vec{B}\cdot d\vec{l} = \mu_0(-8.0\text{A} + 5.0)$

$$\oint \vec{B}\cdot d\vec{l} = (4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(-3.0\text{A}) = -1.2\pi\times10^{-6}\,\text{T}\cdot\text{m}.$$

(ii) For path 2,

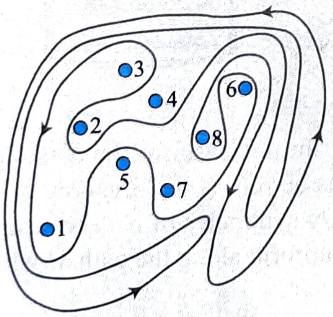


we find, $\oint \vec{B}\cdot d\vec{l} = \mu_0(-8.0\text{A} - 8.0\text{A} - 5.0\text{A})$

$$= (4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(-21.0\text{A}) = -8.4\pi\times10^{-6}\,\text{T}\cdot\text{m}.$$

ILLUSTRATION 2.32

In the figure shown, eight wires cut the page perpendicularly at the points. A wire labeled with the integer n (n = 1, 2,..., 8) carries the current ni, where i = 4.0 mA. For those wires with odd n, the current is out of the page; for those with even n, it is into the page. Evaluate $\oint \vec{B}\cdot d\vec{l}$ along the closed path indicated and in the direction shown.



Sol. A close look at the path reveals that only currents 1, 3, 6 and 7 are enclosed. Thus, noting the different current directions described in the problem. Applying right hand rule, the currents numbered 1, 3 and 7 are positive while the current numbered 6 will be negative. Now applying Ampere's law.

$$\oint \vec{B}\cdot d\vec{l} = \mu_0(7i - 6i + 3i + i) = 5\mu_0 i$$

$$= 5(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(4.0\times10^{-3}\,\text{A})$$

$$= 8.0\pi\times10^{-9}\,\text{T}\cdot\text{m}.$$

MAGNETIC INDUCTION DUE TO A CYLINDRICAL WIRE

Outside the Wire

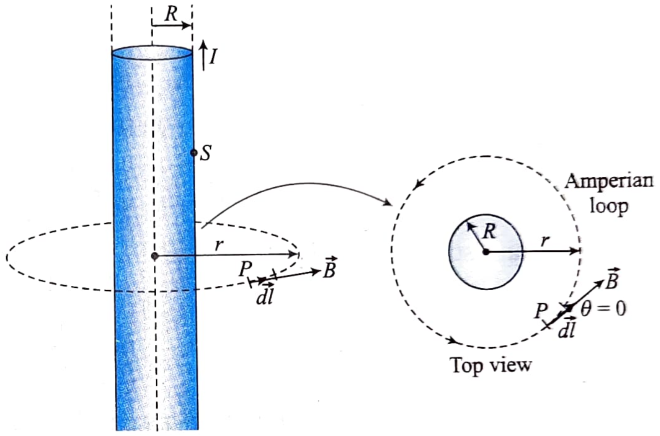
Figure shows a long straight cylindrical wire of radius R carrying a current I. To determine the magnetic induction at outside point P located a distance $r(r > R)$ from its axis we consider a circular Ampere's path of radius r on which applying Ampere's law gives

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{enclosed}}$$

$$\Rightarrow \quad \oint B\,dl\cos(0°) = \mu_0 I$$

$$\Rightarrow \quad B\oint dl = \mu_0 I \Rightarrow B\cdot 2\pi r = \mu_0 I$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{2\pi r} \qquad \text{...(i)}$$



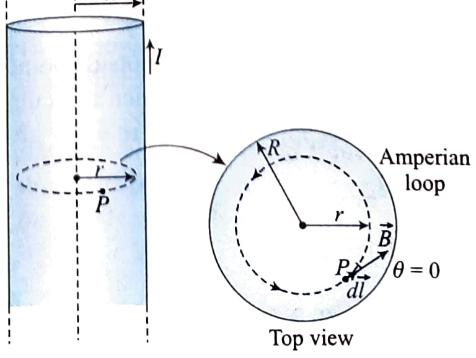
If point P is located on the surface of cylindrical wire we can use $r = R$ for which the magnetic induction is given by replacing x with R in Eq. (i) as

$$B_S = \frac{\mu_0 I}{2\pi R} \qquad \text{...(ii)}$$

Inside the Wire

To determine the magnetic induction at an interior point P at a distance x from the axis of wire again we consider a circular path as shown in figure and apply Ampere's law on this path which gives

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{enclosed}} \qquad \text{...(iii)}$$



As the current I is considered to be uniformly distributed over the cross section of cylindrical wire, the current enclosed in the circular path of radius r as shown in above figure is given as

Thus from Eq. (iii) we have

$$I_{\text{enclosed}} = \frac{I}{\pi R^2}\times \pi r^2 \Rightarrow I_{\text{enclosed}} = \frac{Ir^2}{R^2}$$

Thus from Eq. (iii) we have

$$\Rightarrow \quad \oint B dl \cos(0^\circ) = \mu_0\left(\frac{Ir^2}{R^2}\right)$$

$$\Rightarrow \quad B\oint dl = \mu_0\left(\frac{Ir^2}{R^2}\right) \Rightarrow B\cdot 2\pi r = \left(\frac{Ir^2}{R^2}\right)$$

$$\Rightarrow \quad B = \frac{\mu_0 Ir}{2\pi R^2} \qquad \text{...(iv)}$$

We can express Eq. (iv) in terms of current density,

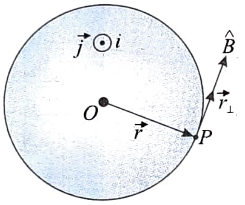
$$B = \frac{1}{2}\mu_0 jr, \quad \left(\text{As } j = \frac{i}{\pi R^2}\right) \qquad \text{...(v)}$$

If position vector of a certain point is considered as $\vec{r}$ then direction of magnetic field at that point will be perpendicular to $\vec{r}$, hence Eq. (v) can be written in vector notation as

$$\vec{B} = \frac{1}{2}\mu_0 \vec{j}\,\vec{r}_\perp \qquad \text{...(vi)}$$

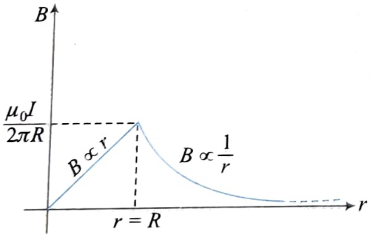
Equation (vi) can also be written as,

$$\vec{B} = \frac{1}{2}\mu_0(\vec{j}\times\vec{r})$$



Here $\vec{j}$ is current density vector.

Now we can plot the variation curve of magnetic induction magnitude in surrounding of the cylindrical current carrying wire which is shown in figure.



MAGNETIC INDUCTION DUE TO A HOLLOW CYLINDRICAL WIRE

Figure shows a hollow thin walled long cylindrical wire carrying a current I.

Outside the Wire

To determine the magnetic induction at outside point P located a distance $r(r > R)$ from its axis we consider a circular Ampere's path of radius r on which applying Ampere's law gives

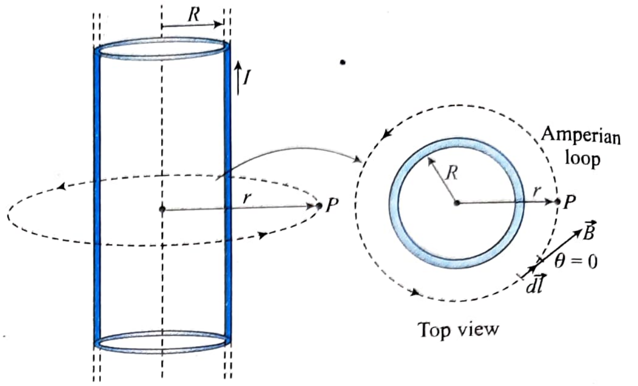
$$\oint \vec{B}\cdot\vec{dl} = \mu_0 I_{\text{enclosed}}$$

$$\Rightarrow \quad \oint B dl \cos(0^\circ) = \mu_0 I$$

$$\Rightarrow \quad B\oint dl = \mu_0 I \Rightarrow B\cdot 2\pi r = \mu_0 I$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{2\pi r} \qquad \text{...(i)}$$

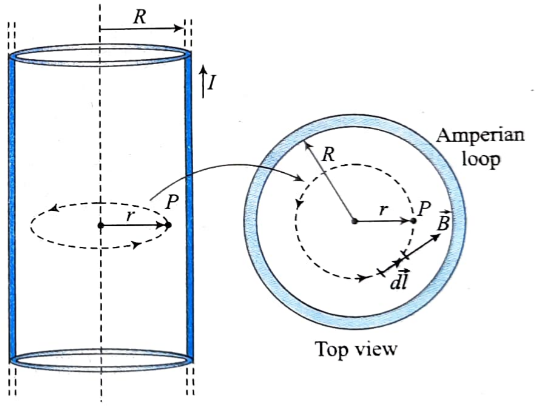
$$\text{For } r = R,\ B_S = \frac{\mu_0 I}{2\pi R} \qquad \text{...(ii)}$$



Inside the Wire

For interior points of the hollow cylindrical wire if we consider a point P as shown in figure and apply Ampere's law on the circular path M as shown, then we have

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 I_{\text{enclosed}}$$



In this case for the path shown enclosed current will be zero as current is flowing only on the surface of the hollow cylinder. By symmetry of path we can assume that magnetic induction is uniform along the path M which gives

$$\oint \vec{B}\cdot\vec{dl} = 0$$

$$B_{\text{inside}} = 0 \qquad \text{...(iii)}$$

Using Eqs. (i), (ii) and (iii), we can plot the variation curve of magnetic induction magnitude in surrounding of the cylindrical current carrying wire which is shown in figure.

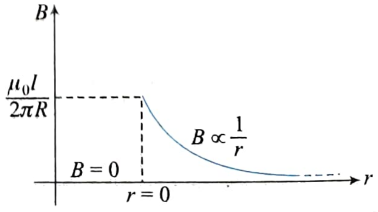


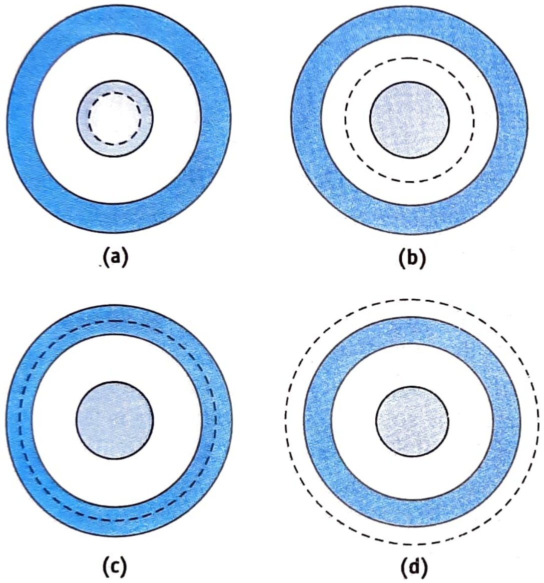
ILLUSTRATION 2.33

Consider a coaxial cable which consists of an inner wire of radius a surrounded by an outer shell of inner and outer radii b and c, respectively. The inner wire carries an electric current i_0 and the outer shell carries an equal current in opposite direction. Find the magnetic field at a distance x from the axis where **(a)** $x < a$, **(b)** $a < x < b$, **(c)** $b < x < c$ and **(d)** $x > c$. Assume that the current density is uniform in the inner wire and also uniform in the outer shell.

Sol. A cross section of the cable is shown in figure. Draw a circle of radius x with the center at the axis of the cable. The parts (a), (b), (c), and (d) of the figure correspond to the four parts of the problem. By symmetry, the magnetic field at each point of a circle will have the same magnitude and will be tangential to it. The circulation of B along this circle is, therefore,

$$\oint \vec{B}\cdot d\vec{l} = B2\pi x$$

in each of the four parts of the figure.



(a) (b)

(c) (d)

(a) The current enclosed within the circle in part (a) is

$$i = \frac{i_0}{\pi a^2}\pi x^2 = \frac{i_0}{a^2}x^2.$$

Ampere's law $\oint \vec{B}\cdot d\vec{l} = \mu_0 i$ gives

$$B2\pi x = \frac{\mu_0 i_0 x^2}{a^2} \quad \text{or,} \quad B = \frac{\mu_0 i_0 x}{2\pi a^2}$$

The direction will be along the tangent to the circle.

(b) The current enclosed within the circle in part (b) is i_0 so that $B\,2\pi x = \mu_0\, i_0$ or, $B = \dfrac{\mu_0 i_0}{2\pi x}$

(c) Current density of outer shell: $J = \dfrac{i_0}{\pi c^2 - \pi b^2}$

So current from $x = 0$ to $x = x$:

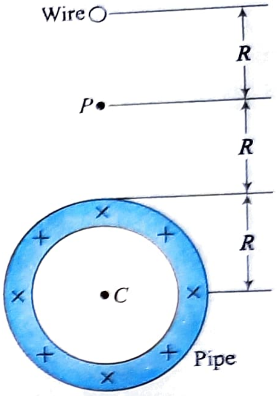
$$I = i_0 - J(\pi x^2 - \pi b^2)$$

$$= i_0 - i_0\left(\frac{x^2-b^2}{c^2-b^2}\right) = \frac{i_0(c^2-x^2)}{c^2-b^2}$$

$$B = \frac{\mu_0 I}{2\pi x} = \frac{\mu_0 i_0(c^2-x^2)}{2\pi x(c^2-b^2)}$$

(d) For $x > c$, magnetic field will be zero, because net current is zero.

ILLUSTRATION 2.34



In figure, a long circular pipe with outside radius R = 2.5 cm carries a (uniformly distributed) current i = 8.0 mA into the page. A wire runs parallel to the pipe at a distance of $3R$ from center to center. Find the **(a)** magnitude and **(b)** direction (into or out of the page) of the current in the wire such that the net magnetic field at point P has the same magnitude as the net magnetic field at the center of the pipe but is in the opposite direction.

Sol.

(a) The field at the center of the pipe (point C) is due to the wire alone, with a magnitude of

$$B_C = \frac{\mu_0 i_{\text{wire}}}{2\pi(3R)} = \frac{\mu_0 i_{\text{wire}}}{6\pi R}.$$

For the wire we have $B_{P,\text{wire}} > B_{C,\text{wire}}$. Thus, for $B_P = B_C$. The current in the wire, i_{wire} must be into the page.

$$B_P = B_{P,\text{ wire}} - B_{P,\text{ pipe}} = \frac{\mu_0 i_{\text{wire}}}{2\pi R} - \frac{\mu_0 i}{2\pi(2R)}.$$

Here $B_C = -B_P$. we obtain

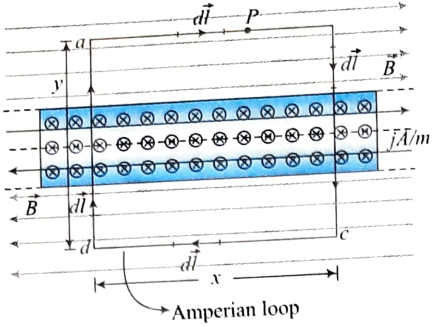
$$\frac{\mu_0 i_{\text{wire}}}{6\pi R} = \frac{\mu_0 i_{\text{wire}}}{2\pi R} - \frac{\mu_0 i}{2\pi(2R)} \Rightarrow i_{\text{wire}} = \frac{3}{8}i = 3.0\times10^{-3}\text{ A}$$

(b) From right hand rule it is clear that the direction of current is into the page.

MAGNETIC INDUCTION DUE TO A LARGE AND THICK CURRENT CARRYING SHEET

A large thick sheet of thickness d, has the current flowing in the sheet is having a current density J A/m². We know the direction of magnetic field above and below the sheet is given by right hand thumb rule as shown in figure. Let us calculate the magnitude of the magnetic field at point P outside the sheet. Here we consider the rectangular path $abcd$ with point P located on the line ab of the path as shown in figure. Let the dimensions of the path are $ab = x$ and $bc = y$. Now applying Ampere's law on this closed path,

$$\oint_{abcd} \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{enclosed}}$$



As current density in the cross section of sheet is j A/m², the current in length x of the sheet which is enclosed by the path $abcd$ is given as jxd ampere which gives

$$\oint_{abcd} \vec{B}\cdot\vec{dl} = \mu_0(jxd) \qquad \text{...(i)}$$

For calculation the integral on LHS of Eq. (i), we split the integral for the different parts of the closed path $abcd$.

$$\int_a^b \vec{B}\cdot\vec{dl} + \int_b^c \vec{B}\cdot\vec{dl} + \int_c^d \vec{B}\cdot\vec{dl} + \int_d^a \vec{B}\cdot\vec{dl} = \mu_0(jxd) \qquad \text{...(ii)}$$

The second and fourth term in LHS, the magnetic field is perpendicular to the element along paths bc and da thus dot product will result zero and for paths ab and cd magnetic field is uniform and parallel to the element which gives

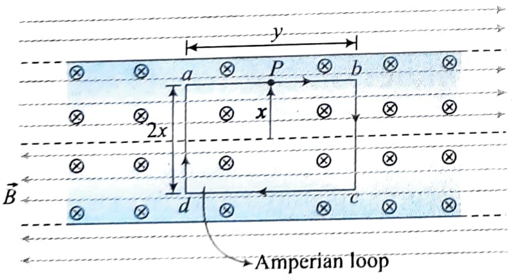
$$B\int_a^b dl + B\int_c^d dl = \mu_0(jxd)$$

$$\Rightarrow \quad Bx + Bx = \mu_0(jxd) \Rightarrow 2Bx = \mu_0(jxd)$$

$$\Rightarrow \quad B = \frac{1}{2}\mu_0 jd \qquad \text{...(iii)}$$

For calculating the magnetic field at interior points of the thick sheet at a point P at a distance x from the central plane of the sheet as shown in figure. We again consider a rectangular path $abcd$ as shown with dimensions $ab = y$ and $bc = 2x$. On applying Ampere's law on this path,

$$\oint_{abcd} \vec{B}\cdot\vec{dl} = \mu_0 I_{\text{enclosed}}$$



As current density in the cross section of sheet is j A/m², the current enclosed by the Ampere's path $abcd$ is given as $j(2xy)$ which gives

$$\oint_{abcd} \vec{B}\cdot\vec{dl} = \mu_0(2jxy) \qquad \text{...(iv)}$$

To solve the line integral on LHS of Eq. (iv), we split the integral for the different parts of the closed path $abcd$ similar to the analysis we did in previous part.

$$\int_a^b \vec{B}\cdot\vec{dl} + \int_b^c \vec{B}\cdot\vec{dl} + \int_c^d \vec{B}\cdot\vec{dl} + \int_d^a \vec{B}\cdot\vec{dl} = \mu_0(2jxd) \qquad \text{...(v)}$$

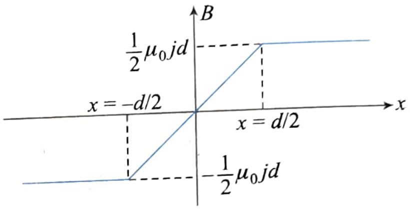
The magnetic field is perpendicular to the element along paths bc and da thus dot product will result in zero and for paths ab and cd magnetic field is uniform and parallel to the element this gives

$$B\int_a^b dl + B\int_c^d dl = \mu_0(2jxd)$$

$$\Rightarrow \quad By + By = \mu_0(2jxd) \Rightarrow 2By = \mu_0(2jxd)$$

$$\Rightarrow \quad B = \mu_0 jd \qquad \text{...(vi)}$$

Now we can plot the variation of magnetic induction with distance from the central plane of a thick sheet which is shown in figure.

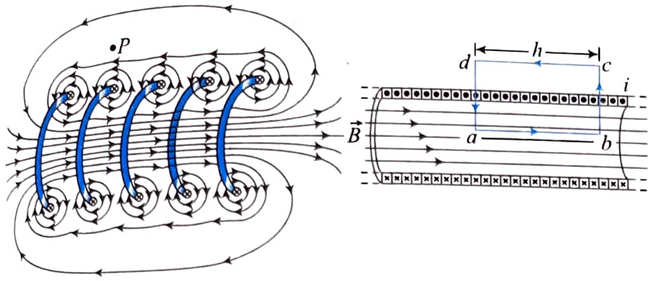


MAGNETIC FIELD OF A SOLENOID

We now turn our attention to another situation in which Ampere's law proves useful. It concerns the magnetic field produced by the current in a long, tightly wound helical coil of wire. Such a coil is called a **solenoid**. We assume that the length of the solenoid is much greater than the diameter.

Figure (a) shows a section through a portion of a "stretched-out" solenoid. The solenoid's magnetic field is the vector sum of the fields produced by the individual turns (windings) that make up the solenoid. For points very close to a turn, the wire behaves magnetically almost like a long straight wire, and the lines of $\vec{B}$ there are almost concentric circles. Figure (a) suggests that the field tends to cancel between adjacent turns. It also suggests that, at points inside the solenoid and reasonably far from the wire, $\vec{B}$ is approximately parallel to the (central) solenoid axis. In the limiting case of an ideal solenoid, which is infinitely long and consists of tightly packed (close-packed) turns of square wire, the field inside the coil is uniform and parallel to the solenoid axis.

At points above the solenoid, such as P in Fig. (a), the magnetic field set up by the upper parts of the solenoid turns (these upper turns are marked ⊙) is directed to the left (as drawn near P) and tends to cancel the field set up at P by the lower parts of the turns (these lower turns are marked ⊗), which is directed to the right (not drawn). In the limiting case of an ideal solenoid, the magnetic field outside the solenoid is zero. Taking the external field to be zero is an excellent assumption for a real solenoid if its length is much greater than its diameter and if we consider external points such as point P that are not at either end of the solenoid. The direction of the magnetic field along the solenoid axis is given by a curled-straight right-hand rule: Grasp the solenoid with your right hand so that your fingers follow the direction of the current in the windings; your extended right thumb then points in the direction of the axial magnetic field.



(a) (b)

Ampere's law: Let us now apply Ampere's law,

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}} \qquad \text{...(i)}$$

to the ideal solenoid of Fig. (b), where B is uniform within the solenoid and zero outside it, using the rectangular Amperian loop *abcda*. We write $\oint \vec{B} \cdot d\vec{s}$ as the sum of four integrals, one for each loop segment:

$$\oint \vec{B} \cdot d\vec{s} = \int_a^b \vec{B} \cdot d\vec{s} + \int_b^c \vec{B} \cdot d\vec{s} + \int_c^d \vec{B} \cdot d\vec{s} + \int_b^a \vec{B} \cdot d\vec{s} \qquad \text{...(ii)}$$

The first integral on the right of Eq.(ii) is Bh, where B is the magnitude of the uniform field $\vec{B}$ inside the solenoid and h is the (arbitrary) length of the segment from a to b. The second and fourth integrals are zero because for every element ds of these segments, $\vec{B}$ either is perpendicular to ds or is zero, and thus the product $\vec{B} \cdot d\vec{s}$ is zero. The third integral, which is taken along a segment that lies outside the solenoid, is zero because $B = 0$ at all external points. Thus, $\oint \vec{B} \cdot d\vec{s}$ for the entire rectangular loop has the value Bh.

Net current: The net current i_{enc} encircled by the rectangular Amperian loop in Fig. (b) is not the same as the current i in the solenoid windings because the windings pass more than once through this loop. Let n be the number of turns per unit length of the solenoid; then the loop encloses nh turns and $i_{\text{enc}} = i(nh)$.

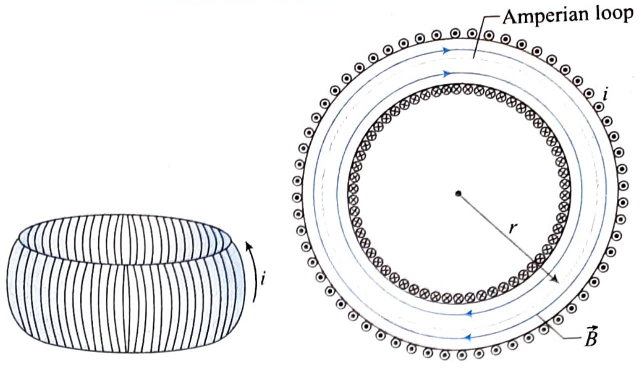
Now Ampere's law then gives us, $Bh = \mu_0 inh$

$$\text{or} \quad B = \mu_0 in \quad \text{(ideal solenoid).} \qquad \text{...(iii)}$$

Although we derived Eq. (iii) for an infinitely long ideal solenoid, it holds quite well for actual solenoids if we apply it only at interior points and well away from the solenoid ends. A solenoid thus provides a practical way to set up a known uniform magnetic field for experimentation, just as a parallel-plate capacitor provides a practical way to set up a known uniform electric field.

MAGNETIC FIELD OF A TOROID

Figure (a) shows a **toroid**, which we may describe as a (hollow) solenoid that has been curved until its two ends meet, forming a sort of hollow bracelet. Let us find the magnetic field B is set up inside the toroid (inside the hollow of the bracelet)? We can find out from Ampere's law and the symmetry of the bracelet.



(a) (b)

From the symmetry, we see that the lines of $\vec{B}$ form concentric circles inside the toroid, directed as shown in Fig. (b). Let us choose a concentric circle of radius r as an Amperian loop and traverse it in the clockwise direction. Ampere's law yields

$$(B)(2\pi r) = \mu_0 iN,$$

where i is the current in the toroid windings (and is positive for those windings enclosed by the Amperian loop) and N is the total number of turns. This gives

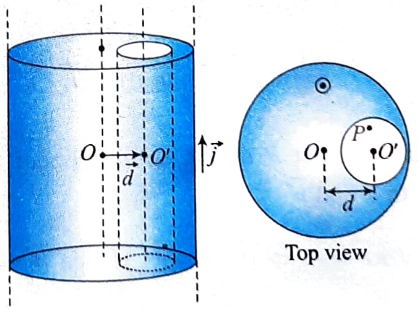
$$B = \frac{\mu_0 iN}{2\pi r} \qquad \text{...(i)}$$

In contrast to the situation for a solenoid, B is not constant over the cross section of a toroid.

The direction of the magnetic field within a toroid follows from our curled-straight right-hand rule: Grasp the toroid with the fingers of your right hand curled in the direction of the current in the windings; your extended right thumb points in the direction of the magnetic field.

ILLUSTRATION 2.35

Inside a long straight uniform wire of round cross-section, there is a long round cylindrical cavity whose axis is parallel to the axis of the wire and displaced from the axis by a distance $\vec{d}$. A direct current of density $\vec{J}$ flows along the wire. Find the magnetic induction at a general point inside the cavity.



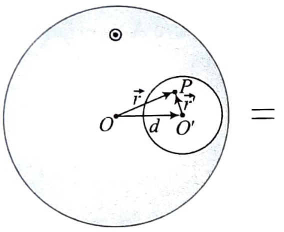
Top view

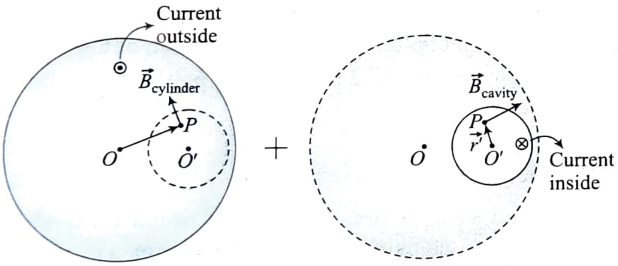
Sol. The magnetic field at a point within the hole is the sum of the fields due to two current distributions. The first is that of the solid cylinder obtained by filling the hole and has a current density that is the same as that in the original cylinder (with the hole). The second is the solid cylinder that fills the hole. It has a current density with the same magnitude as that of the original cylinder but is in the opposite direction. If these two situations are superposed the total current in the region of the hole is zero. Now, a solid cylinder carrying current i, which is uniformly distributed over a cross section, produces a magnetic field with magnitude at a distance r from its axis, inside the cylinder. Here R is the radius of the cylinder.

$$B_{\text{cylinder}} = \frac{\mu_0 i r}{2\pi R^2} \qquad \ldots\text{(i)}$$

Let us consider a cross-section of the cylinder perpendicular to its axis as shown in figure. The cylindrical cavity is shown at a distance d from the axis O of the cylinder. We will calculate the magnetic field at the point P inside the cavity. Magnetic field at point P inside cavity is given by subtracting the magnetic field due to opposite current considered in cavity region from the total magnetic field at P due to complete solid cylindrical wire which is given as

$$\vec{B}_P = \vec{B}_{\text{cylinder}} - \vec{B}_{\text{cavity}}$$





Magnetic field at P due to complete cylinder only

Magnetic field at P due to cavity only

Expressions for the magnetic field due to wire and that due to the opposite current in cavity is given by

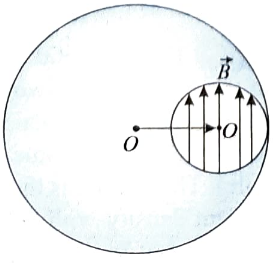
$$\vec{B}_P = \frac{1}{2}\mu_0 j \overrightarrow{r_\perp} - \frac{1}{2}\mu_0 j \overrightarrow{r'_\perp} = \frac{1}{2}\mu_0 j(\overrightarrow{r'_\perp} - \overrightarrow{r_\perp})$$

The magnetic field at P due to solid cylinder and cavity is perpendicular to $\vec{r}$ and $\vec{r'}$

In $\Delta OO'P$: $\vec{d} + \vec{r'} = \vec{r} \Rightarrow \vec{d} = \vec{r} - \vec{r'}$. Hence $\overrightarrow{d_\perp} = \overrightarrow{r'_\perp} - \overrightarrow{r_\perp}$

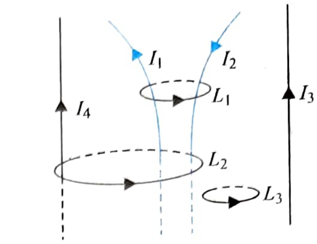
$$\vec{B}_P = \frac{1}{2}\mu_0 j(\overrightarrow{d_\perp})$$

It means the magnetic field inside the cavity is uniform and in the direction normal to the line joining OO' as shown in figure.

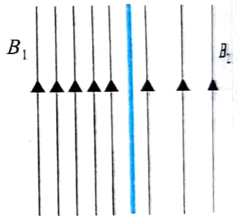


CONCEPT APPLICATION EXERCISE 2.4

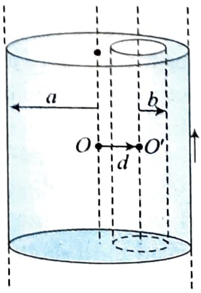
1. Find the values of $\oint \vec{B}\cdot\vec{dl}$ for the loops L_1, L_2, and L_3 in figure. (The sense of $\vec{dl}$ is mentioned in the figure.)



2. The length of solenoid is 0.1 m and its diameter is very small. A wire is wound over it in two layers. The number of turns in inner layer is 50 and that of outer layer is 40. The strength of current flowing in two layers in opposite direction is 3 A. Then find magnetic induction at the middle of the solenoid.
3. A straight long solenoid has produced a magnetic field 'B' at its centre. If it cut is two equal parts and same number of turns wound on one part in double layer. Find magnetic field produced by new solenoid at its centre.
4. A long straight solid conductor of radius 5 cm carries a current of 3 A, which is uniformly distributed over its circular cross-section. Find the magnetic field induction at a distance 4 cm from the axis of the conductor. Relative permeability of the conductor = 1000.
5. A conducting current-carrying plane is placed in an external uniform magnetic field. As a result, the magnetic induction becomes equal to B_1 on one side of the plane and B_2 on the other side. Find the magnetic force acting per unit area of the plane.

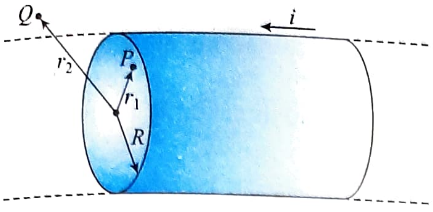


6. Find the current density as a function of distance r from the axis of a radially symmetrical parallel stream of electrons if the magnetic induction inside the stream varies as $B = br^{a}$, where b and a are positive constants.
7. Figure shows a cross section of a long cylindrical conductor of radius a containing a long cylindrical hole of radius b. The central axes of the cylinder and hole are parallel and are distance d apart; current i is uniformly distributed over the tinted area. What is the magnitude of the magnetic field at the center of the hole?



8. A long cylindrical conductor of radius R carries a current I as shown in figure. The current density J is a function of

radius according to, $J = br$, where b is a constant. Find an expression for the magnetic field B.



(a) at a distance $r_1 < R$ and

(b) at a distance $r_2 > R$ measured from the axis.

ANSWERS

1. $\mu_0(I_1 - I_2)$, $\mu_0(I_1 - I_2 + I_4)$, 0 **2.** $12\pi \times 10^{-5}$ T **3.** $2B$

4. 9.6×10^{-3} T **5.** $\dfrac{B_1^2 - B_2^2}{2\mu_0}$ **6.** $\dfrac{b(\alpha+1)r^{\alpha-1}}{\mu_0}$

7. $\dfrac{\mu_0 id}{2\pi(a^2 - b^2)}$ **8.** (a) $\dfrac{\mu_0 br_1^2}{3}$ (b) $\dfrac{\mu_0 bR^3}{3r_2}$

Solved Examples

EXAMPLE 2.1

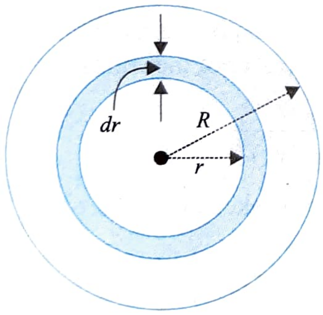
A non-conducting thin disc of radius R charged uniformly over one side with surface density s rotates about its axis with an angular velocity ω. Find

(a) the magnetic induction at the center of the disc;

(b) the magnetic moment of the disc.

Sol.

(a) Consider an elemental area of the disc of radius r and thickness dr



$$dq = \sigma \cdot 2\pi r\, dr$$

$$\therefore \quad \text{Current } dI = \frac{dq}{T} = \sigma \cdot 2\pi r\, dr \cdot \frac{\omega}{2\pi}$$

$$\therefore \quad dB = \text{Magnetic field by this element}$$

$$= \frac{\mu_0\, dI}{2r}$$

$$\text{or} \quad B = \int dB = \frac{\mu_0}{4\pi} \cdot 2\pi\sigma\omega \int_0^R dr = \frac{\mu_0}{2}\sigma\omega R$$

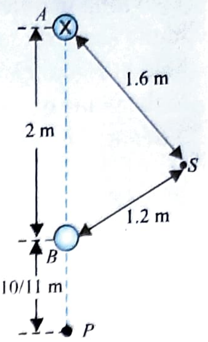
(b) Now dM = Magnetic moment of element

$$= dI\,\pi r^2 = \omega\sigma r\, dr\,\pi r^2$$

$$M = \int dM = \omega\sigma\pi \int_0^R r^3 dr = \frac{1}{4}\omega\pi\sigma R^4$$

EXAMPLE 2.2

Two long straight parallel wires are 2 m apart, perpendicular to the plane of the paper. The wire A carries a current of 9.6 A, directed into the plane of the paper. The wire B carries a current such that the magnetic field of induction at point P, at a distance of 10/11 m from wire B is zero. Find:



(a) The magnitude and direction of the current in B.

(b) The magnitude of the magnetic field of induction at point S.

(c) The force per unit length on wire B.

Sol.

(a) Direction of current at B should be perpendicular to paper outwards. Let current in this wire be i_B. Then

$$\frac{\mu_0}{2\pi}\frac{i_A}{\left(2 + \frac{10}{11}\right)} = \frac{\mu_0}{2\pi}\frac{i_B}{(10/11)}$$

$$\text{or} \quad \frac{i_B}{i_A} = \frac{10}{32}$$

$$\text{or} \quad i_B = \frac{10}{32} \times i_A = \frac{10}{32} \times 9.6 = 3\text{A}$$

(b) Since $AS^2 + BS^2 = AB^2$

$$\therefore \quad \angle ASB = 90°$$

At S: B_1 = Magnetic field due to i_A

$$= \frac{\mu_0}{2\pi}\frac{i_A}{1.6} = \frac{(2\times 10^{-7})(9.6)}{1.6} = 12 \times 10^{-7}\ \text{T}$$

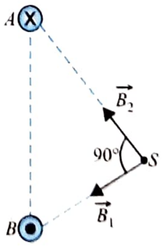
$$B_2 = \text{Magnetic field due to } i_B = \frac{\mu_0}{2\pi}\frac{i_B}{1.2} = \frac{(2\times 10^{-7})(3)}{1.2}$$

$$= 5 \times 10^{-7}\ \text{T}$$

Since B_1 and B_2 are mutually perpendicular, net magnetic field at S would be:

$$B = \sqrt{B_1^2 + B_2^2} = \sqrt{(12\times 10^{-7})^2 + (5\times 10^{-7})^2}$$

$$= 13 \times 10^{-7}\ \text{T}$$



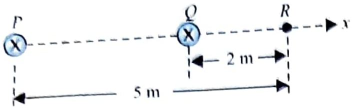
(c) Force per unit length on wire B:

$$\frac{F}{l} = \frac{\mu_0}{2\pi}\frac{i_A i_B}{r} \qquad (r = AB = 2\text{ m})$$

$$= \frac{(2\times 10^{-7})(9.6\times 3)}{2} = 2.88 \times 10^{-6}\ \text{N m}^{-1}$$

EXAMPLE 2.3

Two long parallel wires carrying currents 2.5 A and I (ampere) in the same direction (directed into the plane of the paper) are held at P and Q, respectively, such that they are perpendicular to the plane of paper. Points P and Q are located at a distance of 5 m and 2 m, respectively from a collinear point R (see figure).



(a) An electron moving with a velocity of 4×10^5 m s^{-1} along the positive x direction experiences a force of magnitude 3.2×10^{-20} N at the point R, find the value of I.

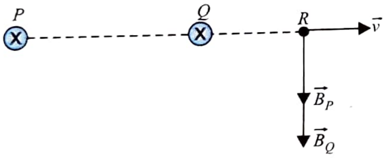
(b) Find all the positions at which a third long parallel wire carrying a current of magnitude 2.5 A may be placed, so that the magnetic induction at R is zero.

Sol.

(a) Magnetic field at R due to both wires P and Q will be downward as shown in figure. Therefore, net field at R will be sum of these two.

$$B = B_P + B_Q = \frac{\mu_0}{2\pi}\frac{I_P}{5} + \frac{\mu_0}{2\pi}\frac{I_Q}{2} = \frac{\mu_0}{2\pi}\left(\frac{2.5}{5} + \frac{I}{2}\right)$$

$$= \frac{\mu_0}{4\pi}(I+1) = 10^{-7}(I+1)$$



Net force on the electron $F = Bqv \sin 90°$

or $(3.2 \times 10^{-20}) = (10^{-7})(I + 1)(1.6 \times 10^{-19})(4 \times 10^5)$

or $I + 1 = 5$

$\therefore$ $I = 4$ A

(b) Net field at R due to wires P and Q is

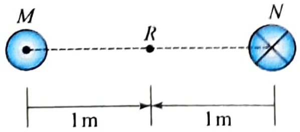
$$B = 10^{-7}(I + 1)\ T = 5 \times 10^{-7}\ \text{T}$$

Magnetic field due to third wire carrying a current of 2.5 A should be 5×10^{-7} T in upward direction so that net field at R becomes zero. Let distance of this wire from R be r. Then,

$$\frac{\mu_0}{2\pi}\frac{2.5}{r} = 5 \times 10^{-7} \quad \text{or} \quad \frac{(2 \times 10^{-7})(2.5)}{r} = 5 \times 10^{-7}\ \text{m}$$

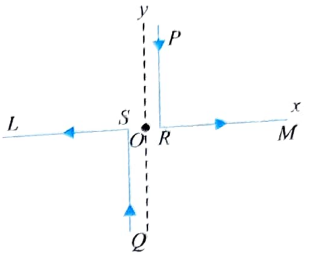
or $r = 1$ m

So, the third wire can be put at M or N as shown in figure. If it is placed at M, then current in it should be outwards and if placed at N, then current will be inward.



EXAMPLE 2.4

A pair of stationary and infinitely long bent wires is placed in the x–y plane as shown in figure. Each wire carries current of 10 A. Segments L and M are along the x-axis. Segments P and Q are parallel to the y-axis such that $OS = OR = 0.02$ m. Find the magnitude and direction of the magnetic induction at origin O.



Sol. As point O is along the length of segments L and M, so the field at O due to these segments will be zero. Also, point O is near one end of a long wire.

The resultant field at O, $B_R = B_P + B_Q$, this field will be into the plane of paper.

$$\Rightarrow \quad B_R = \frac{\mu_0}{4\pi}\frac{I}{RO} + \frac{\mu_0}{4\pi}\frac{I}{SO}$$

But $RO = SO = 0.02$ m

Hence, $B_R = 2 \times \dfrac{\mu_0}{4\pi} \times \dfrac{10}{0.02} = 2 \times 10^{-7}\dfrac{10}{0.02} = 10^{-4}$ Wb m^{-2}

EXAMPLE 2.5

A current I flows along a round loop. Find the integral $\int \vec{B} \cdot d\vec{r}$ along the axis of the loop within the range from $-\infty$ to $+\infty$. Explain the result obtained.

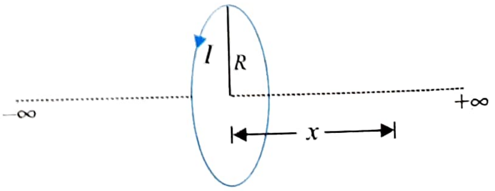
Sol. The magnetic field at point P on the axis of a coil at a dsitance x from the center of the coil is

$$B_x = \frac{\mu_0 I R^2}{2(R^2 + x^2)^{3/2}}$$

the axis of the loop is taken along the x-axis.

$$\int_{-\infty}^{\infty} \vec{B} \cdot d\vec{r} = \int_{-\infty}^{\infty} B_x dx = \frac{\mu_0 I R^2}{2}\int_{-\infty}^{\infty}\frac{dx}{(R^2 + x^2)^{3/2}}$$

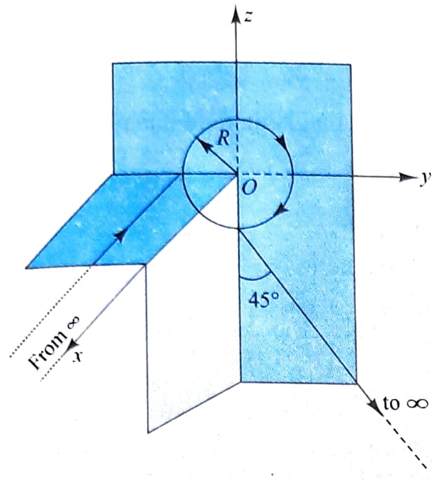
$$= \frac{\mu_0 I R^2}{2}\int_{-\pi/2}^{\pi/2}\frac{R\sec^2\theta}{R^3\sec^3\theta} = \frac{\mu_0 I}{2}\int_{-\pi/2}^{\pi/2}\cos\theta\, d\theta = \mu_0$$



The integral on the LHS can be thought of as the line integra of B over a closed path (a straight line from $-\infty$ to $+\infty$ and a semi circle of infinite radius where the magnetic feild vanishes). Then as per Ampere's law, this must be μ_0 times the current enclosed by the path which is I.

EXAMPLE 2.6

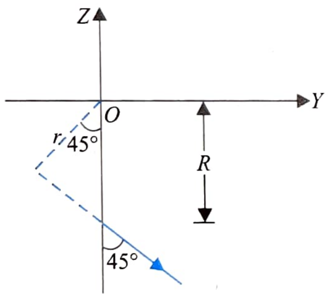
Calculate magnetic induction at point O if the wire carrying a current I has the shape as shown in figure.



(The radius of the curved part of the wire is equal to R and linear parts of the wire are very long.)

Sol. Circuit segment shown in figure can be considered in three parts.

(a) A circular loop in y–z plane. Since this loop is made of uniform wire, therefore magnetic induction at O due to it is $B_1 = 0$.



(b) A straight part, parallel to x-axis. Magnetic induction due to it is

$$B_2 = \frac{\mu_0 I}{4\pi R}(-\hat{k}) = -\frac{\mu_0 I}{4\pi R}\hat{k}$$

(c) A straight part in y–z plane. Perpendicular distance of O from axis of this straight part is $r = R\cos 45°$ as shown in figure.

Angles subtended by lines joining O and ends of this straight part with perpendicular drawn from O are $\alpha = -45°$ and $\beta = 90°$.

Magnetic induction at O due to this part is

$$B_3 = \frac{\mu_0 I}{4\pi R}(\sin\alpha + \sin\beta)$$

$$\vec{B}_3 = \frac{\mu_0 I}{4\pi R}(\sqrt{2} - 1)\hat{i}$$

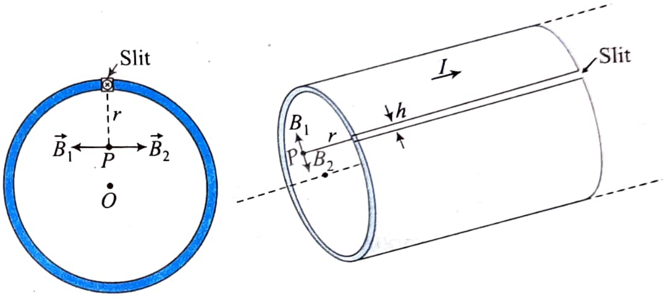
$$\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 = \frac{\mu_0 I}{4R}(\sqrt{2} - 1)\hat{i} - \frac{\mu_0 I}{4\pi R}\hat{k}$$

EXAMPLE 2.7

A current I flows along a lengthy thin walled tube of radius R with a longitudinal slit of width h. Find the magnetic induction inside the tube under the condition $h << R$.

Sol. The magnetic induction due to a current carrying thin walled tube at an internal point is always zero. However if a longitudinal cut of width h is made in the tube as shown in figure, then symmetry will no longer exist and inside magnetic induction will not be zero. A net magnetic induction will exists at an internal point. We consider a point P inside the tube at a distance r from the slit as shown. The net magnetic induction due to the tube at point P is equal to the magnetic induction produced at P due to the current in the portion of tube which is removed to make the slit. As slit width is very small the current in the slit width h is given as

$$I_S = \frac{I}{2\pi R} \times h$$



Front view

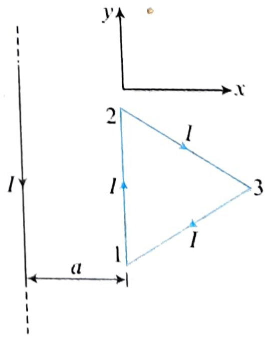
The magnetic induction at point P due to the current in the slit portion of tube is B_1 as shown in figure which is given as

$$B_1 = \frac{\mu_0 I_S}{2\pi r} = \frac{\mu_0}{2\pi}\frac{\left(\frac{I}{2\pi R}\right)h}{r} = \frac{\mu_0 I h}{4\pi^2 R r}$$

The magnetic induction due to the tube with the slit is given as B_2 which is equal and opposite to B_1 so that when tube is complete both the fields will nullify each other.

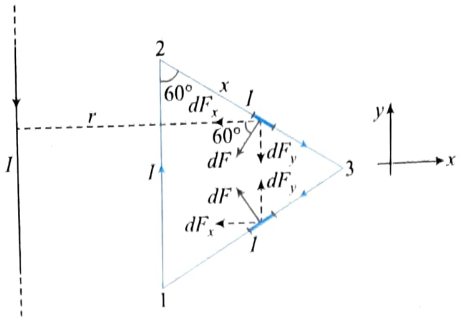
EXAMPLE 2.8

An equilateral triangular frame with side a carrying a current I is placed at a distance a from an infinitely long straight wire carrying a current I as shown in the figure. One side of the frame is parallel to the wire. The whole system lies in the xy-plane. Find the magnetic force F acting on the frame.



Sol. The force on wire 1-2 due to long wire

$$F_1 = \left(\frac{\mu_0}{2\pi}\frac{I}{a}\right)(Ia) \text{ in positive } x\text{-direction.}$$



Now force between long wire and wire 2-3-1. Let us consider the current elements on wire 2-3 and 3-1 as shown in figure, the distance of this element from long wire

$$r = a + x\sin 60° = a + \sqrt{3}\frac{x}{2}$$

The magnetic field due to long wire at the position of this element,

$$B_r = \frac{\mu_0}{2\pi}\frac{I}{r} = \frac{\mu_0}{2\pi}\frac{I}{a+\sqrt{3}\frac{x}{2}} = \frac{\mu_0}{\pi}\frac{I}{2a+\sqrt{3}x}$$

Force on this element, $dF = \frac{\mu_0}{\pi}\cdot\frac{I}{2\pi+\sqrt{3}x}\cdot I\cdot(dx)$ ...(i)

y-component of force on this element will get cancelled.

x-component of force on this element $(dF)_x = dF\cos 60°(-\hat{i})$

Hence net horizontal component of force on the wire 2-3-1.

$$(F_x)_{2-3-1} = \int (dF)_x = \int dF\cos 60°(-\hat{i}) = \left[2\int_{x=0}^{x=a}(dF\cos 60°)\right](-\hat{i})$$

$$= \left[2\int_0^a \frac{\mu_0}{\pi}\cdot\frac{I}{2\pi+\sqrt{3}x}\cdot I\cdot(dx)\frac{1}{2}\right](-\hat{i})$$

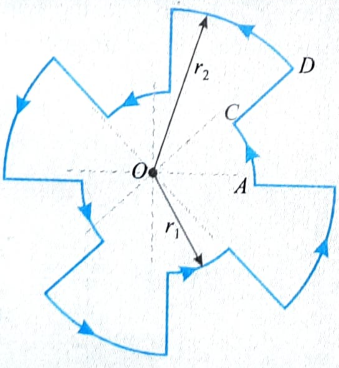
$$= \left(\frac{\mu_0 I^2}{\pi}\right)\cdot\frac{1}{\sqrt{3}}[\ln(2a+\sqrt{3}x)]_0^a(-\hat{i})$$

$$\Rightarrow \quad F_{231} = \frac{\mu_0 I^2}{\sqrt{3}\pi}\ln\left(\frac{2+\sqrt{3}}{2}\right)(-\hat{i}) \text{ and } F_{12} = \frac{\mu_0 I^2}{2\pi}(\hat{i})$$

$$\therefore \quad F_{\text{Total}} = \frac{\mu_0 I^2}{\pi}\left[\frac{1}{2} - \frac{1}{\sqrt{3}}\ln\left(\frac{2+\sqrt{3}}{2}\right)\right](\hat{i})$$

EXAMPLE 2.9

A current of 10 A flows around a closed path in a circuit which is in the horizontal plane as shown in figure. The circuit consists of eight alternating arcs of radii $r_1 = 0.08$ m and $r_2 = 0.12$ m. Each arc subtends the same angle at the center.



(a) Find the magnetic field produced by this circuit at the center.

(b) An infinitely long straight wire carrying a current of 10 A is passing through the center of the above circuit vertically with the direction of the current being into the plane of the circuit. What is the force acting on the wire at the center due to the current in the circuit? What is the force acting on arc AC and the straight segment CD due to the current at the center?

Sol.

(a) Magnetic field at the center due to the straight parts is zero.

Magnetic field due to the arcs of radius r_1:

$$= 4\left(\frac{\mu_0 i}{2r_1}\right)\left(\frac{1}{8}\right) = \left(\frac{\mu_0 i}{4r_1}\right)$$

Similarly, the magnetic field due to the arcs of radius r_2:

$$= \left(\frac{\mu_0 i}{4r_2}\right)$$

$\Rightarrow$ Net magnetic field

$$= \left(\frac{\mu_0 i}{4}\right)\left\{\frac{1}{r_1} + \frac{1}{r_2}\right\} = 6.54 \times 10^{-5}\ \text{T}$$

(b) (i) As the current in the wire at the center is antiparallel to the direction of magnetic field, the force on the wire everywhere will be zero.

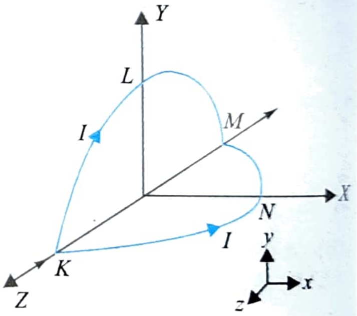
(ii) Further due to the current at the center, the magnetic field at AC will be tangential and hence the force on AC will be zero.

(iii) Force on $CD = \int_{r_1}^{r_2}\frac{\mu_o i}{2\pi x}i\,dx = \frac{\mu_o i^2}{2\pi}\ln\left(\frac{r_2}{r_1}\right)$

$= 8.11 \times 10^{-6}$ N (Vertically downward)

EXAMPLE 2.10

A circular loop of radius R is bent along the diameter and given a shape as shown in figure. One of the semicircles (KNM) lies in the x–z plane and the other one (KLM) in the y–z plane with their centers at the origin. Current I is flowing through each of the semicircles as shown in the figure.



(a) A particle of charge q is released at the origin with a velocity $v = -v_0\hat{i}$. Find the instantaneous force $\vec{F}$ on the particle. Assume that space is gravity free.

(b) If an external uniform magnetic field $B\hat{j}$ is applied, determine the forces $\vec{F}_1$ and $\vec{F}_2$ on the semicircles KLM and KNM due to this field and the net force $\vec{F}$ on the loop.

Sol. R = Radius of circular loop. Given that semicircle KNM lies in the x–z plane while the semicircle KLM lies in the y–z plane. Both the semicircles have their centers located at the origin.

(a) Charge on the particle released at the origin $= q$

Velocity of the particle, $\vec{v} = -v_0\hat{i}$

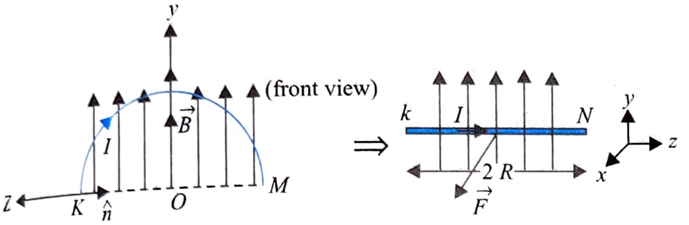
Magnetic field at center O due to current carrying loop KLM lying in y–z plane.

$\vec{B}_1 = \frac{\mu_0 I}{4R}(-\hat{i})$. The direction of $\vec{B}_1$ will be along –ve x-axis.

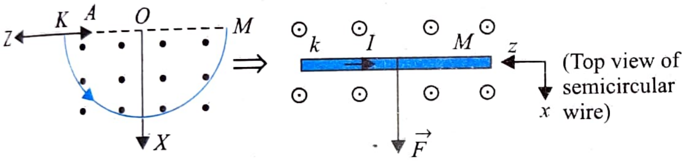
Similarly, magnetic field at center O due to current carrying loop KNM lying in x–z plane, $\vec{B}_2 = \frac{\mu_0 I}{4R}(\hat{j})$.

Thus, the two fields at O are mutually perpendicular in vector form. Total field at O can be expressed as

$$\vec{B} = -\hat{i}B_1 + \hat{j}B_2 = \frac{\mu_o I}{4R}\left(-\hat{i}+\hat{j}\right)$$



Hence, instantaneous force acting on the charged particle, released at O.



$$\vec{F} = q\left(\vec{v}\times\vec{B}\right) = q\left[-v_0\hat{i}\times\left(-\hat{i}+\hat{j}\right)\frac{\mu_0 I}{4R}\right]$$

$$= \frac{q\mu_0 I v_0}{4R}\left[\hat{i}\times\left(\hat{i}-\hat{j}\right)\right] = -\frac{q\mu_0 I v_0}{4R}\hat{k},$$

(b) External uniform magnetic field $\vec{B}_{\text{ext}} = B\hat{j}$

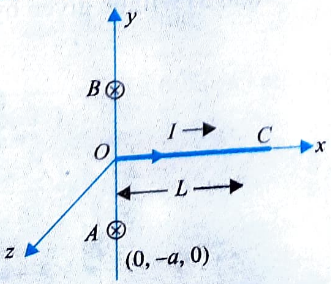
As semicircular wires are placed in uniform magnetic field, these loops can be reduced to straight wires each of length $2R$ placed along z-axis (by joining initial point K and final point M).

$$\vec{F}_{KLM} = \vec{F}_{KMN} = \vec{F}_{KM} = BI2R\hat{i}$$

Net force due to both the wires:

$$\vec{F} = \vec{F}_{KLM} + \vec{F}_{KMN} = 4BIR\hat{i}$$

EXAMPLE 2.11



A straight segment OC (of length L metre) of a circuit carrying a current I amp is placed along the x-axis. Two infinitely long straight wires A and B, each extending from $z = -\infty$ to $+\infty$ are fixed at $y = -a$ metre and $y = +a$ metre, respectively, as shown in figure. If wires A and B each carry a current I amp into the plane of the paper, obtain an expression for the force acting on segment OC. What will be the force on OC if the current in wire B is reversed?

Sol. Let us take an element of thickness dx at a distance x from origin on the wire OC. Magnetic field B_A produced at P $(x,0,0)$ due to wires placed at A and B are

$$B_A = \frac{\mu_0 I}{2\pi R},\ B_B = \frac{\mu_0 I}{2\pi R}$$

Components of B_A and B_B along x-axis cancel, while those along y-axis add up to give total field,

$$B = 2\left(\frac{\mu_0 I}{2\pi R}\right)\cos\theta = \frac{2\mu_0 I}{2\pi R}\frac{x}{R} = \frac{\mu_0 I}{\pi}\frac{x}{(a^2+x^2)}$$

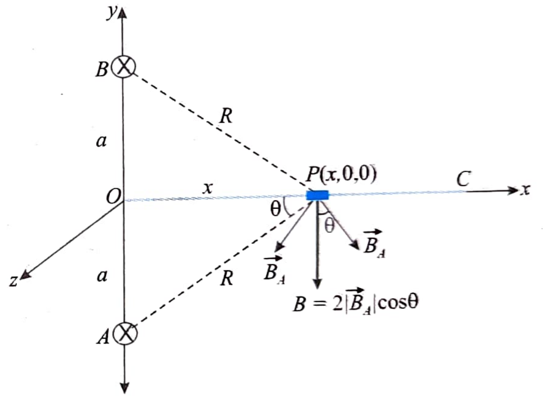
The force dF acting on the current element is

$$d\vec{F} = I(d\vec{l}\times\vec{B})$$

$$dF = \frac{\mu_0 I^2}{\pi}\frac{xdx}{a^2+x^2} \qquad [\because \ \sin 90° = 1]$$

Hence, net force on wire OC, $F = \int dF$

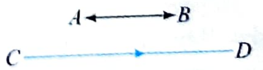
i.e., $F = \frac{\mu_0 I^2}{\pi}\int_0^L \frac{xdx}{a^2+x^2} = \frac{\mu_0 I^2}{2\pi}\ln\frac{a^2+L^2}{a^2}$ along –z direction.



If the current in B is reversed, the magnetic field due to the two wires would be only along x-direction and the force on the current carrying wire along x-direction will be zero.

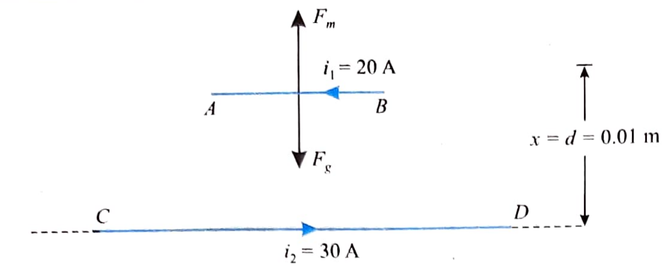
EXAMPLE 2.12

A long horizontal wire AB, which is free to move in a vertical plane and carries a steady current of 20 A, is in equilibrium at a height of 0.01 m over another parallel long wire CD which is fixed in a horizontal plane and carries a steady current of 30 A, as shown in figure. Show that when AB is slightly depressed, it executes simple harmonic motion. Find the period of oscillations.



Sol. Let m be the mass per unit length of wire AB. At a height x above the wire CD, magnetic force per unit length on wire AB will be given by

$$F_m = \frac{\mu_0}{2\pi}\frac{i_1 i_2}{x} \quad \text{(upward)} \qquad ...(i)$$



Weight per unit length of wire AB is

$$F_g = mg \quad \text{(downwards)}$$

Here, m = mass per unit length of wire AB

At $x = d$, wire is in equilibrium, i.e.,

$$F_m = F_g; \text{ or } \frac{\mu_0}{2\pi}\frac{i_1 i_2}{d} = mg$$

When AB is depressed, x decreases therefore, F_m will increase, while F_g remains the same. Let AB be displaced by dx downwards. Differentiating Eq. (i) w.r.t. x, we get

$$\frac{dF_m}{dx} = -\frac{\mu_0}{2\pi}\frac{i_1 i_2}{x^2}dx = \frac{-\mu_0}{2\pi}\frac{i_1 i_2}{d^2}dx \Rightarrow dF_m = -\left(\frac{mg}{d}\right)dx \quad \text{...(ii)}$$

i.e., restoring force, $dF_m \propto -dx$

Hence, the motion of wire is simple harmonic.

From Eqs. (ii) and (iii), we can write

$$dF_m = -\left(\frac{mg}{d}\right).dx \qquad (x = d)$$

$$\therefore \quad \text{Acceleration of wire } a = \frac{dF_m}{m} = -\left(\frac{g}{d}\right)\cdot dx$$

Hence, period of oscillation

$$T = 2\pi\sqrt{\left|\frac{dx}{a}\right|} = 2\pi\sqrt{\frac{|\text{displacement}|}{|\text{acceleration}|}}$$

or $$T = 2\pi\sqrt{\frac{d}{g}} = 2\pi\sqrt{\frac{0.01}{9.8}} \quad \text{or} \quad T = 0.2 \text{ s}$$

EXAMPLE 2.13

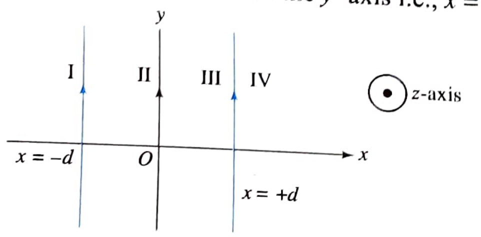
Three infinitely long thin wires, each carrying current i in the same direction, are in the x–y plane of a gravity free space. The central wire is along the y–axis while the other two are along $x = \pm d$.

(a) Find the locus of the points for which the magnetic field B is zero.

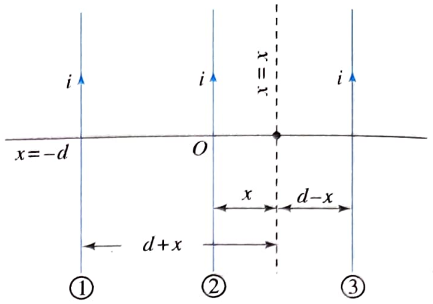
(b) If the central wire is displaced along the z-direction by a small amount and released, show that it will execute simple harmonic motion. If the linear density of the wire is l, find the frequency of oscillation.

Sol.

(a) Magnetic field will be zero on the y–axis i.e., $x = 0 = z$



magnetic field cannot be zero in region I and region IV because in region I magnetic field is along positive z-direction due to all the three wires, while in region IV magnetic field is along negative z-axis due to all the three wires. It can be zero only in region II and III.



Let magnetic field be zero on line ($z = 0$) and $x = x$. Then magnetic field on this line due to wires (1) and (2) will be along negative z-axis and due to wire (3) along positive z-axis. Thus,

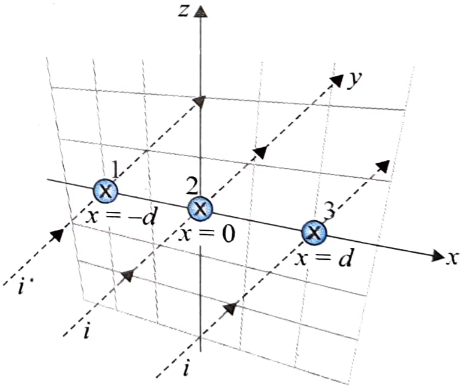
$$B_1 + B_2 = B_3$$

or $$\frac{\mu_0}{2\pi}\frac{i}{d+x} + \frac{\mu_0}{2\pi}\frac{i}{x} = \frac{\mu_0}{2\pi}\frac{i}{d-x}$$

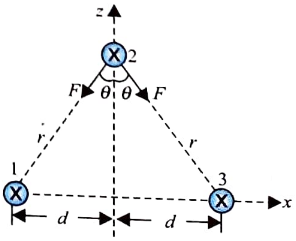
or $$\frac{1}{d+x} + \frac{1}{x} = \frac{1}{d-x}$$

This equation gives $x = \pm\dfrac{d}{\sqrt{3}}$ where magnetic field is zero.

(b) In this part we change our coordinate axes system just for better understanding.



There are three wires (1), (2) and (3) as shown in figure. If we displace wire (2) toward the z-axis, then force of attraction per unit length between wires (1 and 2) and (
and 3) will be given by



$$F = \frac{\mu_0}{2\pi}\frac{i^2}{r}$$

The components of F along x-axis will be cancelled out. Net resultant force will be towards negative z-axis (or mean position) and will be given by

$$F_{\text{net}}=\frac{\mu_0}{2\pi}\frac{i^2}{r}(2\cos\theta)=2\left\{\frac{\mu_0}{2\pi}\frac{i^2}{r}\right\}\frac{z}{r}=\frac{\mu_0}{\pi}\frac{i^2}{(z^2+d^2)}z$$

If $z << d$, then $z^2 + d^2 = d^2$ and $F_{\text{net}}=-\left(\frac{\mu_0}{\pi}\frac{i^2}{d^2}\right).z$

Negative sign implies that F_{net} is restoring in nature

Therefore, $F_{\text{net}} \propto -z$

i.e., the wire will oscillate simple harmonically.

Let a be the acceleration of wire in this position and λ is the mass per unit length of this wire then

$$F_{\text{net}}=\lambda.a=-\left(\frac{\mu_0}{\pi}\frac{i^2}{d^2}\right)z \quad \text{or} \quad a=-\left(\frac{\mu_0 i^2}{\pi\lambda d^2}\right).za$$

Therefore, frequency of oscillation

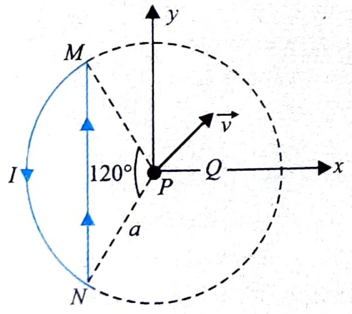
$$f=\frac{1}{2\pi}\sqrt{\frac{a}{z}}=\frac{1}{2\pi}\frac{i}{d}\sqrt{\frac{\mu_0}{\pi\lambda}}=f=\frac{i}{2\pi d}\sqrt{\frac{\mu_0}{\pi\lambda}}$$

EXAMPLE 2.14

A wire loop carrying a current is placed in the x-y plane as shown in figure.

(a) If a particle with charge $+Q$ and mass m is placed at the center P and given a velocity $\vec{v}$ along NP, find its instantaneous acceleration.

(b) If an external uniform magnetic induction field $\vec{B}=B\hat{i}$ is applied, find the force and torque acting on the loop.



Sol.

(a) As in case of current-carrying straight conductor and arc, the magnitude of B is given by

$$B_1=\frac{\mu_0 I}{4\pi d}(\sin\alpha+\sin\beta) \text{ and } B_2=\frac{\mu_0 I\phi}{4\pi r}$$

So in accordance with right hand screw rule,

$$\vec{B}_1=\left(\vec{B}_w\right)=\frac{\mu}{4\pi}\frac{I}{(a\cos 60^o)}\times 2\sin 60^\circ(-\hat{k}) \text{ and}$$

$$\vec{B}_2=\left(\vec{B}\right)_{MN}=\frac{\mu_0 I}{2a}\left[\frac{2\pi/3}{2\pi}\right](\hat{k})$$

and hence net $\vec{B}$ at P due to the given loop

$$\vec{B}=\vec{B}_w+\vec{B}_A \Rightarrow \vec{B}=\frac{\mu_o}{4\pi}\frac{2I}{a}\left[\sqrt{3}-\frac{\pi}{3}\right]\left(-\hat{k}\right) \quad \text{...(i)}$$

Now as force on charged particle in a magnetic field is given by $\vec{F}=q\left(\vec{v}\times\vec{B}\right)$

so here, $\vec{F}=qvB\sin 90^\text{o}$ along PF

i.e. $\vec{F}=\frac{\mu_0}{4\pi}\frac{2QvI}{a}\left[\sqrt{3}-\frac{\pi}{3}\right]$ along PF

and so $\vec{a}=\frac{\vec{F}}{m}=10^{-7}\frac{2QvI}{ma}\left[\sqrt{3}-\frac{\pi}{3}\right]$ along PF

(b) As $d\vec{F}=Id\vec{L}\times\vec{B}$, so $\vec{F}=\int Id\vec{L}\times\vec{B}$

As here I and $\vec{B}$ are constant

$$\vec{F}=\left[\oint d\vec{L}\right]\times\vec{B}=0 \qquad \left[\text{as} \oint d\vec{L}=0\right]$$

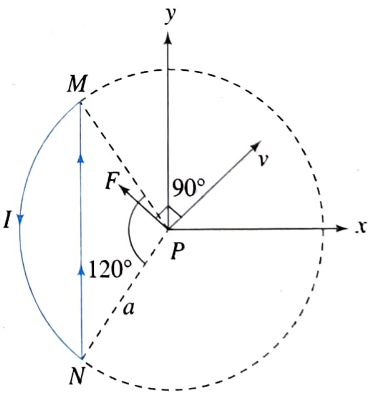
Further as area of coil,

$$\vec{S}=\left[\frac{1}{3}\pi a^2-\frac{1}{2}.2a\sin 60^\circ\times a\cos 60^\circ\right]\hat{k}=a^2\left[\frac{\pi}{3}-\frac{\sqrt{3}}{4}\right]\hat{k}$$

so $\quad \vec{M}=I\vec{S}=Ia^2\left[\frac{\pi}{3}-\frac{\sqrt{3}}{4}\right]\hat{k}$

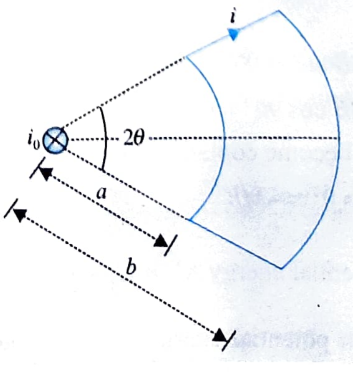
and hence $\vec{\tau}=\vec{M}\times\vec{B}=Ia^2B\left[\frac{\pi}{3}-\frac{\sqrt{3}}{4}\right]\left(\hat{k}\times\hat{i}\right)$

$$\Rightarrow \quad \vec{\tau}=Ia^2B\left[\frac{\pi}{3}-\frac{\sqrt{3}}{4}\right]\hat{j}\text{ Nm}$$



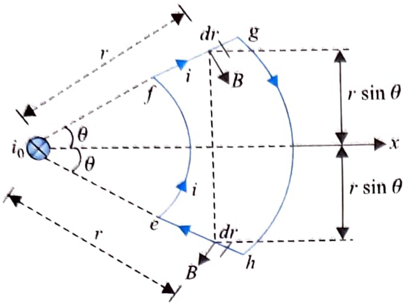
EXAMPLE 2.15

A loop, carring a current i, lying in the plane of the paper, is in the field of a long straight wire with current i_0 (inward) as shown in figure. Find the torque acting on the loop.



Sol. Here the field is tangential at every point on the circular portions and hence the forces acting on these segments are zero.

Now consider two small elements of length dr at a distance r from the current element symmetrically as shown in figure.



The magnitude of the force experienced by each element is

$$dF = i\,B\,dr \qquad (\because \; d\vec{F} = i d\vec{l} \times \vec{B})$$

$$\left\{\text{where } B = \frac{\mu_0 i_0}{2\pi r}\right\}$$

$$dF = \frac{\mu_0 i i_0}{2\pi r}\,dr$$

Force on fg will be in inward direction and on eh will be in outward direciton.

So, torque about x-axis: $d\tau = (2r\sin\theta)\dfrac{\mu_0 i_0 i}{2\pi r}\,dr$

To obtain the total torque, we integrate the above expression within proper limit

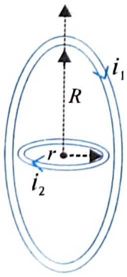
$$\tau = \frac{\mu_0 i_0 i}{\pi}\sin\theta\int_a^b dr = \frac{\mu_0 i_0 i}{\pi}[\sin\theta](b-a)$$

EXAMPLE 2.16

A coil of radius R carries current i_1. Another concentric coil of radius r ($r << R$) carries current i_2. Planes of two coils are mutually perpendicular and both the coils are free to rotate about common diameter. Find maximum kinetic energy of smaller coil when both the coils are released, masses of coils are M and m, respectively.

Sol. Magnetic induction at center due to current in larger coil is

$$B = \frac{\mu_0 i_1}{2R} \qquad \text{...(i)}$$



Magnetic dipole moment of smaller coil

$$M = \pi r^2 i_2 \qquad \text{...(ii)}$$

Initially M and B are at 90°.

So, $U_1 = -MB\cos 90° = 0$

When the coils become coplanar

$U_2 = -MB\cos 0° = -MB$

Decrease in potential energy $\Delta U = U_1 - U_2 = MB = \dfrac{\mu_0 i_1 i_2 \pi r^2}{2R}$

This decrease in potential energy converts into kinetic energy. Here the kinetic energies are maximum when the coils become coplanar. The two coils rotate due to their mutual interaction only and if one coil rotates clockwise, then the other coil rotates anticlockwise.

Let ω_1 and ω_2 be the angular velocities of larger and smaller coils when they become coplanar. According to law of conservation of angular momentum,

$$I_1\omega_1 = I_2\omega_2$$

and according to law of conservation of energy

$$\frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2 = \Delta U$$

From above two equations, we get

$$\frac{1}{2}I_1\left(\frac{I_2\omega_2}{I_1}\right)^2 + \frac{1}{2}I_2\omega_2^2 = \Delta U \Rightarrow \frac{1}{2}I_2\omega_2^2\left(\frac{I_2}{I_1}+1\right) = \Delta U$$

$$\Rightarrow \quad \frac{1}{2}I_2\omega_2^2 = \frac{\Delta U I_1}{I_1 + I_2}$$

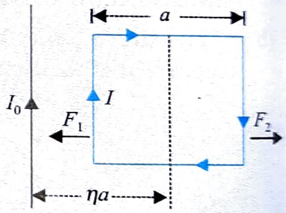
So maximum kinetic energy of smaller coil is given by

$$\Rightarrow \quad \frac{1}{2}I_2\omega_2^2 = \left(\frac{\mu_0 i_1 i_2 \pi r^2}{2R}\right)\left(\frac{(1/2)MR^2}{(1/2)MR^2 + (1/2)mr^2}\right)$$

$$= \frac{\mu_0 i_1 i_2 \pi r^2 MR}{2(MR^2 + mr^2)}$$

EXAMPLE 2.17

A square frame carrying a current $I = 0.90$ A is located in the same plane as a long straight wire carrying a current $I_0 = 5$ A. The frame side has a length $a = 8$ cm. The axis of the frame passing through the mid-points of the opposite sides is parallel to the wire and is separated from it by a distance $h = 15$ times greater than the side of the frame. Find:



(a) ampere force acting on the frame.

(b) the mechanical work to be performed in order to turn the frame through 180° about its axis, with the current maintained constant.

Sol.

(a) Force of attraction between parallel currents

$$F_1 = \left[\frac{\mu_0}{4\pi}\frac{2I\,I_0}{(2\eta-1)\dfrac{a}{2}}\right]a$$

Similarly force of repulsion between antiparallel currents

$$F_2 = \left[\frac{\mu_0}{4\pi}\frac{2I\,I_0}{(2\eta+1)\dfrac{a}{2}}\right]a$$

Net force of attraction between the square frame and the long straight wire

$$F = F_1 - F_2 = \frac{2\mu_0}{\pi}\frac{I I_0}{4\eta^2 - 1}$$

Putting values, $F = 4 \times 10^{-9}\ N = 40$ nN

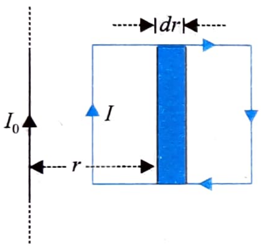
(b) Initial energy; $U_1 = -\int d\vec{M}\cdot\vec{B}$

$$U_1 = -\int_{(2\eta-1)a/2}^{(2\eta+1)a/2} (I\,a\,dr)\frac{\mu_0 I_0}{2\pi r}\cos 0°$$

$$= -\frac{\mu_0 I I_0 a}{2\pi}\int_{(2\eta-1)a/2}^{(2\eta+1)a/2}\frac{dr}{r} = -\frac{\mu_0 I I_0 a}{2\pi}\ln\left[\frac{2\eta+1}{2\eta-1}\right]$$

Final energy: $U_2 = -\int dM\,B\cos 180°$

$$= \int dM\,B = \frac{\mu_0 I I_0 a}{2\pi}\ln\left(\frac{2\eta+1}{2\eta-1}\right)$$

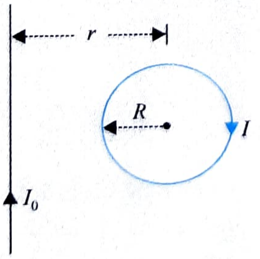


Work done: $W = U_2 - U_1$

$$= \frac{\mu_0 I I_0 a}{\pi}\ln\left[\frac{2\eta+1}{2\eta-1}\right] = 9.6 \times 10^{-9}\ \text{J} = 9.6\ \text{nJ}$$

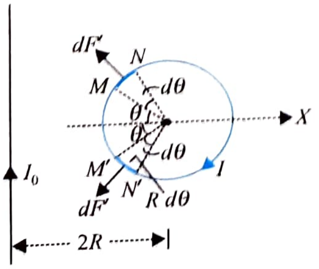
EXAMPLE 2.18

A long straight wire is coplanar with a current carrying circular loop of radius R as shown in figure. Current flowing through wire and the loop is I_0 and I, respectively. If distance between center of loop and wire is $r = 2R$, calculate force of attraction between the wire and the loop.



Sol. Consider two equal elemental arc lengths MN and $M'N'$ of lengths $Rd\theta$. The distance of each elemental length from straight wire is

$$x = (r - R\cos\theta)$$
$$= R(2 - \cos\theta) \quad \text{...(i)}$$



The magnetic induction at these elemental arc lengths due to current I_0 in the wire is given by

$$B = \frac{\mu_0 I_0}{2\pi x} = \frac{\mu_0 I_0}{2\pi R(2-\cos\theta)} \quad \text{...(ii)}$$

The magnitude of the force on each elemental length

$$dF' = BI(dl) = BI(Rd\theta)$$

$$= \frac{\mu_0 I_0 I\,R\,d\theta}{2\pi R(2-\cos\theta)} = \frac{\mu_0 I_0 I\,d\theta}{2\pi(2-\cos\theta)}$$

The direction of dF' is shown in figure.

The resultant force dF is given by

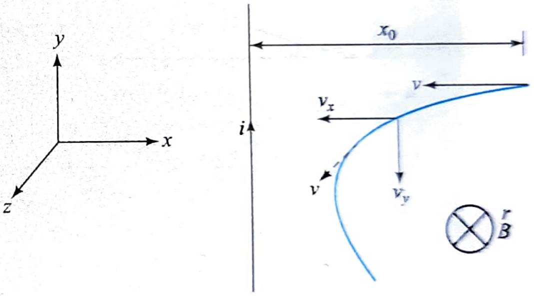
$$dF = dF'\cdot 2\cos\theta = \frac{\mu_0 I_0 I\,d\theta}{2\pi(2-\cos\theta)}\times 2\cos\theta$$

$$= \frac{\mu_0 I_0 I}{\pi}\left[\frac{\cos\theta\,d\theta}{2-\cos\theta}\right]$$

$$F = \frac{\mu_0 I_0 I}{\pi}\int_0^{\pi}\left(\frac{\cos\theta}{2-\cos\theta}\right)d\theta = \mu_0 I_0 I\left(\frac{2-\sqrt{3}}{\sqrt{3}}\right)$$

EXAMPLE 2.19

A long straight wire carries a current i. A particle having a positive charge q and mass m, kept at distance x_0 from the wire is projected towards it with speed v. Find the closest distance of approach of charged particle to the wire.



Sol. Magnetic field due to long wire is

$$\vec{B} = -\frac{\mu_0 i\hat{k}}{2\pi x}$$

Let the velocity of the charged particle at any instant be

$$\vec{v} = (v_x\hat{i} + v_y\hat{j})$$

Magnetic force on the charge,

$$\vec{F} = q(\vec{v}\times\vec{B}) = -\frac{\mu_0 q i v_y}{2\pi x}\hat{i} + \frac{\mu_0 q i v_x}{2\pi x}\hat{j} \quad \text{...(i)}$$

From Eq. (i), y component of acceleration is

$$a_y = \frac{dv_y}{dt} = \frac{\mu_0 q i v_x}{2\pi m x} = \frac{\mu_0 q}{2\pi m x}\,i\,\frac{dx}{dt}$$

At the minimum separation, velocity of particle is $-v\hat{j}$.
[Closest distance will be when velocity becomes parallel to wire or in –ve y-direction]

$$\int_0^{-v} dv_y = \frac{\mu_0 q i}{2\pi m}\int_{x_0}^{x_{\min}}\frac{dx}{x} \Rightarrow -v = \frac{\mu_0 q i}{2\pi m}\ln\left|\frac{x_{\min}}{x_0}\right|$$

Thus, $x_{\min} = x_0 e^{-2\pi m v/\mu_0 q i}$

EXAMPLE 2.20

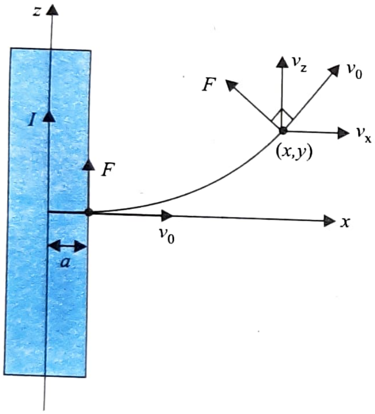
From the surface of a round wire of radius a carrying a direct current i, a positive charge q escapes with a velocity v_0 perpendicular to the surface. Find what the maximum distance of the electron will be from the axis of the wire before it turns back due to the action of the magnetic field generated by the current.

Sol. Let at any instant velocity of charge is v, then force on the charge due to magnetic field caused by wire is given by

$$\vec{F} = q(\vec{v}\times\vec{B})$$

$$ma_x\hat{i} + ma_y\hat{j} = q(v_x\hat{i} + v_z\hat{k})\times\frac{\mu_0 I}{2\pi x}\hat{j}$$

$$\Rightarrow \quad ma_x\hat{i} + ma_y\hat{j} = qv_x\frac{\mu_0 I}{2\pi x}\hat{k} - qv_z\frac{\mu_0 I}{2\pi x}\hat{i}$$



$$\Rightarrow \quad ma_x = -\frac{q\mu_0 I}{2\pi}\frac{v_z}{x}$$

$$\Rightarrow \quad a_x = -\frac{q\mu_0 I}{2\pi m}\frac{v_z}{x}$$

[But $v_0^2 = v_x^2 + v_z^2 \Rightarrow v_z = \pm\sqrt{v_0^2 - v_x^2}$

Here we have to take $v_z = +\sqrt{v_0^2 - v_x^2}$ because a_x is negative]

$$\Rightarrow \quad v_x\frac{dv_x}{dx} = -\frac{q\mu_0 I}{2\pi m}\frac{\sqrt{v_0^2 - v_x^2}}{x}$$

$$\Rightarrow \quad \int_{v_0}^{0}\frac{v_x dv_x}{\sqrt{v_0^2 - v_x^2}} = -\frac{q\mu_0 I}{2\pi m}\int_{a}^{x_{\max}}\frac{dx}{x}$$

$$\Rightarrow \quad \left[-\sqrt{v_0^2 - v_x^2}\right]_{v_0}^{0} = -\frac{q\mu_0 I}{2\pi m}\ln\left(\frac{x_{\max}}{a}\right)$$

$$\Rightarrow \quad v_0 = \frac{q\mu_0 I}{2\pi m}\ln\left(\frac{x_{\max}}{a}\right)$$

$$\Rightarrow \quad x_{\max} = a\,e^{\frac{2\pi m v_0}{q\mu_0 I}}$$

when x is maximum, x component of velocity is zero.

Exercises

Single Correct Answer Type

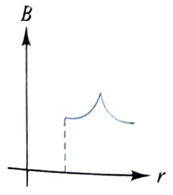
1. A point charge is moving in clockwise direction in a circle with constant speed. Consider the magnetic field produced by the charge at a fixed point P (not at the center of circle) on the axis of the circle. Then,
(1) it is constant in magnitude only
(2) it is constant in direction only
(3) it is constant both in direction and magnitude
(4) it is constant neither in magnitude nor in direction

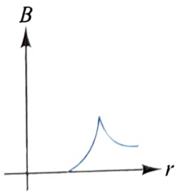
2. A stream of electrons is projected horizontally to the right. A straight conductor carrying a current is supported parallel to the electron stream and above it. If the current in the conductor is from left to right, what will be the effect on the electron stream?
(1) The electron stream will be pulled upward
(2) The electron stream will be pulled downward
(3) The electron stream will be retarded
(4) The electron stream will be speeded up toward the right

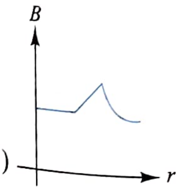
3. An electron is ejected from the surface of a long, thick straight conductor carrying a current, initially in a direction perpendicular to the conductor. The electron will
(1) ultimately return to the conductor
(2) move in a circular path around the conductor
(3) gradually move away from the conductor along a spiral
(4) move in a helical path, with the conductor as the axis

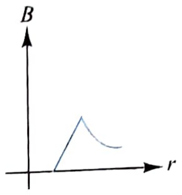
4. A circular loop is kept in that vertical plane which contains the north-south direction. It carries a current that is toward north at the topmost point. Let A be a point on the axis of the circle to the east of it and B a point on this axis to the west of it. The magnetic field due to the loop
(1) is toward east at A and toward west at B
(2) is toward west at A and toward east at B
(3) is toward east at both A and B
(4) is toward west at both A and B

5. A current i is uniformly distributed over the cross section of a long hollow cylindrical wire of inner radius R_1 and outer radius R_2. Magnetic field B varies with distance r from the axis of the cylinder as

(1) 

(2) 

(3) 

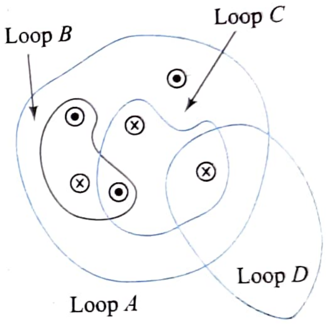
(4) 

6. A circular current carrying coil has a radius R. The distance from the center of the coil on the axis where the magnetic induction will be $(1/8)^{\text{th}}$ of its value at the center of the coil, is
(1) $R/\sqrt{3}$ (2) $R\sqrt{3}$
(3) $2R\sqrt{3}$ (4) $\left(2\sqrt{3}\right)R$

7. A square conducting loop of side length L carries a current I. The magnetic field at the center of the loop is
(1) independent of L
(2) proportional to L
(3) inversely proportional to L
(4) linearly proportional to L

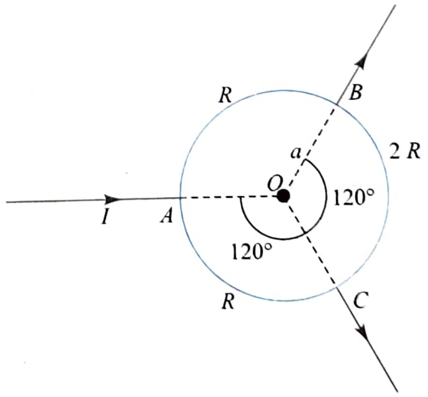
8. A current of $1/(4\pi)$ ampere is flowing in a long straight conductor. The line integral of magnetic induction around a closed path enclosing the current carrying conductor is
(1) 10^{-7} Wb m^{-1} (2) $4\pi \times 10^{-7}$ Wb m^{-1}
(3) $16\pi^2 \times 10^{-7}$ Wb m^{-1} (4) zero

9. Consider six wires coming into or out of the page, all with the same current. Rank the line integral of the magnetic field (from most positive to most negative) taken counterclockwise around each loop shown.



(1) $B > C > D > A$
(2) $B > C = D > A$
(3) $B > A > C = D$ (4) $C > B = D > A$

10. The resistances of three parts of a circular loop are as shown in figure. The magnetic field at the center O is (current enters at A and leaves at B and C as shown)



(1) $\dfrac{\mu_0 I}{6a}$ (2) $\dfrac{\mu_0 I}{3a}$

(3) $\dfrac{2}{3}\dfrac{\mu_0 I}{a}$ (4) zero

11. A very long straight conducting wire, lying along the z-axis, carries a current of 2 A. The integral $\oint \vec{B} \cdot d\vec{l}$ is computed along the straight line PQ, where P has the coordinates

(2 cm, 0, 0) and Q has the coordinates (2 cm, 2 cm, 0). The integral has the magnitude (in SI units)

(1) $\frac{\pi}{2}\times 10^{-7}$ (2) $8\pi\times 10^{-7}$

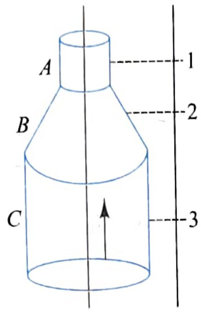
(3) $2\pi\times 10^{-7}$ (4) $\pi\times 10^{-7}$

12. Five very long, straight insulated wires are closely bound together to form a small cable. Currents carried by the wires are: $I_1 = 20$ A, $I_2 = -6$ A, $I_3 = 12$ A, $I_4 = -7$ A, $I_5 = 18$ A. (Negative currents are opposite in direction to the positive.) The magnetic field induction at a distance of 10 cm from the cable is (current enters at A and leaves at B and C as shown)

(1) 5 μT (2) 15 μT

(3) 74 μT (4) 128 μT

13. A long, straight, hollow conductor (tube) carrying a current has two sections A and C of unequal cross sections joined by a conical section B. 1, 2, and 3 are points on a line parallel to the axis of the conductor. The magnetic fields at 1, 2 and 3 have magnitudes B_1, B_2 and B_3. Then,



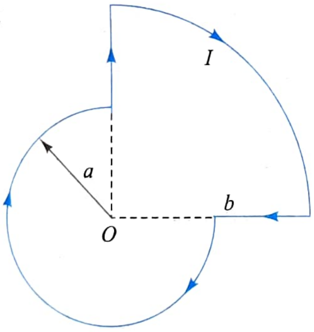
(1) $B_1 = B_2 = B_3$

(2) $B_1 = B_2 \neq B_3$

(3) $B_1 < B_2 < B_3$

(4) B_2 cannot be found unless the dimensions of the section B are known

14. The magnetic induction at center O (figure) is



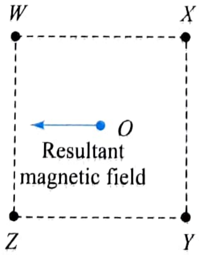
(1) $\frac{\mu_0 I}{2a}+\frac{\mu_0 I}{2b}\otimes$

(2) $\frac{3\mu_0 I}{8a}+\frac{\mu_0 I}{8b}\otimes$

(3) $\frac{3\mu_0 I}{8a}-\frac{\mu_0 I}{8b}\otimes$

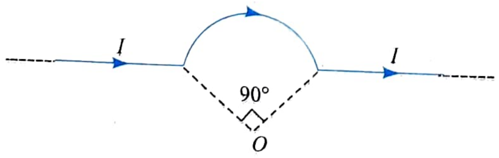
(4) $\frac{3\mu_0 I}{8a}+\frac{\mu_0 I}{8b}\odot$

15. Four parallel conductors, carrying equal currents, pass vertically through the four corners of a square $WXYZ$. In two conductors, the current is flowing into the page, and in the other two out of the page. In what directions must the currents flow to produce a resultant magnetic field in the direction shown at O, the center of the square ?



	Into the page	Out of the page
(1)	W and Y	X and Z
(2)	X and Z	W and Y
(3)	W and Z	X and Y
(4)	W and X	Y and Z

16. The magnetic field at center O of the arc in figure is



(1) $\frac{\mu_0 I}{4\pi\times r}\left[\sqrt{2}+\pi\right]$ (2) $\frac{\mu_0 I}{2\pi r}\left[\frac{\pi}{4}+(\sqrt{2}-1)\right]$

(3) $\frac{\mu_0}{4\pi}\times\frac{I}{r}\left[\sqrt{2}-\pi\right]$ (3) $\frac{\mu_0}{4\pi}\times\frac{I}{r}\left[\sqrt{2}+\frac{\pi}{4}\right]$

17. A wire of length l is used to form a coil. The magnetic field at its center for a given current in it is minimum if the coil has

(1) 4 turns (2) 2 turns

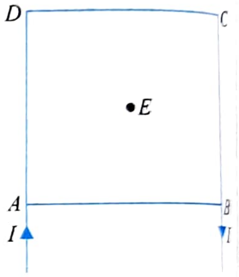
(3) 1 turn (4) data is not sufficient

18. A steady current is flowing in a circular coil of radius R, made up of a thin conducting wire. The magnetic field at the center of the loop is B_L. Now, a circular loop of radius R/n is made from the same wire without changing its length, by unfolding and refolding the loop, and the same current is passed through it. If new magnetic field at the center of the coil is B_C, then the ratio B_L/B_C is

(1) $1 : n^2$ (2) $n^{1/2}$

(3) $n : 1$ (4) none of these

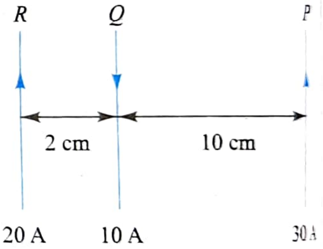
19. Current I enters at A in a square loop of uniform resistance and leaves at B. The ratio of magnetic field at E, the center of square, due to segment AB to that due to DC is



(1) 1 (2) 2

(3) 3 (4) 4

20. Three long, straight and parallel wires are arranged as shown in figure. The force experienced by 10 cm length of wire Q is



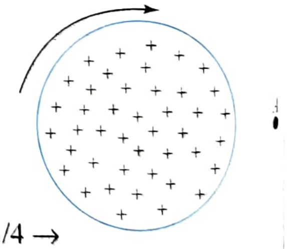
(1) 1.4×10^{-4} N toward the right

(2) 1.4×10^{-4} N toward the left

(3) 2.6×10^{-4} N toward the right

(4) 2.6×10^{-4} N toward the left

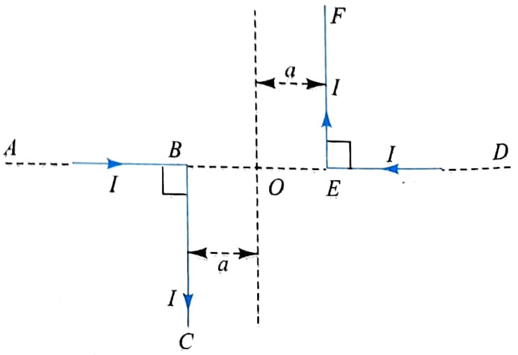
21. A positively charged disk is rotated clockwise as shown in figure. The direction of the magnetic field at point A in the plane of the disk is



(1) ⊗ (into the page) (2) 3/4 →

(3) ← 3/4 (4) ⊙ (out of the page)

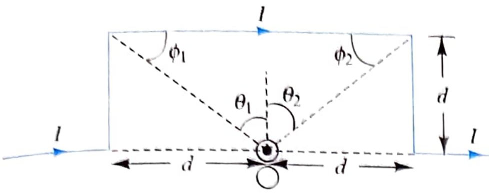
22. Two long thin wires ABC and DEF are arranged as shown in figure. They carry equal currents I as shown. The magnitude of the magnetic field at O is



(1) zero (2) $\mu_0 I/4\pi a$

(3) $\mu_0 I/2\pi a$ (4) $\mu_0 I/2\sqrt{2}\ \pi a$

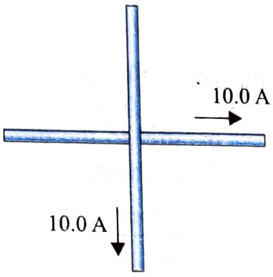
23. The magnetic field at O due to current in the infinite wire forming a loop as shown in figure is



(1) $\dfrac{\mu_0 I}{2\pi d}(\cos\phi_1+\cos\phi_2)$ (2) $\dfrac{\mu_0}{4\pi}\dfrac{2I}{d}(\tan\theta_1+\tan\theta_2)$

(3) $\dfrac{\mu_0}{4\pi}\dfrac{I}{d}(\sin\phi_1+\sin\phi_2)$ (4) $\dfrac{\mu_0}{4\pi}\dfrac{I}{d}(\cos\theta_1+\sin\theta_2)$

24. Two very long, straight wires carrying, currents as shown in figure. Find all locations where the net magnetic field is zero.



(1) $y=\sqrt{2}\,x$

(2) $y=x$

(3) $y=-x$

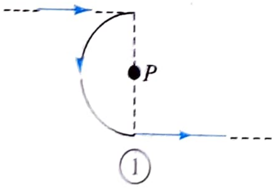
(4) $y=-(x/2)$

25. A current I flows through a thin wire shaped as regular polygon of n sides which can be inscribed in a circle of radius R. The magnetic field induction at the center of polygon due to one side of the polygon is

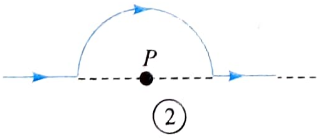
(1) $\dfrac{\mu_0 I}{\pi R}\left(\tan\dfrac{\pi}{n}\right)$ (2) $\dfrac{\mu_0 I}{4\pi R}\tan\dfrac{\pi}{n}$

(3) $\dfrac{\mu_0 I}{2\pi R}\left(\tan\dfrac{\pi}{n}\right)$ (4) $\dfrac{\mu_0 I}{2\pi R}\left(\cos\dfrac{\pi}{n}\right)$

26. Figure shows three cases: in all cases the circular part has radius r and straight ones are infinitely long. For the same current the ratio of field B at center P in the three cases $B_1 : B_2 : B_3$ is



①



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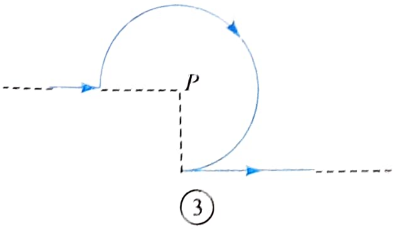


③

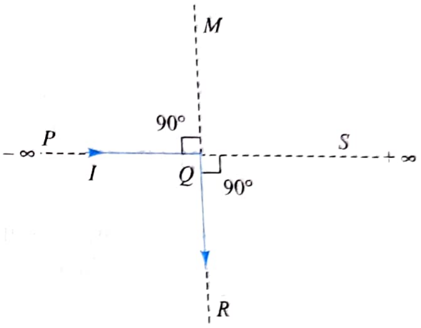
(1) $\left(-\dfrac{\pi}{2}\right):\left(\dfrac{\pi}{2}\right):\left(\dfrac{3\pi}{4}-\dfrac{1}{2}\right)$

(2) $\left(-\dfrac{\pi}{2}+1\right):\left(\dfrac{\pi}{2}+1\right):\left(\dfrac{3\pi}{4}+\dfrac{1}{2}\right)$

(3) $\left(-\dfrac{\pi}{2}\right):\left(\dfrac{\pi}{2}\right):\left(\dfrac{3\pi}{4}\right)$

(4) $\left(-\dfrac{\pi}{2}-1\right):\left(\dfrac{\pi}{2}-\dfrac{1}{4}\right):\left(\dfrac{3\pi}{4}+\dfrac{1}{2}\right)$

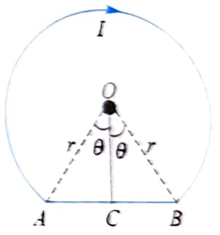
27. An infinitely long conductor PQR is bent to form a right angle as shown in figure. A current I flows through PQR. The magnetic field due to this current carrying conductor at the point M is B_1. Now, another infinitely long straight conductor QS, is connected at Q so that the current is $\dfrac{1}{2}I$ in QR as well as in QS, the current in PQ remaining unchanged. The magnetic field at M is now B_2. The ratio B_1/B_2 is given by



(1) 1/2 (2) 1

(3) 2/3 (4) 2

28. A wire is bent in the form of a circular arc with a straight portion AB. Magnetic induction at O when current I flowing in the wire, is



(1) $\dfrac{\mu_0}{2r}(\pi-\theta+\tan\theta)$

(2) $\dfrac{\mu_0 I}{2\pi r}(\pi+\theta-\tan\theta)$

(3) $\dfrac{\mu_0 I}{2\pi r}(\pi-\theta+\tan\theta)$

(4) $\dfrac{\mu_0 I}{2\pi r}(-\tan\theta+\pi-\theta)$.

29. Two concentric coils, each of radius equal to 2π cm, are placed at right angles to each other. Currents of 3 A and 4 A, respectively, are flowing through the two coils. The magnetic induction, in Wb m^{-2}, at the center of the coils will be $[\mu_0 = 4\pi\times 10^{-7}\ \text{Wb(Am}^{-1})]$

(1) 5×10^{-5} (2) 7×10^{-5}

(3) 12×10^{-5} (4) 10^{-5}

30. The field due to a wire of n turns and radius r which carries a current I is measured on the axis of the coil at a small distance h from the center of the coil. This is smaller than the field at the center by the fraction:

(1) $\dfrac{3}{2}\dfrac{h^2}{r^2}$ (2) $\dfrac{2}{3}\dfrac{h^2}{r^2}$

(3) $\dfrac{3}{2}\dfrac{r^2}{h^2}$ (4) $\dfrac{2}{3}\dfrac{r^2}{h^2}$

31. A coaxial cable consists of a thin inner conductor fixed along the axis of a hollow outer conductor. The two conductors carry equal currents in opposite directions. Let B_1 and B_2 be the magnetic fields in the region between the conductors and outside the conductor, respectively. Then,

(1) $B_1 \neq 0, B_2 \neq 0$ (2) $B_1 = B_2 = 0$

(3) $B_1 \neq 0, B_2 = 0$ (4) $B_1 = 0, B_2 \neq 0$

32. Two identical wires A and B have the same length l and carry the same current I. Wire A is bent into a circle of radius R and wire B is bent to form a square of side a. If B_1 and B_2 are the values of magnetic induction at the center of the circle and the center of the square, respectively, then the ratio B_1/B_2 is

(1) $(\pi^2/8)$ (2) $\left(\pi^2/8\sqrt{2}\right)$

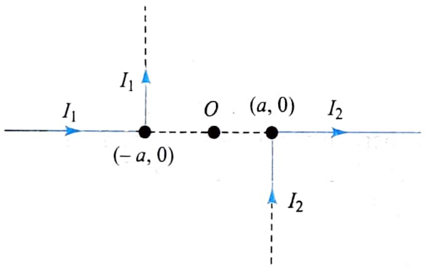
(3) $(\pi^2/16)$ (4) $\left(\pi^2/16\sqrt{2}\right)$

33. Two parallel wires carrying equal currents in opposite directions are placed at $x = \pm a$ parallel to y-axis with $z = 0$. Magnetic field at origin O is B_1 and at P $(2a, 0, 0)$ is B_2. Then, the ratio B_1/B_2 is

(1) 3 (2) 1/2

(3) 1/3 (4) 2

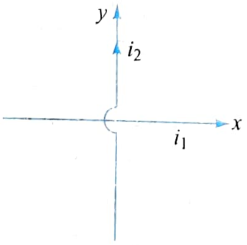
34. Currents I_1 and I_2 flow in the wires shown in figure. The field is zero at distance x to the right of O. Then



(1) $x = \left(\dfrac{I_1}{I_2}\right)a$ (2) $x = \left(\dfrac{I_2}{I_1}\right)a$

(3) $x = \left(\dfrac{I_1 - I_2}{I_1 + I_2}\right)a$ (4) $x = \left(\dfrac{I_1 + I_2}{I_1 - I_2}\right)a$

35. Two infinite wires, carrying currents i_1 and i_2, are lying along x- and y-axes, as shown in the x–y plane. Then,



(1) locus of points where B is zero is a circle

(2) locus of points where B is zero is a line

(3) B decays hyperbolically along any line parallel to x-axis

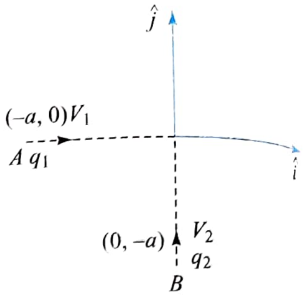
(4) B decays hyperbolically along any line parallel to y-axis

36. Two very long straight parallel wires carry steady currents i and $2i$ in opposite directions. The distance between the wires is d. At a certain instant of time, a point charge q is at a point equidistant from the two wires in the plane of the wires. Its instantaneous velocity $\vec{v}$ is perpendicular to this plane. The magnitude of the force due to the magnetic field acting on the charge at this instant is

(1) $\dfrac{\mu_0 iqv}{2\pi d}$ (2) $\dfrac{\mu_0 iqv}{\pi d}$

(3) $\dfrac{3\mu_0 iqv}{2\pi d}$ (4) zero

37. Two positive charges q_1 and q_2 are moving with velocities v_1 and v_2 when they are at points A and B, respectively, as shown in figure. The magnetic force experienced by charge q_1 due to the other charge q_2 is



(1) $\dfrac{\mu_0 q_1 q_2 v_1 v_2}{8\sqrt{2}\,\pi a^2}$ (2) $\dfrac{\mu_0 q_1 q_2 v_1 v_2}{4\sqrt{2}\,\pi a^2}$

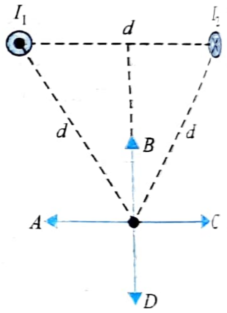
(3) $\dfrac{\mu_0 q_1 q_2 v_1 v_2}{2\sqrt{2}\,\pi a^2}$ (4) $\dfrac{\mu_0 q_1 q_2 v_1 v_2}{\sqrt{2}\,\pi a^2}$

38. An infinitely long current carrying wire carries current i. A charge of mass m and charge q is projected with speed v parallel to the direction of current at a distance r from it. Then, the radius of curvature at the point of projection is

(1) $\dfrac{2rmv}{q\mu_0 i}$ (2) $\dfrac{2\pi rmv}{q\mu_0 i}$

(3) r (4) cannot be determined

39. Figure shows two long wires carrying equal currents I_1 and I_2 flowing in opposite directions. Which of the arrows labeled A to D correctly represents the direction of the magnetic field due to the wires at a point located at an equal distance d from each wire?

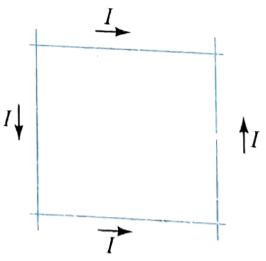


(1) A (2) B

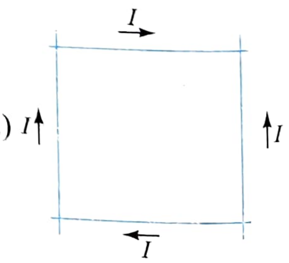
(3) C (4) D

40. In the figures given show four different sets of wires that cross each other without actually touching. The magnitude of the current is the same in all four cases and the directions of current flow are as indicated. For which configuration will the magnetic field at the center of the square formed by the wires be equal to zero?

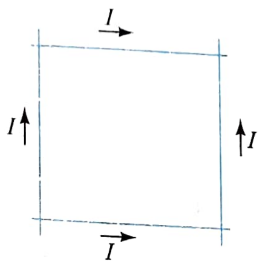
(1)



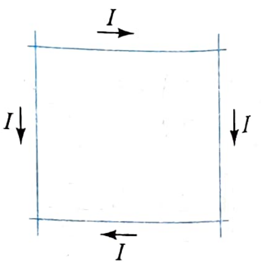
(2)



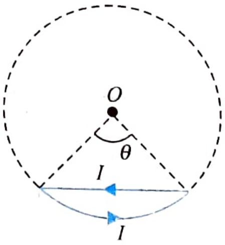
(3)



(4)

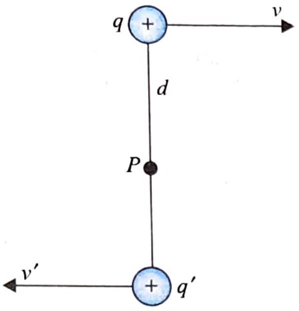


41. Figure shows a small loop carrying a current I. The curved portion is an arc of a circle of radius R and the straight portion is a chord to the same circle subtending an angle θ. The magnetic induction at center O is



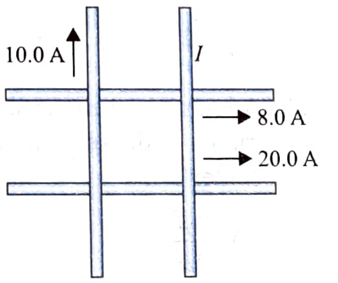
(1) zero
(2) always inward irrespective of the value of θ
(3) inward as long as θ is less than π
(4) always outward irrespective of the value of θ

42. Positive point charges $q = +8.00\ \mu C$ and $q' = +3.00\ \mu C$ are moving relative to an observer at point P, as shown in figure. The distance d is 0.120 m. When the two charges are at the locations shown in the figure, what are the magnitude and direction of the net magnetic field they produce at point P? (Take $v = 4.50 \times 10^6\ \text{ms}^{-1}$ and $v' = 9.00 \times 10^6\ \text{m s}^{-1}$.)



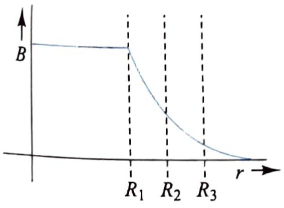
(1) 4.38×10^{-4} T, into the page
(2) 4.38×10^{-4} T, out of the page
(3) 2.16×10^{-4} T, into the page
(4) 2.16×10^{-4} T, out of the page

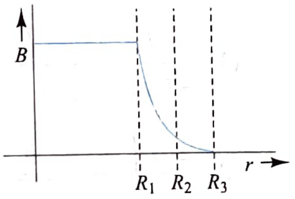
43. Four very long, current carrying wires in the same plane intersect to form a square 40.0 cm on each side, as shown in figure. Find the magnitude and direction of the current I so that the magnetic field at the center of the square is zero.

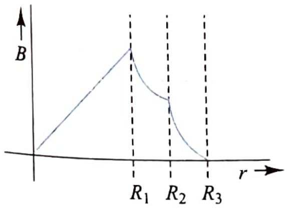


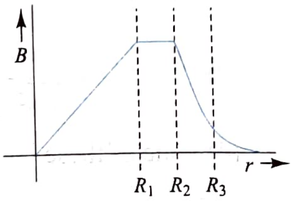
(1) 4.0 A toward the bottom of the page.
(2) 2.0 A toward the bottom of the page.
(3) 2.5 A toward the bottom of the page.
(4) 3.6 A toward the top of the page.

44. A coaxial cable is made up of two conductors. The inner conductor is solid and is of radius R_1 and the outer conductor is hollow of inner radius R_2 and outer radius R_3. The space between the conductors is filled with air. The inner and outer conductors are carrying currents of equal magnitudes and in opposite directions. Then the variation of magnetic field with distance from the axis is best plotted as

(1)


(2)


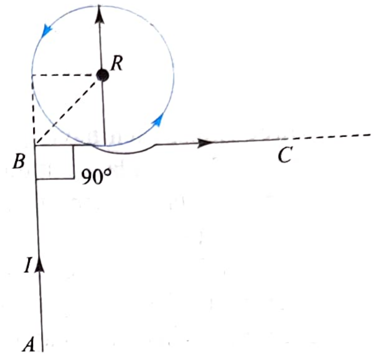
(3)


(4)


45. A particle is moving with velocity $\vec{v} = \hat{i} + 3\hat{j}$ and it produces an electric field at a point given by $\vec{E} = 2\hat{k}$. It will produce magnetic field at that point equal to (all quantities are in SI units)

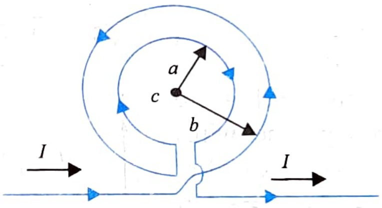
(1) $(6\hat{i} - 2\hat{j})\mu_0 \varepsilon_0$
(2) $(6\hat{i} + 2\hat{j})\mu_0 \varepsilon_0$
(3) zero
(4) cannot be determined from the given data

46. The magnetic field at the center of the circular loop as shown in figure, when a single wire is bent to form a circular loop and also extends to form straight sections, is



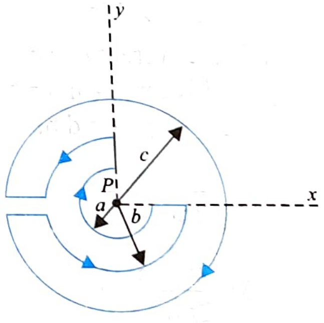
(1) $\dfrac{\mu_0 I}{2R} \odot$
(2) $\dfrac{\mu_0 I}{2R}\left(1 + \dfrac{1}{\pi\sqrt{2}}\right) \odot$
(3) $\dfrac{\mu_0 I}{2R}\left(1 - \dfrac{1}{\pi\sqrt{2}}\right) \otimes$
(4) $\dfrac{\mu_0 I}{R}\left(1 - \dfrac{1}{\pi\sqrt{2}}\right) \otimes$

47. An otherwise infinite, straight wire has two concentric loops of radii a and b carrying equal currents in opposite directions as shown in figure. The magnetic field at the common center is zero for



(1) $\dfrac{a}{b} = \dfrac{\pi - 1}{\pi}$
(2) $\dfrac{a}{b} = \dfrac{\pi}{\pi + 1}$
(3) $\dfrac{a}{b} = \dfrac{\pi - 1}{\pi + 1}$
(4) $\dfrac{a}{b} = \dfrac{\pi + 1}{\pi - 1}$

48. For $c = 2a$ if, the magnetic field at point P will be zero when

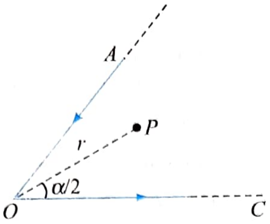


(1) $a = b$
(2) $a = \dfrac{3}{5}b$
(3) $a = \dfrac{5}{3}b$
(4) $a = \dfrac{1}{3}b$

49. The magnetic field at the center of an equilateral triangular loop of side $2L$ and carrying a current i is

(1) $\dfrac{9\mu_0 i}{4\pi L}$
(2) $\dfrac{3\sqrt{3}\mu_0 i}{4\pi L}$
(3) $\dfrac{2\sqrt{3}\mu_0 i}{\pi L}$
(4) $\dfrac{3\mu_0 i}{4\pi L}$

50. Two wires AO and OC carry equal currents i as shown in figure. One end of both the wires extends to infinity. Angle AOC is α. The magnitude of magnetic field at point P on the bisector of these two wires at a distance r from point O is



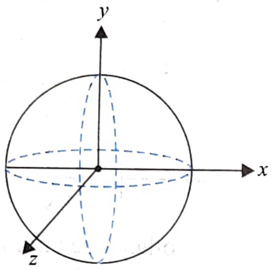
(1) $\frac{\mu_0}{2\pi}\frac{i}{r}\cot\left(\frac{\alpha}{2}\right)$

(2) $\frac{\mu_0}{4\pi}\frac{i}{r}\cot\left(\frac{\alpha}{2}\right)$

(3) $\frac{\mu_0}{2\pi}\frac{i}{r}\frac{\left(1+\cos\frac{\alpha}{2}\right)}{\sin\left(\frac{\alpha}{2}\right)}$

(4) $\frac{\mu_0}{4\pi}\frac{i}{r}\left(\frac{\alpha}{2}\right)$

51. Three rings, each having equal radius R, are placed mutually perpendicular to each other and each having its center at the origin of coordinate system. If current I is flowing through each ring, then the magnitude of the magnetic field at the common center is



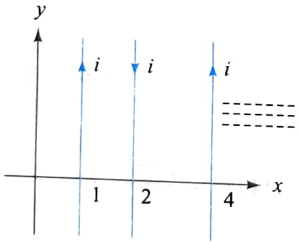
(1) $\sqrt{3}\frac{\mu_0 I}{2R}$

(2) zero

(3) $(\sqrt{2}-1)\frac{\mu_0 I}{2R}$

(4) $(\sqrt{3}-\sqrt{2})\frac{\mu_0 I}{2R}$

52. Equal currents $i = 1$ A are flowing through the wires parallel to y-axis located at $x = +1$ m, $x = +2$ m, $x = +4$ m and so on..., etc. but in opposite directions as shown in figure. The magnetic field (in tesla) at origin would be



(1) $-1.33 \times 10^{-7}\,\hat{k}$

(2) $1.33 \times 10^{-7}\,\hat{k}$

(3) $2.67 \times 10^{-7}\,\hat{k}$

(4) $-2.67 \times 10^{-7}\,\hat{k}$

53. Same current i is flowing in three infinitely long wires along positive x-, y- and z-directions. The magnetic field at a point $(0, 0, -a)$ would be

(1) $\frac{\mu_0 i}{2\pi a}(\hat{j}-\hat{i})$

(2) $\frac{\mu_0 i}{2\pi a}(\hat{i}+\hat{j})$

(3) $\frac{\mu_0 i}{2\pi a}(\hat{i}-\hat{j})$

(4) $\frac{\mu_0 i}{2\pi a}(\hat{i}+\hat{j}+\hat{k})$

54. A long cylindrical wire of radius 'a' carries a current i distributed uniformly over its cross section. If the magnetic fields at distances $r < a$ and $R > a$ from the axis have equal magnitude, then

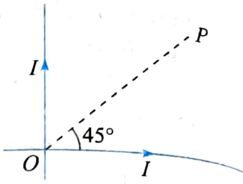
(1) $a = \frac{R+r}{2}$

(2) $a = \sqrt{Rr}$

(3) $a = Rr/R + r$

(4) $a = R^2/r$

55. Two infinite long wires, each carrying current I, are lying along x- and y-axis, respectively. A charged particle, having a charge q and mass m, is projected with a velocity u along the straight line OP. The path of the particle is (neglect gravity)



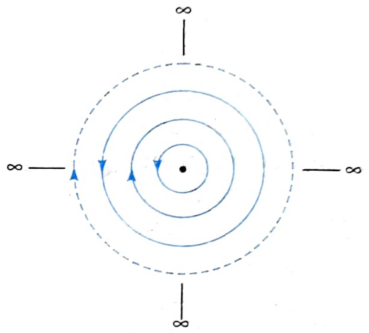
(1) straight line

(2) circle

(3) helix

(4) cycloid

56. In figure, infinite conducting rings each having current i in the direction shown are placed concentrically in the same plane as shown in the figure. The radii of rings are r, $2r$, 2^2r, 2^3r, ...,∞. The magnetic field at the center of rings will be



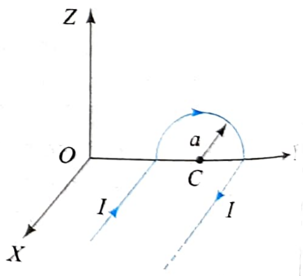
(1) zero

(2) $\frac{\mu_0 i}{r}$

(3) $\frac{\mu_0 i}{2r}$

(4) $\frac{\mu_0 i}{3r}$

57. A long wire bent as shown in figure carries current I. If the radius of the semicircular portion is a, the magnetic field at center C is



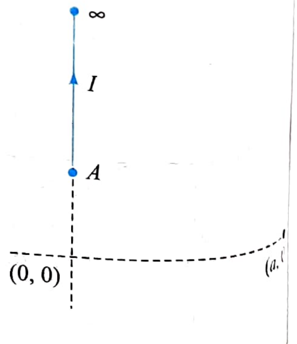
(1) $\frac{\mu_0 I}{4a}$

(2) $\frac{\mu_0 I}{4\pi a}\sqrt{\pi^2+4}$

(3) $\frac{\mu_0 I}{4a}+\frac{\mu_0 I}{4\pi a}$

(4) $\frac{\mu_0 I}{4\pi a}\sqrt{(\pi^2-4)}$

58. An infinitely long wire carrying current I is along Y-axis such that its one end is at point A $(0, b)$ while the wire extends upto +∞. The magnitude of magnetic field strength at point $(a, 0)$ is



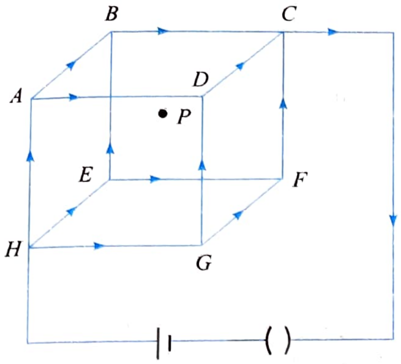
(1) $\frac{\mu_0 I}{4\pi a}\left(1+\frac{b}{\sqrt{a^2+b^2}}\right)$

(2) $\frac{\mu_0 I}{4\pi a}\left(1-\frac{b}{\sqrt{a^2+b^2}}\right)$

(3) $\frac{\mu_0 I}{4\pi a}\left(\frac{b}{\sqrt{a^2+b^2}}\right)$

(4) none of these

59. A steady current is set up in a cubic network composed of wires of equal resistance and length d as shown in figure. What is the magnetic field at center P due to the cubic network?



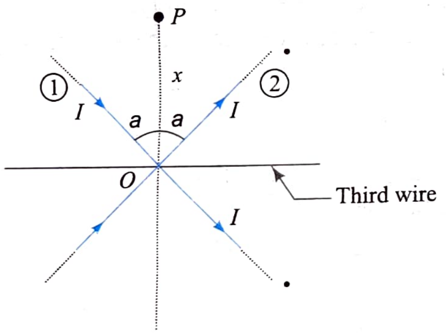
(1) $\frac{\mu_0}{4\pi}\frac{2I}{d}$

(2) $\frac{\mu_0}{4\pi}\frac{3I}{\sqrt{2}d}$

(3) 0

(4) $\frac{\mu_0}{4\pi}\frac{\theta\pi I}{d}$

60. Three infinite current carrying conductors are placed as shown in figure. Two wires carry same current while current in third wire is unknown. The three wires are electrically insulated from each other and all of them are in the plane of paper. Which of the following is correct about point P which is also in the same plane?



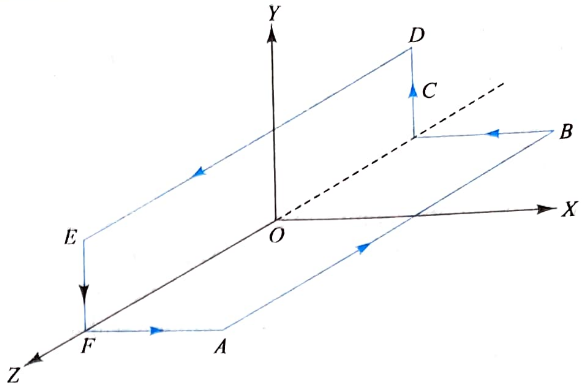
(1) Magnetic field intensity at P is zero for all values of x, whatever is the current in the third wire.

(2) If the current in the third wire is $\frac{2I}{\sin\alpha}$ (left to right), then magnetic field will be zero at P for all values of x.

(3) If the current in the third wire is $\frac{2I}{\sin\alpha}$ (right to left), then magnetic field will be zero at P for all values of x.

(4) None of these

61. In figure, $ABCDEFA$ was a square loop of side l, but is folded in two equal parts so that half of it lies in the x-z plane and the other half lies in the y-z plane. The origin O is center of the frame also. The loop carries current 'i'. The magnetic field at the center is



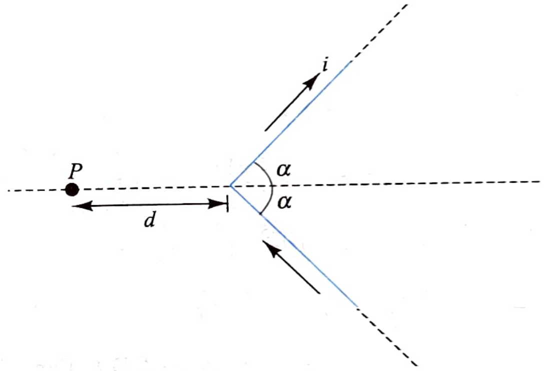
(1) $\frac{\mu_0 i}{2\sqrt{2}\,\pi\ell}(\hat{i}-\hat{j})$

(2) $\frac{\mu_0 i}{4\pi\ell}(-\hat{i}+\hat{j})$

(3) $\frac{\sqrt{2}\,\mu_0 i}{\pi\ell}(\hat{i}+\hat{j})$

(4) $\frac{\mu_0 i}{\sqrt{2}\,\pi\ell}(\hat{i}+\hat{j})$

62. If the magnetic field at P can be written as $K\tan\left(\frac{\alpha}{2}\right)$, then K is



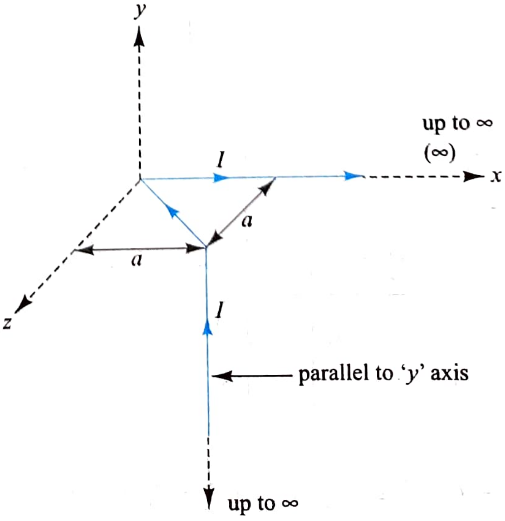
(1) $\frac{\mu_0 I}{4\pi d}$

(2) $\frac{\mu_0 I}{2\pi d}$

(3) $\frac{\mu_0 I}{\pi d}$

(4) $\frac{2\mu_0 I}{\pi d}$

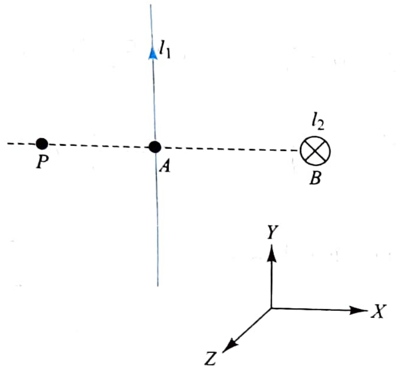
63. The magnetic field at the origin due to the current flowing i the wire is



(1) $-\dfrac{\mu_0 I}{8\pi a}\left(\hat{i}+\hat{k}\right)$ (2) $\dfrac{\mu_0 I}{2\pi a}\left(\hat{i}+\hat{k}\right)$

(3) $\dfrac{\mu_0 I}{8\pi a}\left(-\hat{i}+\hat{k}\right)$ (4) $\dfrac{\mu_0 I}{4\pi a\sqrt{2}}\left(\hat{i}-\hat{k}\right)$

64. Two infinitely long linear conductors are arranged perpendicular to each other and are in mutually perpendicular planes as shown in figure. If $I_1 = 2$A along the y-axis, $I_2 = 3$A along –ve z-axis and $AP = AB = 1$ cm, the value of magnetic field strength $\vec{B}$ at P is



(1) $\left(3\times10^{-5}\text{T}\right)\hat{j}+\left(-4\times10^{-5}\text{T}\right)\hat{k}$

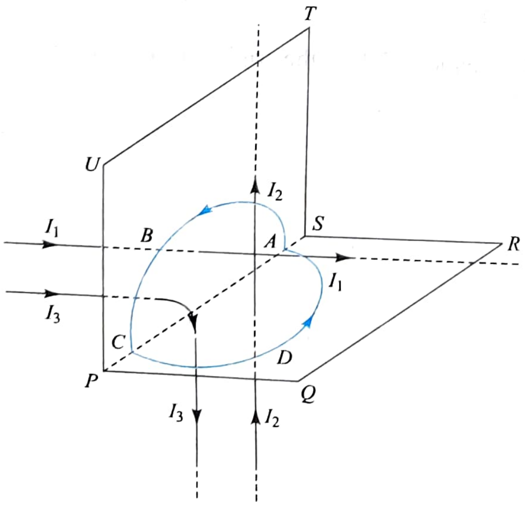
(2) $\left(3\times10^{-5}\text{T}\right)\hat{j}+\left(4\times10^{-5}\text{T}\right)\hat{k}$

(3) $\left(4\times10^{-5}\text{T}\right)\hat{j}+\left(3\times10^{-5}\text{T}\right)\hat{k}$

(4) $\left(-3\times10^{-5}\text{T}\right)\hat{j}+\left(4\times10^{-5}\text{T}\right)\hat{k}$

65. Figure shows an Amperian path $ABCDA$. Part ABC is in vertical plane $PSTU$ while part CDA is in horizontal plane $PQRS$. Direction of circulation along the path is shown by an arrow near point B and at D.

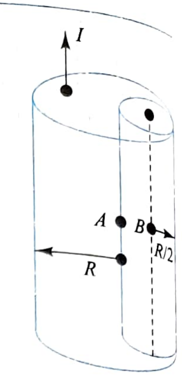
$\oint \vec{B}\cdot d\vec{\ell}$ for this path according to Ampere's law will be



(1) $(I_1 - I_2 + I_3)\mu_0$ (2) $(-I_1 + I_2)\mu_0$

(3) $I_3\mu_0$ (4) $(I_1 + I_2)\mu_0$

66. From a cylinder of radius R, a cylinder of radius $R/2$ is removed, as shown in figure. Current flowing in the remaining cylinder is I. Then, magnetic field strength is



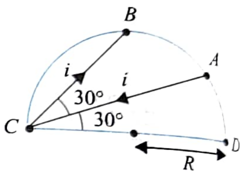
(1) zero at point A

(2) zero at point B

(3) $\dfrac{\mu_0 I}{2\pi R}$ at point A

(4) $\dfrac{\mu_0 I}{3\pi R}$ at point B

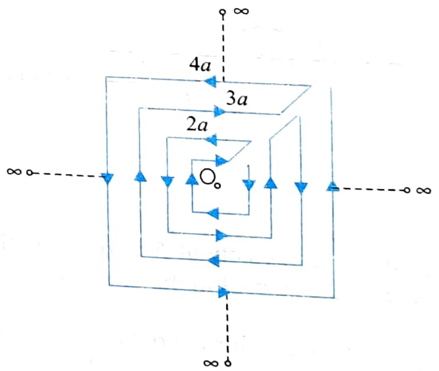
67. A current carrying wire is placed in the grooves of an insulating semicircular disc of radius 'R', as shown in figure. The current enters at point A and leaves from point B. Determine the magnetic field at point D.



(1) $\dfrac{\mu_0\, i}{8\pi R\sqrt{3}}$ (2) $\dfrac{\mu_0\, i}{4\pi R\sqrt{3}}$

(3) $\dfrac{\sqrt{3}\mu_0\, i}{4\pi R}$ (4) none of these

68. Determine the magnetic field at the centre of the current carrying wire arrangement shown in figure. The arrangement extends to infinity. (The wires joining the successive squares are along the line passing through the centre.)



(1) $\dfrac{\mu_0\, i}{\sqrt{2}\pi a}$ (2) 0

(3) $\dfrac{2\sqrt{2}\mu_0\, i}{\pi a}\ln 2$ (4) none of these

69. A wire is wound on a long rod of material of relative permeability $\mu_r = 4000$ to make a solenoid. If the current through the wire is 5 A and number of turns per unit length is 1000 per metre, then the magnetic field inside the solenoid is

(1) 4 π mT (2) 8π mT

(3) 4 π T (4) 8 π T

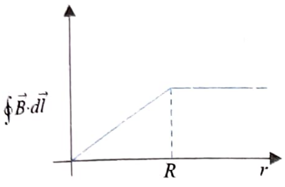
70. Axis of a solid cylinder of infinite length and radius R lies along y-axis. It carries a uniformly distributed current i along $+y$ direction. Magnetic field at a point $(R/2, y, R/2)$ is

(1) $\frac{\mu_0 I}{4\pi R}(\hat{i}-\hat{k})$ (2) $\frac{\mu_0 I}{2\pi R}(\hat{j}-\hat{k})$

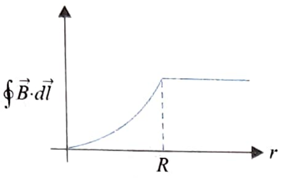
(3) $\frac{\mu_0 I}{4\pi R}\hat{j}$ (4) $\frac{\mu_0 I}{4\pi R}(\hat{i}+\hat{k})$

71. A cylindrical wire of radius R is carrying uniformly distributed current i over its cross-section. If a circular loop of radius r is taken as amperian loop, then the variation value of $\oint \vec{B}\cdot\vec{d\ell}$ over this loop with radius 'r' of loop will be best represented by

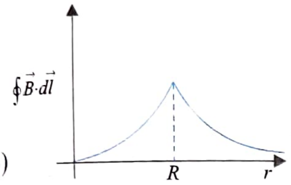
(1)



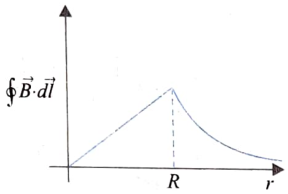
(2)



(3)



(4)



72. Two infinitely long, thin, insulated, straight wires lie in the x–y plane along the x- and y-axes respectively. Each wire carries a current I, respectively in the positive x-direction and positive y-direction. The magnetic field will be zero at all points on the straight line:

(1) $y = x$ (2) $y = -x$

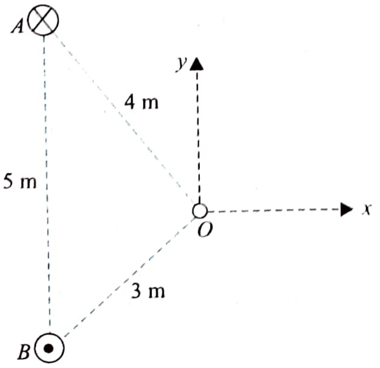
(3) $y = x - 1$ (4) $y = -x + 1$

73. Two infinitely long straight parallel wires carry equal currents 'i' in the same direction and are 2 m apart. Magnetic induction at a point which is at same normal distance $\sqrt{2}$ m from each wire has magnitude

(1) $\frac{\mu_0 i}{2\pi}\sqrt{2}$ (2) $\frac{\mu_0 i}{4\pi}\sqrt{2}$

(3) $\frac{\mu_0 i}{2\sqrt{2}\pi}$ (4) $\frac{\mu_0 i}{2\pi}$

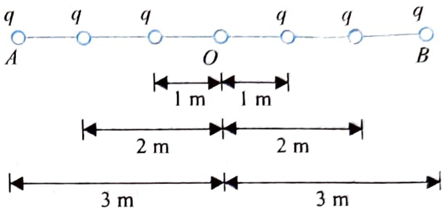
74. Two infinitely long straight parallel wires are 5 m apart, perpendicular to the plane of paper. One of the wires, as it passes perpendicular to the plane of paper, intersects it at A and carries current i in the downward direction. The other wire intersects the plane of paper at point B and carries current i in the outward direction O in the plane of paper as shown in figure. With x and y axes as shown, magnetic induction at O in the component form can be expressed as



(1) $\vec{B} = \frac{\mu_0 i}{2\pi}\left[-\hat{i}+\frac{3\hat{j}}{5}\right]$ (2) $\vec{B} = \frac{\mu_0 i}{5\pi}\left[-\hat{i}+\frac{7\hat{j}}{24}\right]$

(3) $\vec{B} = \frac{\mu_0 i}{4\pi}\left[-3\hat{i}+\frac{7\hat{j}}{18}\right]$ (4) $\vec{B} = \frac{\mu_0 i}{7\pi}\left[-2\hat{i}+\frac{3\hat{j}}{5}\right]$

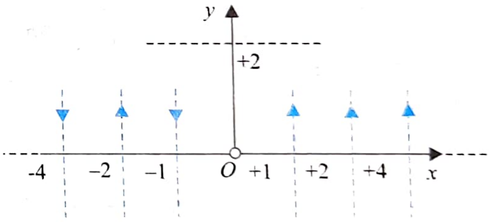
75. In figure, AB is a non-conducting rod. Equal charges of magnitude q are fixed at various points on the rod as shown. The rod is rotated uniformly about an axis passing through O and perpendicular to its length such that linear speed at the end A or B of the rod is 3 ms^{-1}. Magnetic field at O is



(1) $\frac{11\mu_0 q}{12\pi}$ (2) $\frac{3\mu_0 q}{7\pi}$

(3) $\frac{\mu_0 q}{2\pi}$ (4) $\frac{6\mu_0 q}{13\pi}$

76. Infinite wires, each carrying a current 0.5 A, are kept parallel to y-axis and intersecting x-axis at $x = \pm 1$ m, ± 2 m, ± 4 m, ± 8 m, etc. Wires kept at positive values of x carry current in the same direction while wires kept at negative values of x carry current successively in opposite directions as shown in figure.



A wire is kept parallel to x-axis and intersecting y-axis at $y = 2$ m. It carries a current i. Assuming this wire to be insulated from others then the current i in it such that magnetic induction at O is zero is:

(Assume each wire to be infinitely long)

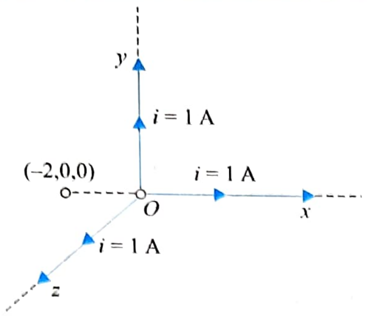
(1) $\frac{8}{3}$ A along positive x-axis

(2) $\frac{4}{3}$ A along negative x-axis

(3) $\frac{5}{3}$ A along negative x-axis

(4) $\frac{7}{3}$ A along positive x-axis

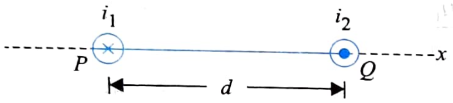
77. Three infinitely long wires, each carrying a current 1 A, are placed such that one end of each wire is at the origin, and, one of these wires is along x-axis, the other along y-axis and the third along z-axis. Magnetic induction at point (–2 m, 0, 0) due to the system of these wires can be expressed as



(1) $\frac{\mu_0}{4\pi}(\hat{j}+\hat{k})$ (2) $\frac{\mu_0}{4\pi}(\hat{j}-\hat{k})$

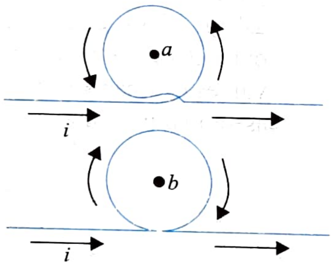
(3) $\frac{\mu_0}{8\pi}(-\hat{j}+\hat{k})$ (4) $\frac{\mu_0}{8\pi}(\hat{j}+\hat{k})$

78. Two parallel wires P and Q placed at a separation $d = 6$ cm on x-axis carry electric current $i_1 = 5$ A and $i_2 = 2$ A in opposite directions as shown in figure. Find the point on the line PQ where the resultant magnetic field is zero.



(1) 4 cm left of P (2) 4 cm right of Q
(3) middle of PQ (4) At no position on x-axis

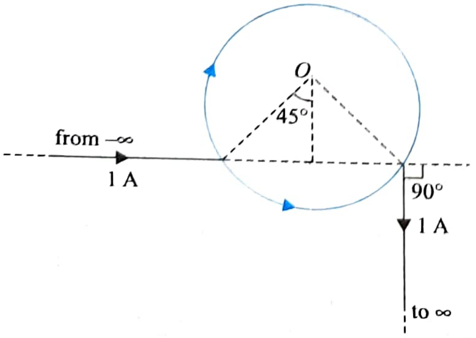
79. In figure, the conductors carry equal currents i. All straight segments are very long, and the two circular loops have equal radii. However, the currents around the loops have opposite senses. The ratio of the magnetic fields at a and b, at the centres of the two loops, is



(1) $\frac{B_a}{B_b}=\frac{\pi+1}{-\pi+1}$ (2) $\frac{B_a}{B_b}=\frac{\pi-1}{\pi+1}$

(3) $\frac{B_a}{B_b}=\frac{\pi-2}{\pi+2}$ (4) $\frac{B_a}{B_b}=\frac{\pi+2}{\pi-2}$

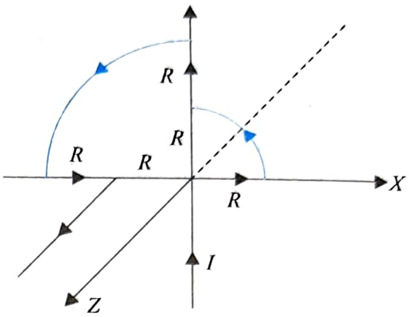
80. What is the magnitude of magnetic field at the centre O of loop of radius $\sqrt{2}$ m made of uniform wire when a current of 1 A enters in the loop and is taken out of it by two long wires as shown in figure?



(1) $\frac{\mu_0}{\pi}\left[1-\frac{1}{\sqrt{2}}\right]T$ (2) $\frac{\mu_0}{\sqrt{2}\pi}\left[1-\frac{1}{\sqrt{2}}\right]T$

(3) zero (4) None of these

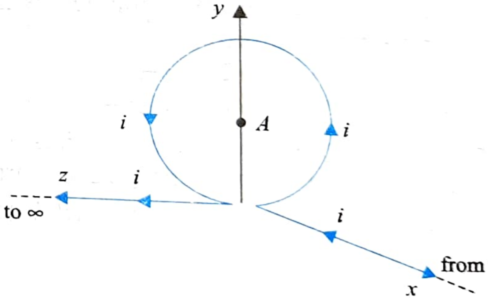
81. Find the magnetic field at the origin in the figure shown in figure.



(1) $\frac{\mu_0 I}{4R}\left(\frac{3}{4}\hat{k}+\frac{1}{\pi}\hat{j}\right)$ (2) $\frac{\mu_0 I}{4R}\left(\frac{3}{2}\hat{k}+\frac{1}{\pi}\hat{j}\right)$

(3) $\frac{\mu_0 I}{4R}\left(\frac{3}{4}\hat{k}+\frac{1}{2\pi}\hat{j}\right)$ (4) None of these

82. Find the magnitude of the magnetic induction B of a magnetic field generated by a system of thin conductors along which a current i is flowing at a point A (O, R, O), that is the centre of a circular conductor of radius R. The ring is in yz plane.



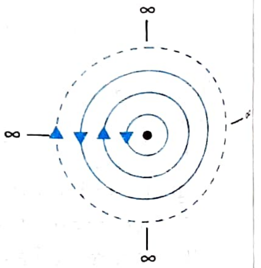
(1) $B=\frac{\mu_0 i}{4\pi R}\sqrt{(2\pi^2-2\pi+1)}$

(2) $B=\frac{\mu_0 i}{4\pi R}\sqrt{2(2\pi^2-2\pi+1)}$

(3) $B=\frac{\mu_0 i}{2\pi R}\sqrt{(2\pi^2-2\pi+1)}$

(4) None of these

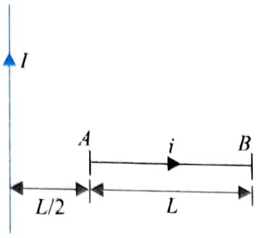
83. In figure, infinite conducting rings each having current i in the direction shown are placed concentrically in the same plane as shown in the figure. The radius of rings are r, $2r$, 2^2r, 2^3r ... ∞. The magnetic field at the centre of rings will be



(1) Zero (2) $\frac{\mu_0 i}{r}$

(3) $\frac{\mu_0 i}{2r}$ (4) $\frac{\mu_0 i}{3r}$

84. A conductor AB of length l carrying current i is placed perpendicular to a long straight conductor carrying a current I as shown in figure. Force on AB will be



(1) $\frac{3\mu_0 Ii}{2\pi}$ (2) $\frac{\mu_0 Ii}{2\pi}\log_e 3$

(3) $\frac{\mu_0 Ii}{2\pi}\log_e 2$ (4) $\frac{2\mu_0 Ii}{3\pi}$

85. A long straight non-conducting string carries a charge density of 40 μC m^{-1}. It is pulled along its length at a speed of 300 m/sec. What is the magnetic field at a normal distance of 5 mm from the moving string?

(1) $B = 4.8 \times 10^{-7}$ T (2) $B = 3.2 \times 10^{-7}$ T

(3) $B = 2.5 \times 10^{-7}$ T (4) $B = 5 \times 10^{-7}$ T

Multiple Correct Answers Type

1. A steady electric current is flowing through a cylindrical conductor. Then,

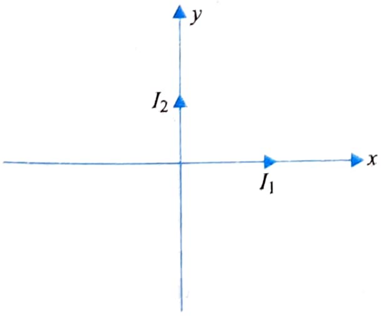
(1) the electric field at the axis of the conductor is zero

(2) the magnetic field at the axis of the conductor is zero

(3) the electric field in the vicinity of the conductor is zero

(4) the magnetic field in the vicinity of the conductor is zero

2. Two thin long wires carry currents I_1 and I_2 along x- and y-axes, respectively, as shown in figure. Consider the points only in x–y plane.



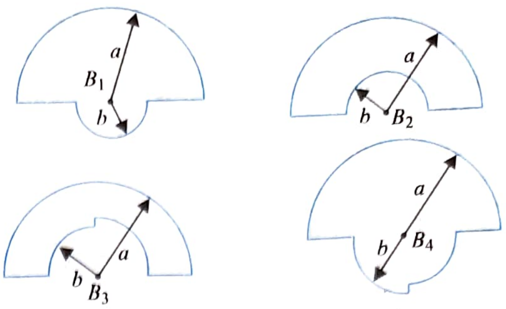
(1) Magnetic field is zero at least at one point in each quadrant

(2) Magnetic field can be zero somewhere in the first quadrant

(3) Magnetic field can be zero somewhere in the second quadrant

(4) Magnetic field is non-zero in second quadrant

3. In the loops shown in figure, all curved sections are either semicircles or quarter circles. All the loops carry same current. The magnetic fields at the centers have magnitudes B_1, B_2, B_3, and B_4. Then,



(1) B_4 is maximum (2) B_3 is minimum

(3) $B_4 > B_1 > B_2 > B_3$ (4) $B_1 > B_4 > B_3 > B_2$

4. Two circular coils of radii 5 cm and 10 cm carry equal currents of 2 A. The coils have 50 and 100 turns, respectively, and are placed in such a way that their planes as well as their centers coincide. Magnitude of magnetic field at the common center of coils is

(1) $8\pi \times 10^{-4}$ T if current in the coils are in same sense

(2) $4\pi \times 10^{-4}$ T if current in the coils are in opposite sense

(3) zero if current in the coils are in opposite sense

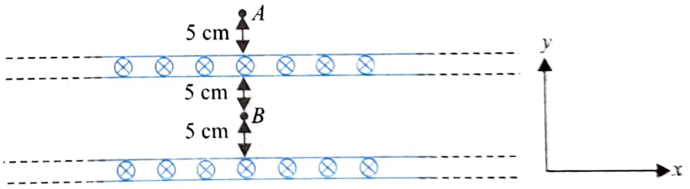
(4) $8\pi \times 10^{-4}$ T if current in the coils are in opposite sense

5. A long straight wire carries the current along +ve x-direction. Consider four points in space $A(0, 1, 0)$, $B(0, 1, 1)$, $C(1, 0, 1)$, and $D(1, 1, 1)$. Which of the pairs will have the same magnitude of magnetic field?

(1) A and B (2) A and C

(3) B and C (4) B and D

6. Figure shows cross section of two large parallel metal sheets carrying electric currents along their surfaces. The current in each sheet is $10/\pi$ A/m along the width. Consider two points A and B, as shown in the figure with their positions.



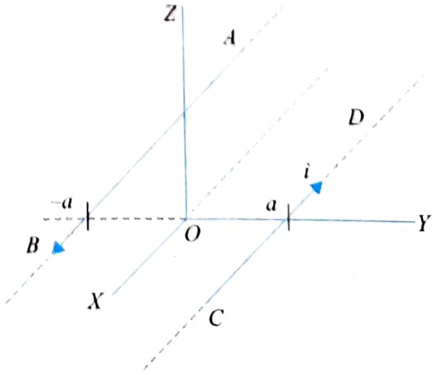
(1) Magnetic field at A is 4 μT along x-direction.

(2) Magnetic field at A is 4 μT along negative x-direction.

(3) Magnetic field at B is zero.

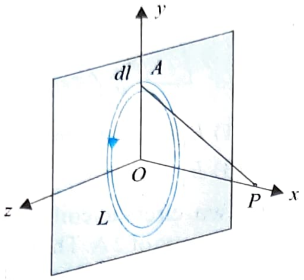
(4) Magnetic field at B is 2 μT along x-direction.

7. Two long parallel wires, AB and CD, carry equal currents in opposite directions. They lies in the xy plane, parallel to the x-axis, and pass through the points $(0, -a, 0)$ and $(0, a, 0)$ respectively. The resultant magnetic field is



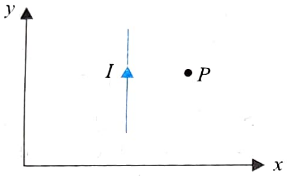
(1) zero on the x-axis
(2) maximum on the x-axis.
(3) directed along the z-axis at the origin, but not at other points on the z-axis.
(4) directed along the z-axis at all points on the z-axis.

8. L is a circular loop (in y-z plane) carrying an anticlockwise current. P is a point on its axis OX. dl is an element of length on the loop at a point A on it. The magnetic field at P



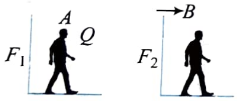
(1) due to L is directed along OX.
(2) due to dl is directed along OX.
(3) due to dl is perpendicular to AP in upward direction
(4) due to dl is perpendicular to AP in downward direction

9. A straight wire carrying current is parallel to the y-axis as shown in figure. The



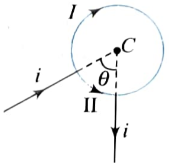
(1) magnetic field at the point P is parallel to the x-axis.
(2) magnetic field at P is along z-axis.
(3) magnetic field are concentric circle with the wire passing through their common centre
(4) magnetic fields to the left and right of the wire are oppositely directed.

10. An observer A and a charge Q are fixed in a stationary frame F_1. An observer B is fixed in a frame F_2, which is moving with respect to F_1.



(1) Both A and B will observe electric fields.
(2) Both A and B will observe magnetic fields.
(3) Neither A nor B will observe magnetic fields.
(4) B will observe a magnetic field, but A will not.

11. L is the circular ring made of a uniform wire. Current enters and leaves the ring through straight conductors which, if produced, would have passed through the centre C of the ring. The magnetic field at C

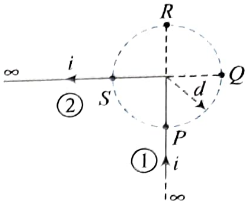


(1) due to the straight conductors is zero
(2) due to the loop is zero
(3) due to path II is proportional to θ
(4) due to the Path I is proportional to $(\pi - \theta)$

12. Two circular coils A and B with their centres lying on the same axis have same number of turns and carry equal currents in the same sense. They are separated by a distance, have different diameters but subtend same angle at a point P lying on their common axis. The coil B lies exactly midway between coil A and the point P. The magnetic field at point P due to coils A and B is B_1 and B_2 respectively.

(1) $B_1 > B_2$
(2) $B_1 < B_2$
(3) $\dfrac{B_1}{B_2} = 2$
(4) $\dfrac{B_1}{B_2} = \dfrac{1}{2}$

13. Figure shows a long wire bent at the middle to form a right angle.



(1) The magnitude of magnetic field at the points P, Q, R and S are same.
(2) The direction of magnetic field at points P and S are same.
(3) The magnitude of magnetic field at points P and S is zero.
(4) The direction of magnetic field at points R and S are opposite to each other.

14. Two coaxial solenoids 1 and 2 of the same length are set so that one is inside the other. The number of turns per unit length are n_1 and n_2. The currents i_1 and i_2 are flowing in opposite directions. The magnetic field inside the inner coil is zero. This is possible when

(1) $i_1 \neq i_2$ and $n_1 = n_2$
(2) $i_1 = i_2$ and $n_1 \neq n_2$
(3) $i_1 = i_2$ and $n_1 = n_2$
(4) i_1n_1 and i_2n_2

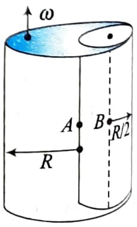
15. Two identical current carrying coaxial loops, carry current I in an opposite sense. A simple amperian loop passes through both of them once. Calling the loop as C,

(1) $\oint B \cdot dl = \mu_0 I$
(2) the value of $\oint B \cdot dl = \pm 2\mu_0 I$ is independent of sense of C
(3) there may be a point on C where, B and dl are perpendicular
(4) B vanishes everywhere on C

16. A small current element of length dl and carrying current is placed at (1, 1, 0) and is carrying current in $+z$ direction. If magnetic field at origin be $\vec{B}_1$ and at point (2, 2, 0) be $\vec{B}_2$, then

(1) $|\vec{B}_1| = |\vec{B}_2|$
(2) $\vec{B}_1 = -\vec{B}_2$
(3) $|\vec{B}_1| = |2\vec{B}_2|$
(4) $\vec{B}_1 = -2\vec{B}_2$

17. From a cylinder of radius R, a cylinder of radius $R/2$ is removed, as shown. Current flowing in the remaining cylinder is I. Magnetic field strength is



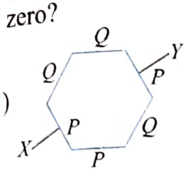
(1) zero at point A
(2) zero at point B
(3) $\dfrac{\mu_0 I}{3\pi R}$ at point A
(4) $\dfrac{\mu_0 I}{3\pi R}$ at point B

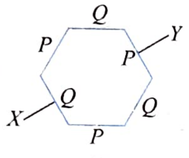
18. A wire is bent into a circular loop of radius R and carries current. The magnetic field at the center of the loop is B. The same wire is bent into a double loop. If both loops carry the same current in the same direction, the magnetic field at the center of the double loop is B_1. If they carry the same current in opposite directions, the magnetic field at their center is B_2. Then

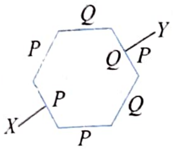
(1) $B_1 = 0$ (2) $B_1 = 4B$

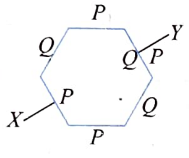
(3) $B_2 = 0$ (4) $B_2 = \frac{B}{4}$

19. In the following hexagons made up of two different material P and Q, current enters and leaves from points X and Y, respectively. In which case(s) the magnetic field at its centre is zero?

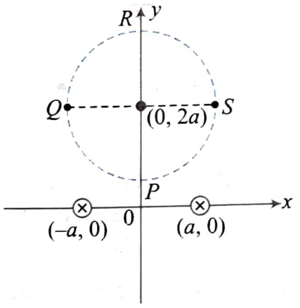
(1) 

(2) 

(3) 

(4) 

20. Two long conducting wires carrying equal currents are placed parallel to the z-axis. A charged particle is made to move in a circular path of radius a with centre of the path at point $(0, 2a)$ in clockwise direction. During the course of motion, it passes through four points P, Q, R and S having coordinates $(0, a)$, $(-a, 2a)$, $(0, 3a)$ and $(a, 2a)$, respectively. The magnitude of force exerted on the particle by the magnetic field created by the wires is maximum when it passes through point.



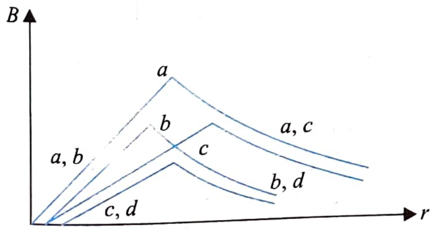
(1) P (2) Q

(3) R (4) S

Linked Comprehension Type

For Problems 1–3

Curves in the graph shown in figure give, as functions of radius distance r, the magnitude B of the magnetic field inside and outside four long wires a, b, c, and d, carrying currents that are uniformly distributed across the cross sections of the wires. Overlapping portions of the plots are indicated by double labels.



1. Which wire has the greatest radius?

(1) a (2) b

(3) c (4) d

2. Which wire has the greatest magnitude of the magnetic field on the surface?

(1) a (2) b

(3) c (4) d

3. The current density in wire a is

(1) greater than in wire c

(2) less than in wire c

(3) equal to that in wire c

(4) not comparable to that of in wire c due to lack of information

For Problems 4–6

Ampere's law provides us an easy way to calculate the magnetic field due to a symmetrical distribution of current. Its mathematical expression is $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{in}$

The quantity on the left hand side is known as line integral of magnetic field over a closed Ampere's loop.

4. Only the current inside the Amperian loop contributes in

(1) finding magnetic field at any point on the Ampere's loop

(2) line integral of magnetic field

(3) in both of the above

(4) in neither of them

5. If the current density in a linear conductor of radius a varies with r according to relation $J = kr^2$, where k is a constant and r is the distance of a point from the axis of conductor, find the magnetic field induction at a point distance r from the axis when $r < a$. Assume relative permeability of the conductor to be unity.

(1) $\frac{\mu_0 k\pi a^4}{4r}$ (2) $\frac{\mu_0 kr^3}{2}$

(3) $\frac{\mu_0 k\pi a^4}{2r}$ (4) $\frac{\mu_0 kr^3}{4}$

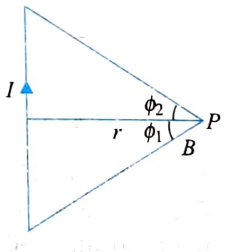
6. In the above question, find the magnetic field induction at a point distance r from the axis when $r > a$. Assume relative permeability of the medium surrounding the conductor to be unity.

(1) $\frac{\mu_0 ka^4}{4r}$ (2) $\frac{\mu_0 kr^3}{2}$

(3) $\frac{\mu_0 k\pi a^4}{2r}$ (4) $\frac{\mu_0 kr^3}{4}$

For Problems 7–8

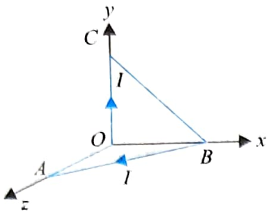
According to Biot–Savarat's law, magnetic field due to a straight current carrying wire at a point at a distance r from it is given by



$$B = \frac{\mu_0}{4\pi}\frac{I}{r}(\sin\phi_1 + \sin\phi_2)$$

The direction of magnetic field being perpendicular to the plane containing the wire and that point.

7. Figure shows a closed loop $AOCBA$ in which current I is flowing as shown. Given $OA = OB = OC = a$. Find the magnetic field at point B due to this loop.



(1) $-\frac{\mu_0 I}{4\pi\sqrt{2}a}\left(\hat{i}+\hat{k}\right)$ (2) $-\frac{\mu_0 I}{4\pi\sqrt{2}a}\left(\hat{j}+\hat{k}\right)$

(3) $\frac{-\mu_0 I}{2\sqrt{2}\pi a}\left(\hat{j}+\hat{k}\right)$ (4) None of these

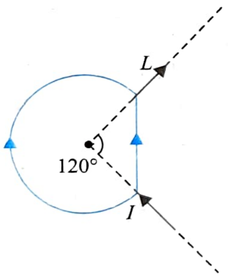
8. Find magnetic field at point O in figure.

(1) $-\frac{\mu_0 I}{4\pi a}\left(\hat{i}+\hat{k}\right)$ (2) $-\frac{\mu_0 I}{4\pi a}\left(\hat{j}+\hat{k}\right)$

(3) $\frac{-\mu_0 I}{2\pi a}\left(\hat{j}+\hat{k}\right)$ (4) $\frac{-\mu_0 I}{2\sqrt{2}\pi a}\left(\hat{j}+\hat{k}\right)$

For Problems 9–10

In figure, the circular and the straight parts of the wire are made of same material but have different diameters. The magnetic field at the center is zero.



9. Ratio of the currents I_1 and I_2 flowing through the circular and straight parts is

(1) $\frac{\sqrt{3}}{2\pi}$ (2) $\frac{2\sqrt{3}}{\pi}$

(3) $\frac{3\sqrt{3}}{2\pi}$ (4) $\frac{3\sqrt{3}}{2\sqrt{2}\,\pi}$

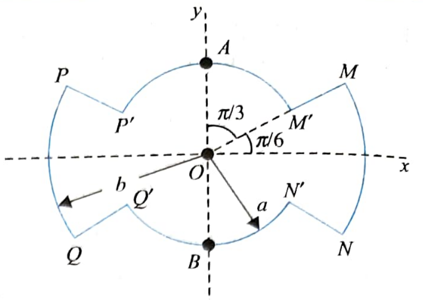
10. The ratio of the diameters of wires of circular and straight parts is

(1) $\frac{1}{\sqrt{2}}$ (2) $\frac{2\sqrt{3}}{\pi}$

(3) $\frac{3\sqrt{3}}{2\pi}$ (4) $\sqrt{2}$

For Problems 11–12

There exists a long conductor along z-axis carrying a current I_0 along positive z-direction. A loop having total resistance R is placed symmetrical about x- and y-axes in x-y plane as shown in figure. Potential difference $V_{BA} = V$ is applied. Radii of arcs are a and b, respectively, as shown in the figure.



11. The magnitude of force experienced by the arc MN is

(1) zero (2) $\frac{\mu_0 V I_0}{\pi R b}$

(3) $\frac{\mu_0 V I_0}{2\pi R b}$ (4) none of these

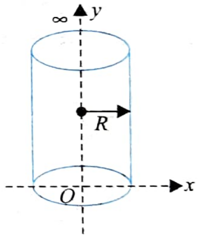
12. The total torque acting on the loop is nearly

(1) $\frac{\mu_0 V I_0}{\pi R}(b-a)\hat{i}$ (2) $-2\frac{\mu_0 V I_0}{\pi R}(b-a)\hat{i}$

(3) $\frac{\mu_0 V I_0}{2\pi R}(b-a)\hat{i}$ (4) none of these

For Problems 13–15

An infinite cylindrical wire of radius R and having current densit varying with its radius r as, $J = J_0[1 - (r/R)]$. Then answer th following questions.



13. Position where magnetic field strength is maximum.

(1) R (2) $3R/4$

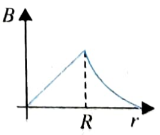
(3) $R/2$ (4) $R/4$

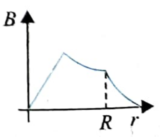
14. Magnitude of maximum magnetic field is

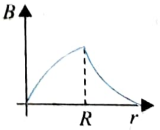
(1) $\frac{J_0 R}{6}\mu_0$ (2) $\frac{3}{16}J_0 R\mu_0$

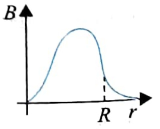
(3) $\frac{5}{16}J_0 R\mu_0$ (4) $\frac{5}{6}J_0 R\mu_0$

15. Graph between the magnetic field and radius is

(1)


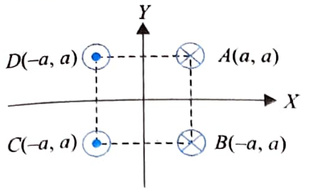
(2)


(3)


(4)


For Problems 16–17

Four long wires each carrying current I as shown in figure placed at points A, B, C and D. Find the magnitude and directi of



16. magnetic field at the centre of the square.

(1) $\frac{\mu_0}{4\pi}\left(\frac{I}{a}\right)$ along Y-axis (2) $\frac{\mu_0}{4\pi}\left(\frac{4I}{a}\right)$ along X-axis

(3) $\frac{\mu_0}{4\pi}\left(\frac{4I}{a}\right)$ along Y-axis (4) None of these

17. force per metre acting on wire at point D.

(1) $\frac{\mu_0}{4\pi}\left(\frac{I^2}{2a}\right)\sqrt{10}, \pi+\tan^{-1}\left(\frac{1}{3}\right)$ with positive X-axis

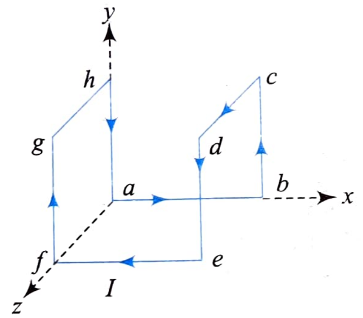
(2) $\frac{\mu_0}{4\pi}\left(\frac{I^2}{2a}\right)\sqrt{10}, \pi+\tan^{-1}\left(\frac{1}{3}\right)$ with negative X-axis

(3) $\frac{\mu_0}{4\pi}\left(\frac{I^2}{a}\right)\sqrt{10}, \pi+\tan^{-1}\left(\frac{1}{3}\right)$ with positive X-axis

(4) None of these

For Problems 18–20

A current I amperes flows through a loop *abcdefgha* along the edge of a cube of width l metres as shown in figure. One corner *a* of the loop lies at origin.



18. This current path (*abcdefgha*) can be treated as a superposition of three square loops carrying current I. Choose the correct option.

(1) *fghaf, fabef, ebcde* (2) *fghaf, fabef, fgdef*

(3) *fghaf, abcha, ebcde* (4) *fgdef, fabef, ebcde*

19. The unit vector in the direction of magnetic field at the centre of cube *abcdefgh* of width λ is given by

(1) $\hat{i}$ (2) $-\hat{j}$

(3) $\frac{2\hat{i}-\hat{j}}{\sqrt{5}}$ (4) $\hat{k}$

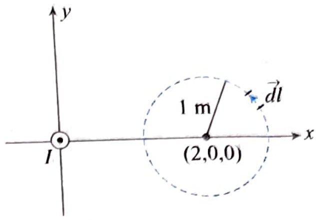
20. Now if a uniform external magnetic field is $\vec{B}=B_0\hat{j}$ is switched on, then the unit vector in the direction of torque due to external magnetic field ($\vec{B}$) acting on the current carrying loop (*abcdefgha*) is

(1) $\hat{k}$ (2) $-\hat{i}$

(3) $\frac{2\hat{i}-\hat{j}}{\sqrt{5}}$ (4) None of these

For Problems 21–23

An infinitely long wire lying along z-axis carries a current I, flowing towards positive z-direction. There is no other current; consider a circle in x-y plane with centre at (2 meter, 0, 0) and radius 1 meter. Divide the circle in small segments and then denote the length of a small segment in anticlockwise direction, as shown.



21. The path integral $\oint \vec{B}\cdot\vec{dl}$ of the total magnetic field $\vec{B}$ along the perimeter of the given circle is

(1) $\frac{\mu_0 I}{8}$ (2) $\frac{\mu_0 I}{2}$

(3) $\mu_0 I$ (4) 0

22. Consider two points $A(3,0,0)$ and $B(2,1,0)$ on the given circle. The path integral $\int_A^B \vec{B}\cdot d\vec{\ell}$ of the total magnetic field $\vec{B}$ along the perimeter of the given circle from A to B is

(1) $\frac{\mu_0 I}{\pi}\tan^{-1}\frac{1}{2}$ (2) $\frac{\mu_0 I}{2\pi}\tan^{-1}\frac{1}{2}$

(3) $\frac{\mu_0 I}{2\pi}\sin^{-1}\frac{1}{2}$ (4) 0

23. The maximum value of path integral $\int \vec{B}\cdot\vec{dl}$ of the total magnetic field $\vec{B}$ along the perimeter of the given circle between any two points on the circle is

(1) $\frac{\mu_0 I}{12}$ (2) $\frac{\mu_0 I}{8}$

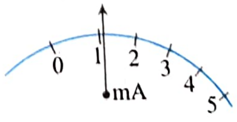
(3) $\frac{\mu_0 I}{6}$ (4) 0

For Problems 24–26

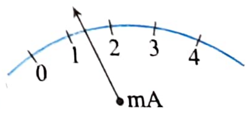
A moving coil galvanometer essentially consists of a rectangular coil in a magnetic field. When a current flows through the coil, a torque is produced and this rotates the coil. Magnitude of this torque is $\tau = BINA\cos\theta$

24. If a uniform magnetic field is used in the galvanometer, then scale of galvanometer looks like

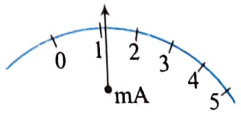
(1)



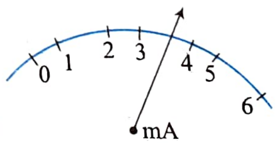
(2)



(3)



(4)



25. A rectangular coil of N turns pivoted about a vertical axis carrying a current I is placed in the region of a uniform horizontal magnetic field B. When the plane of coil makes an angle θ with magnetic field, torque required to prevent it rotating is τ_1. When coil rotates through 90°, then torque required will be τ_2. Magnitude of magnetic field will be

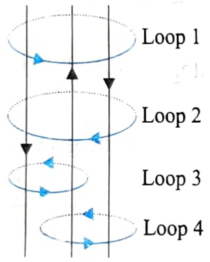
(1) $\dfrac{\sqrt{\tau_1^2+\tau_2^2}}{NIA}$ (2) $\dfrac{2\sqrt{\tau_1^2+\tau_2^2}}{NIA}$

(3) $\dfrac{\tau_1+\tau_2}{2(NIA)}$ (4) $\dfrac{\tau_1+\tau_2}{NIA}$

26. In a practical galvanometer, the field is made radial instead of uniform because
(1) when field is radial, torque is maximum
(2) when field is radial, torque is uniform
(3) when field is radial, a linear scale is possible
(4) a non-linear scale is difficult to read and calibrate

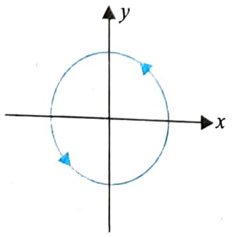
Matrix Match Type

1. Three wires are carrying same constant current i in different directions. Four loops enclosing the wires in different manners are shown in figure. The direction of $d\vec{l}$ is shown in the figure.



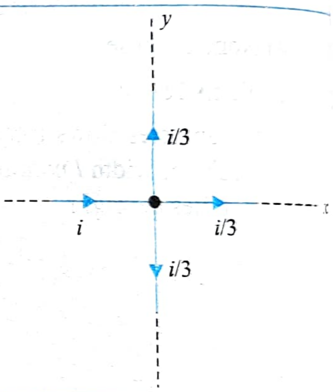
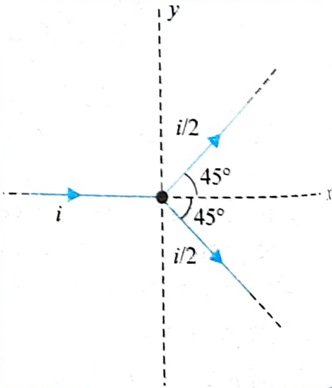
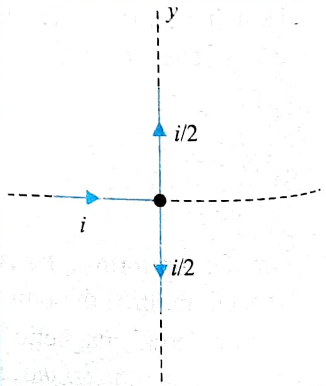
Column I		Column II
i. Along closed loop 1	**a.**	$\oint \vec{B}\,d\vec{l} = \mu_0 i$
ii. Along closed loop 2	**b.**	$\oint \vec{B}\cdot d\vec{l} = -\mu_0 i$
iii. Along closed loop 3	**c.**	$\oint \vec{B}\cdot dl = 0$
iv. Along closed loop 4	**d.**	network done by the magnetic force to move a unit charge along the loop is zero

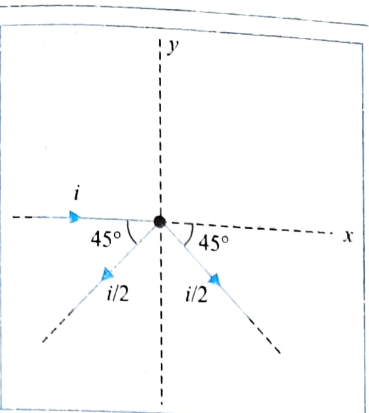
2. A circular current carrying loop having 100 turns and radius 10 cm is placed in x–y plane as shown in figure. A uniform magnetic field $\vec{B} = (-\hat{i}+\hat{k})$ tesla is present in the region. If current in the loop is 5 A, then match the following:



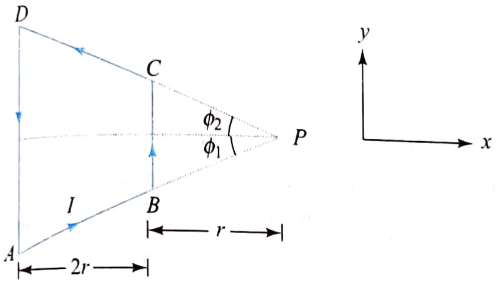
Column I		Column II
i. Magnitude and direction of moment (in A-m) of the loop are	**a.**	Zero
ii. Magnitude and direction of torque (in N-m) on the loop are	**b.**	5π
iii. Magnitude and direction of net force (in N) on the current loop are	**c.**	along positive z-axis
iv. Direction of magnetic field of loop at the center is	**d.**	along negative y-axis

3. Column II gives four situations in which three or four semi-infinite current carrying wires are placed in x-y plane. The magnitude of the direction of current is shown in each figure. Column I gives statements regarding the x- and y-components of magnetic field at point P whose coordinates are $(0, 0, d)$. Match the statements in Column I with the corresponding figures in Column II.

Column I		Column II
i. The x-component of magnetic field at point P is zero in	**a.**	
ii. The z-component of magnetic field at point P is zero in	**b.**	
iii. The magnitude of magnetic field at point P is $\dfrac{\mu_0 i}{4\pi d}$ in	**c.**	

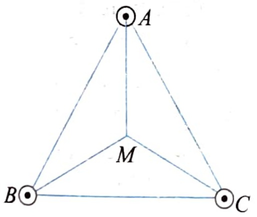
iv.	The magnitude of magnetic field at point P is less than $\frac{\mu_0 i}{2\pi d}$ in	**d.**	

4. Consider a closed loop in the form of a trapezium carrying current I. Match the following regarding the magnitude of magnetic field at point P.



	Column I		Column II
i.	Magnetic field due to AB	**a.**	is greater than that due to DA
ii.	Magnetic field due to BC	**b.**	is greater than that due to CD
iii.	Magnetic field due to DA	**c.**	is not equal to zero
iv.	Magnetic field due to complete figure	**d.**	is zero

5. Consider three infinite current-carrying wires passing through each of the vertices of an equilateral triangle such that the wires are perpendicular to the plane of triangle. All the wires carry identical currents in direction out of the page. M is the centroid of triangle.



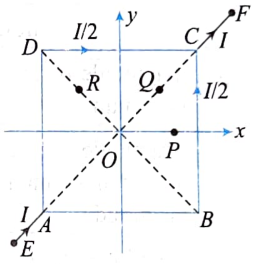
	Column I		Column II
i.	Magnetic field at M for the given situation is	**p.**	Zero
ii.	Magnetic field at M if direction of current in A is reversed keeping its magnitude same is	**q.**	Perpendicular to BC
iii.	Magnetic field at M if current in A is halved keeping its direction same is	**r.**	Along BC
iv.	Magnetic field at M if current in A is doubled keeping its direction same is	**s.**	Along CB

Codes:

	i.	ii.	iii.	iv.
(1)	p	s	s	r
(2)	p	q	r	s
(3)	s	p	r	q
(4)	q	s	s	p

6. A square wire frame $ABCD$ made of uniform conducting wire is placed in x-y plane ($z = 0$ plane) with its centre at origin and side BC is parallel to y-axis. Two straight conducting wires EA and CF also lie in x-y plane and both are parallel to line $y = x$.

Current I is fed to square wire frame through wire EA and current I exits the square wire frame through wire CF. P, Q and R are three points as shown on x-axis, segment AC and segment BD respectively. In each situation of Column I, magnetic field due to only current carrying square wire frame is to be considered (Neglect the magnetic field due to current carrying segments EA and CF). Match the statement in Column I with results in Column II.



	Column I		Column II
i.	The magnetic field at origin	**p.**	has zero magnitude
ii.	The magnetic field at point P	**q.**	has non-zero magnitude
iii.	The magnetic field at point Q	**r.**	is along +ve z-direction
iv.	The magnetic field at point R	**s.**	is along –ve z-direction

Codes:

	i.	ii.	iii.	iv.
(1)	p	q, s	r	p, q
(2)	p, q	p, q	q, s	r, s
(3)	p	q, r	p	q, s
(4)	p	q, r	p, q	q, s

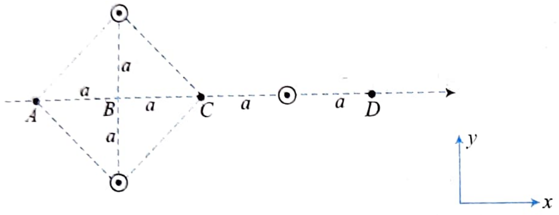
7. DC-measuring instruments are based on magnetic effect of current, whereas AC-measuring instruments are based on heating effect of current. Now, match entries of Column I with entries of Column II with their correct proportionality.

	Column I		Column II
i.	Deflection of AC ammeter	**p.**	V
ii.	Deflection of DC ammeter	**q.**	V^2
iii.	Deflection of AC voltmeter	**r.**	I
iv.	Deflection of DC voltmeter	**s.**	I^2

Codes:

	i.	ii.	iii.	iv.
(1)	q	p	s	r
(2)	s	p	r	q
(3)	p	q	r	s
(4)	s	r	q	p

8. Figure shows arrangement of three long parallel conducting wires, each carrying equal current coming out of page. Four points A, B, C and D are also marked in the figure, showing distances between points and wires. All points lie in the same plane which $\vec{dl}$ is perpendicular to the wires.



	Column I		Column II
i.	At point A,	**p.**	magnitude of the magnetic field is maximum
ii.	At point B,	**q.**	magnitude of the magnetic field is neither maximum nor minimum in magnitude
iii.	At point C,	**r.**	magnitude of the magnetic field is minimum
iv.	At point D,	**s.**	direction of the magnetic field is along negative y axis.

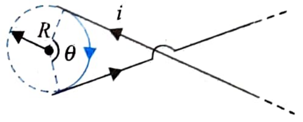
Codes:

	i.	ii.	iii.	iv.
(1)	s, q	p, r	s	r
(2)	p, r	s, q	r	p
(3)	s, q	s, q	r	p
(4)	p, r	p, s	s, q	p

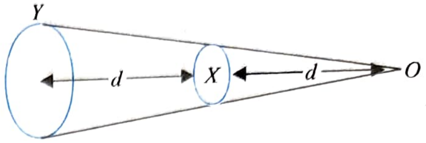
Numerical Value Type

1. Two circular coils made of similar wires but of radii 20 cm and 40 cm are connected in parallel. Find the ratio of the magnetic fields at their centers.

2. A wire carrying current i has the configuration shown in figure. Two semi-infinite straight sections, each tangent to the same circle, are connected by a circular arc, of angle θ, along the circumference of the circle, with all sections lying in the same plane. What must θ (in rad) be in order for B to be zero at the center of circle?

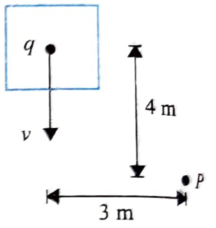


3. Two circular coils X and Y, having equal number of turns and carrying currents in the same sense, subtend same solid angle at point O. If the smaller coil X is midway between O and Y and if we represent the magnetic induction due to bigger coil Y at O as B_y and that due to smaller coil X at O as B_x, then find the ratio B_x/B_y.



4. A square loop of side $a = 6$ cm carries a current $I = 1$ A. Calculate magnetic induction B (in μT) at point P, lying on the axis of loop and at a distance $x = \sqrt{7}$ cm from the center of loop.

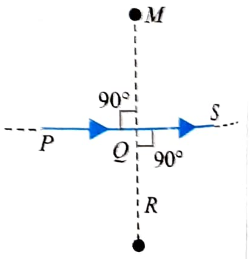
5. An elevator carrying a charge of 0.5 C is moving down with a velocity of 5×10^3 m s^{-1}. The elevator is 4 m from the bottom and 3 m horizontally from P as shown in figure. What magnetic field (in μT) does it produce at point P?



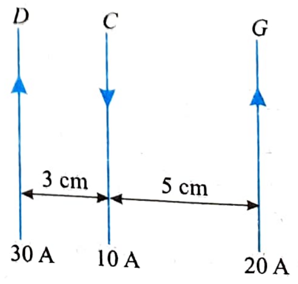
6. A system consists of two paralel planes carrying currents producing a uniform magnetic field of induction B between the planes. Outside this space there is no magnetic field. The magnetic force acting per unit area of each plane is found to be $B^2/N\mu_0$. Find N.

7. Two parallel, long wires carry currents i_1 and i_2 with $i_1 > i_2$. When the currents are in the same direction, the magnetic field at a point midway between the wire is 10 mT. If the direction of i_2 is reversed, the field becomes 30 mT. Find the ratio i_1/i_2.

8. An infinitely long conductor PQR is bent to form a right angle as shown. A current I flows through PQR. The magnetic field due to this current at the point M is B_1. Now, another infinitely long straight conductor QS is connected at Q so that the current is $I/2$ in QR as well as in QS, the current in PQ remaining unchanged. The magnetic field at M is now $B_2 = 4$ mT. Find the value of B_1 (in mT).



9. Consider three long straight parallel wires as shown in figure. The force experienced by a 25 cm length of wire C is $N \times 10^{-4}$ N. Find the value of N?



10. A toroid of total 1000 turns is made by using a tore of average radius 25 cm. The magnetic induction inside the tore is (in $\times 10^{-2}$ T) if a current of 2 A is passed through it and the relative magnetic permeability of the tore material is $\mu_r = 100$?

11. Two horizontal parallel straight conductors, each 20 cm long, are arranged one vertically above the other and carry equal currents in opposite directions. The lower conductor is fixed while the other is free to move in guides remaining parallel to the lower. If the upper conductor weighs 1.0 g, what is the approximate current (in A) that will maintain the conductors at a distance 0.50 cm apart?

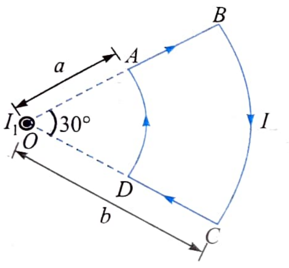
Archives

JEE MAIN

Single Correct Answer Type

Statement for Problems 1 and 2

A current loop $ABCD$ is held fixed on the plane of the paper as shown in the figure. The arcs BC (radius b) and DA (radius a) of the loop are joined by two straight wires AB and CD. A steady current I is flowing in the loop. The angle made by AB and CD at the origin O is 30°. Another straight thin wire with steady current I_1 flowing out of the plane of the paper, is kept at the origin. **(AIEEE 2009)**



1. The magnitude of the magnetic field (B) due to loop $ABCD$ at the origin (O) is

(1) zero

(2) $\frac{\mu_0 I(b-a)}{24ab}$

(3) $\frac{\mu_0 I}{4\pi}\left[\frac{b-a}{ab}\right]$

(4) $\frac{\mu_0 I}{4\pi}\left[2(b-a)+\frac{\pi}{3}(a+b)\right]$

2. Due to the presence of the current I_1 at the origin,

(1) the forces on AB and DC are zero.

(2) the forces on AD and BC are zero.

(3) the magnitude of the net force on the loop is given by

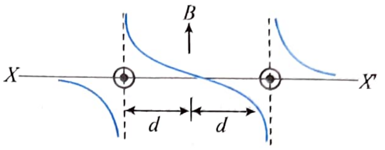
$$\frac{\mu_0 II}{4\pi}\left[2(b-a)+\frac{\pi}{3}(a+b)\right]$$

(4) the magnitude of the net force on the loop is given by

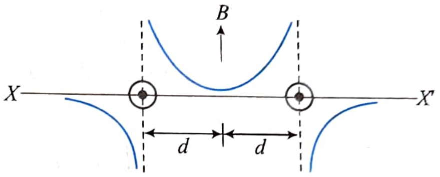
$$\frac{\mu_0 II_1}{24ab}(b-a)$$

3. Two long parallel wires are at a distance $2d$ apart. They carry steady equal currents flowing out of the plane of the paper as shown. The variation of the magnetic field along the line XX' is given by

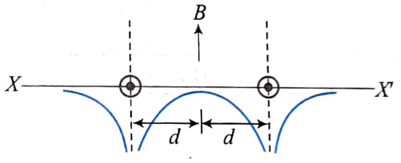
(1)



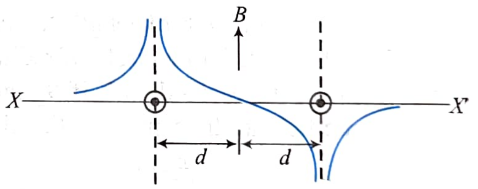
(2)



(3)



(4)



(AIEEE 2010)

4. A current I flows in an infinitely long wire with cross-section in the form of a semicircular ring of radius R. The magnitude of the magnetic induction along its axis is

(1) $\frac{\mu_0 I}{\pi^2 R}$

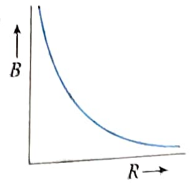
(2) $\frac{\mu_0 I}{2\pi^2 R}$

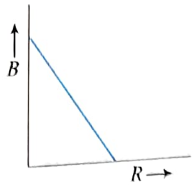
(3) $\frac{\mu_0 I}{2\pi R}$

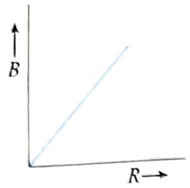
(4) $\frac{\mu_0 I}{4\pi R}$

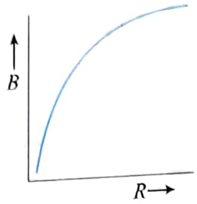
(AIEEE 2011)

5. A charge Q is uniformly distributed over the surface of non-condcting disc of radius R. The disc rotates about an axis perpendicular to its plane and passing through its centre with an angular velocity ω. As a result of this rotation a magnetic field of induction B is obtained at the centre of the disc. If we keep both the amount of charge placed on the disc and its angular velocity to be constant and vary the radius of the disc, then the variation of the magnetic induction at the centre of the disc will be represented by which of the following figures.

(1)


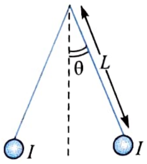
(2)


(3)


(4)


(AIEEE 2012)

6. Two long current-carrying thin wires, both with current I, are held by insulating threads of length L and are in equilibrium as shown in the figure, with threads making an angle θ with the vertical. If wires have mass λ per unit length then the value of I is (g = gravitational acceleration)



(1) $\sin\theta\sqrt{\dfrac{\pi\lambda gL}{\mu_0\cos\theta}}$ (2) $2\sin\theta\sqrt{\dfrac{\pi\lambda gL}{\mu_0\cos\theta}}$

(3) $2\sqrt{\dfrac{\pi gL}{\mu_0}}\tan\theta$ (4) $\sqrt{\dfrac{\pi\lambda gL}{\mu_0}}\tan\theta$

(JEE Main 2015)

7. Two identical wires A and B, each of length 'l', carry the same current I. Wire A is bent into a circle of radius R and wire B is bent to form a square of side 'a'. If B_A and B_B are the values of magnetic field at the centres of the circle and square respectively, then the ratio $\dfrac{B_A}{B_B}$ is

(1) $\dfrac{\pi^2}{8}$ (2) $\dfrac{\pi^2}{16\sqrt{2}}$

(3) $\dfrac{\pi^2}{16}$ (4) $\dfrac{\pi^2}{8\sqrt{2}}$ **(JEE Main 2016)**

JEE ADVANCED

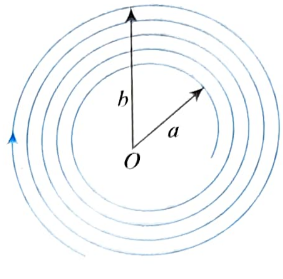
Single Correct Answer Type

1. A steady current I goes through a wire loop PQR having shape of a right angle triangle with $PQ = 3x$, $PR = 4x$, and $QR = 5x$. If the magnitude of the magnetic field at P due to this loop is $k\left(\dfrac{\mu_0 I}{48\pi x}\right)$, the value of k is

(1) 5 (2) 8

(3) 7 (4) 10 **(IIT-JEE 2009)**

2. A long insulated copper wire is closely wound as a spiral of N turns. The spiral has inner radius a and outer radius b. The spiral lies in the X–Y plane and a steady current I flows through the wire. The Z-component of the magnetic field at the center of the spiral is

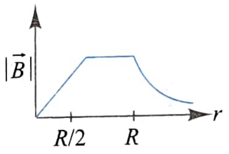


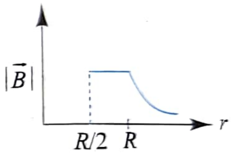
(1) $\dfrac{\mu_0 NI}{2(b-a)}\ln\left(\dfrac{b}{a}\right)$ (2) $\dfrac{\mu_0 NI}{2(b-a)}\ln\left(\dfrac{b+a}{b-a}\right)$

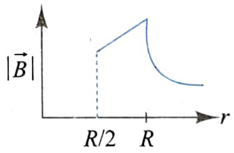
(3) $\dfrac{\mu_0 NI}{2b}\ln\left(\dfrac{b}{a}\right)$ (4) $\dfrac{\mu_0 NI}{2b}\ln\left(\dfrac{b+a}{b-a}\right)$

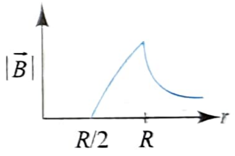
(IIT-JEE 2011)

3. An infinitely long hollow conducting cylinder with inner radius $R/2$ and outer radius R carries a uniform current density along length. The magnitude of the magnetic field, $|\vec{B}|$ as a function of the radial distance r from the axis is best represented by

(1)


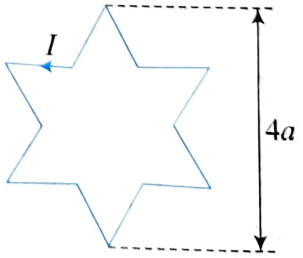
(2)


(3)


(4)


(IIT-JEE 201

4. A symmetric star shaped conducting wire loop is carrying a steady state current I as shown in the figure. The distance between the diametrically opposite vertices of the star is $4a$. The magnitude of the magnetic field at the centre of the loop is:



(1) $\dfrac{\mu_0 I}{4\pi a}3\left[\sqrt{3}-1\right]$ (2) $\dfrac{\mu_0 I}{4\pi a}6\left[\sqrt{3}-1\right]$

(3) $\dfrac{\mu_0 I}{4\pi a}6\left[\sqrt{3}+1\right]$ (4) $\dfrac{\mu_0 I}{4\pi a}3\left[2-\sqrt{3}\right]$

(JEE Advanced 2017)

Multiple Correct Answers Type

1. A steady current I flows along an infinitely long hollow cylindrical conductor of radius R. This cylinder is placed coaxially inside an infinite solenoid of radius $2R$. The solenoid has n turns per unit length and carries a steady current I. Consider a point P at a distance r from the common axis. The correct statement(s) is(are)

(1) In the region $0 < r < R$, the magnetic field is non-zero

(2) In the region $R < r < 2R$, the magnetic field is along the common axis.

(3) In the region $R < r < 2R$, the magnetic field is tangential to the circle of radius r, centered on the axis.

(4) In the region $r > 2R$, the magnetic field is non-zero.

(IIT-JEE 2013)

2. Two infinitely long straight wires lie in the xy-plane along the lines $x = \pm R$. The wire located at $x = +R$ carries a constant current I_1 and the wire located at $x = -R$ carries a constant current I_2. A circular loop of radius R is suspended with its centre at $(0, 0, \sqrt{3}R)$ and in a plane parallel to the xy-plane. This loop carries a constant current I in the clockwise direction as seen from above the loop. The current in the wire is taken to be positive if it is in the $+\hat{j}$ direction. Which of the following statements regarding the magnetic field $\vec{B}$ is (are) true?

(1) If $I_1 = I_2$, then $\vec{B}$ cannot be equal to zero at the origin $(0, 0, 0)$

(2) If $I_1 > 0$ and $I_2 < 0$, then $\vec{B}$ can be equal to zero at the origin $(0, 0, 0)$

(3) If $I_1 < 0$ and $I_2 > 0$, then $\vec{B}$ can be equal to zero at the origin $(0, 0, 0)$

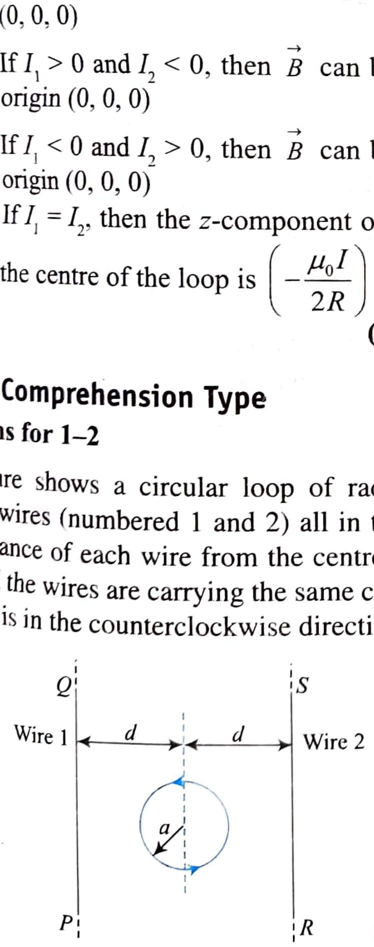
(4) If $I_1 = I_2$, then the z-component of the magnetic field at the centre of the loop is $\left(-\dfrac{\mu_0 I}{2R}\right)$.

(JEE Advanced 2018)

Linked Comprehension Type

Problems for 1–2

The figure shows a circular loop of radius a with two long parallel wires (numbered 1 and 2) all in the plane of the paper. The distance of each wire from the centre of the loop is d. The loop and the wires are carrying the same current I. The current in the loop is in the counterclockwise direction if seen from above.



(JEE Advanced 2014)

1. When $d \approx a$ but wires are not touching the loop, it is found that the net magnetic field on the axis of the loop is zero at a height h above the loop. In that case

(1) Current in wire 1 and wire 2 is the direction PQ and RS, respectively and $h \approx a$

(2) Current in wire 1 and wire 2 is the direction PQ and SR, respectively and $h \approx a$

(3) Current in wire 1 and wire 2 is the direction PQ and SR, respectively and $h \approx 1.2a$

(4) Current in wire 1 and wire 2 is the direction PQ and RS, respectively and $h \approx 1.2a$

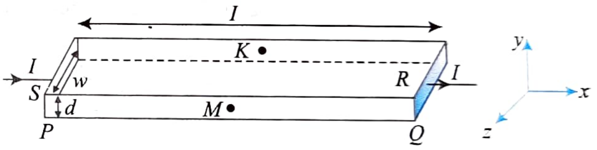
2. Consider $d >> a$, and the loop is rotated about its diameter parallel to the wires by 30° from the position shown in the figure. If the currents in the wires are in the opposite directions, the torque on the loop at its new position will be (assume that the net field due to the wires is constant over the loop)

(1) $\dfrac{\mu_0 I^2 a^2}{d}$ (2) $\dfrac{\mu_0 I^2 a^2}{2d}$

(3) $\dfrac{\sqrt{3}\mu_0 I^2 a^2}{d}$ (4) $\dfrac{\sqrt{3}\mu_0 I^2 a^2}{2d}$

Paragraph for Questions 3 and 4

In a thin rectangular metallic strip a constant current I flows along the positive x-direction, as shown in the figure. The length, width and thickness of the strip are l, w and d, respectively.



A uniform magnetic field $\vec{B}$ is applied on the strip along the positive y-direction. Due to this, the charge carriers experience a net deflection along the z-direction. This results in accumulation of charge carriers on the surface $PQRS$ and is appearance of equal and opposite charges on the face opposite to $PQRS$. A potential difference along the z-direction is thus developed. Charge accumulation continues until the magnetic force is balanced by the electric force. The current is assumed to be uniformly distributed on the cross section of the strip and carried by electrons. **(JEE Advanced 2015)**

3. Consider two different metallic strips (1 and 2) of the same material. Their lengths are the same, width are w_1 and w_2 and thicknesses are d_1 and d_2, respectively. Two points K and M are symmetrically located on the opposite faces parallel to the x-y plane (see figure). V_1 and V_2 are the potential differences between K and M in strips 1 and 2, respectively. Then, for a given current I flowing through them in a given magnetic field strength B, the correct statement(s) is (are)

(1) If $w_1 = w_2$ and $d_1 = 2d_2$, then $V_2 = 2V_1$

(2) If $w_1 = w_2$ and $d_1 = 2d_2$, then $V_2 = V_1$

(3) If $w_1 = 2w_2$ and $d_1 = d_2$, then $V_2 = 2V_1$

(4) If $w_1 = 2w_2$ and $d_1 = d_2$, then $V_2 = V_1$

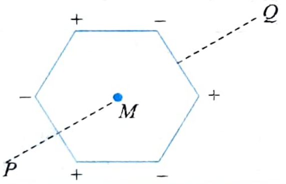
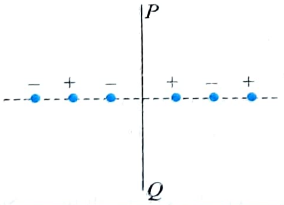
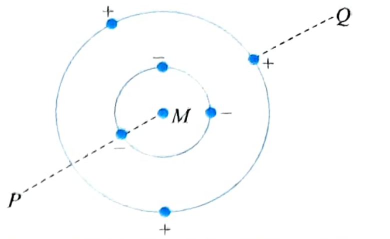
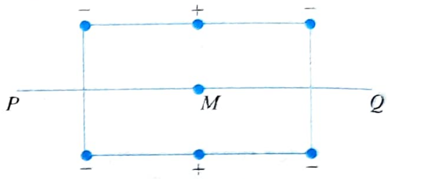
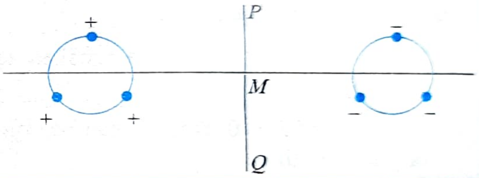
4. Consider two different metallic strips (1 and 2) of the same dimensions (length l, width w and thickness d) with carrier densities n_1 and n_2, respectively. Strip 1 is placed

in magnetic field B_1 and strip 2 is placed in magnetic field B_2, both along positive y-directions. Then V_1 and V_2 are the potential differences developed between K and M in strips 1 and 2, respectively. Assuming that the current I is the same for both the strips, the correct option(s) is (are)

(1) If $B_1 = B_2$ and $n_1 = 2n_2$, then $V_2 = 2V_1$
(2) If $B_1 = B_2$ and $n_1 = 2n_2$, then $V_2 = V_1$
(3) If $B_1 = 2B_2$ and $n_1 = n_2$, then $V_2 = 0.5V_1$
(4) If $B_1 = 2B_2$ and $n_1 = n_2$, then $V_2 = V_1$

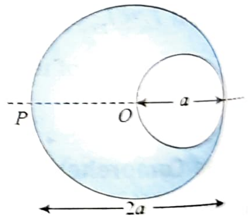
Matrix Match Type

1. Six point charges, each of the same magnitude q, are arranged in different manners as shown in Column II. In each case, point M and line PQ passing through M are shown. Let E be the electric field and V be the electric potential at M (potential at infinity is zero) due to the given charge distribution when it is at rest. Now the whole system is set into rotation with a constant angular velocity about the line PQ. Let B be the magnetic field at M and μ be the magnetic moment of the system in this condition. Assume each rotating charge to be equivalent to a steady urrent. **(IIT-JEE 2009)**

	Column I		Column II
i.	$E = 0$	**a.**	Charges are at the corners of a regular hexagon. M is at the center of the hexagon. PQ is perpendicular to the plane of the hexagon. 
ii.	$V \neq 0$	**b.**	Charges are on a line perpendicular to PQ at equal intervals. M is the midpoint between the two innermost charges 
iii.	$B = 0$	**c.**	Charges are placed on two coplanar insulating rings at equal intervals. M is the common center of the rings. PQ is perpendicular to the plane of the rings 
iv.	$\mu \neq 0$	**d.**	Charges are placed at the corners of a rectangle of sides a and $2a$ and at the midpoints of the longer sides. M is at the center of the rectangle. PQ is parallel to the longer sides 
		e.	Charges are placed on two coplanar, identical insulating rings at equal intervals. M is the midpoint between the centers of the rings. PQ is perpendicular to the line joining the centers and coplanar to the rings. 

Numerical Value Type

1. A cylindrical cavity of diameter a exists inside a cylinder of diameter $2a$ as shown in figure. Both the cylinder and the cavity are infinitely long. A uniform current density J flows along the length. If the magnitude of the magnetic field at point P is given by $\frac{N}{12}\mu_0\, aJ$, then the value of N is **(IIT-JEE 2012)**



Answers Key

EXERCISES

Single Correct Answer Type

1. (1) **2.** (2) **3.** (1) **4.** (4) **5.** (2)
6. (2) **7.** (3) **8.** (1) **9.** (3) **10.** (4)
11. (1) **12.** (3) **13.** (1) **14.** (2) **15.** (4)
16. (2) **17.** (3) **18.** (1) **19.** (3) **20.** (1)
21. (4) **22.** (3) **23.** (1) **24.** (3) **25.** (3)
26. (1) **27.** (3) **28.** (3) **29.** (1) **30.** (1)
31. (3) **32.** (2) **33.** (1) **34.** (3) **35.** (2)
36. (4) **37.** (1) **38.** (2) **39.** (2) **40.** (3)
41. (3) **42.** (1) **43.** (2) **44.** (3) **45.** (1)
46. (2) **47.** (2) **48.** (3) **49.** (1) **50.** (3)
51. (1) **52.** (2) **53.** (1) **54.** (2) **55.** (1)
56. (4) **57.** (2) **58.** (2) **59.** (3) **60.** (3)
61. (3) **62.** (2) **63.** (3) **64.** (2) **65.** (4)

66. (4) **67.** (2) **68.** (3) **69.** (4) **70.** (1)
71. (2) **72.** (1) **73.** (4) **74.** (2) **75.** (1)
76. (1) **77.** (3) **78.** (2) **79.** (1) **80.** (3)
81. (1) **82.** (2) **83.** (4) **84.** (2) **85.** (1)

Multiple Correct Answers Type

1. (2),(3) **2.** (2),(4) **3.** (1),(2),(3)
4. (1),(3) **5.** (2),(4) **6.** (1),(3)
7. (2),(4) **8.** (1),(4) **9.** (2),(3),(4)
10. (1),(4) **11.** (1),(2) **12.** (2),(4)
13. (1),(2),(3) **14.** (3),(4) **15.** (2),(3)
16. (1),(2) **17.** (3),(4) **18.** (2),(3)
19. (2),(3),(4) **20.** (2),(4)

Linked Comprehension Type

1. (3) **2.** (1) **3.** (1) **4.** (2) **5.** (4)
6. (1) **7.** (2) **8.** (3) **9.** (3) **10.** (4)
11. (1) **12.** (2) **13.** (2) **14.** (2) **15.** (4)
16. (3) **17.** (1) **18.** (1) **19.** (2) **20.** (4)
21. (4) **22.** (2) **23.** (3) **24.** (1) **25.** (1)
26. (4)

Matrix Match Type

1. i. → b., d.; ii. → a., d.; iii. → c., d.; iv. → c., d.
2. i. → b., c.; ii. → b., d.; iii. → a.; iv. → c.
3. i. → a., b., c.; ii. → a., b., c., d.; iii. → c.; iv. → a., b., c., d.
4. i. → d.; ii. → a., b., c.; iii. → b., c.; iv. → a., b., c.
5. (1) **6.** (2) **7.** (4) **8.** (3)

Numerical Value Type

1. (4) **2.** (2) **3.** (2) **4.** (9) **5.** (6)
6. (2) **7.** (2) **8.** (3) **9.** (3) **10.** (16)
11. (50)

ARCHIVES

JEE Main

Single Correct Answer Type

1. (2) **2.** (2) **3.** (1) **4.** (1) **5.** (1)
6. (2) **7.** (4)

JEE Advanced

Single Correct Answer Type

1. (3) **2.** (1) **3.** (4) **4.** (2)

Multiple Correct Answers Type

1. (1),(4) **2.** (1),(2),(4)

Linked Comprehension Type

1. (3) **2.** (2) **3.** (1),(4) **4.** (1),(3)

Matrix Match Type

1. i. → a., c., d.; ii. → c., d.; iii. → a., b., e.; iv. → c., d.

Numerical Value Type

1. (5)

3

Permanent Magnets and Magnetic Properties of Matter

INTRODUCTION

In the previous chapter, we studied that electric currents produce magnetic field, i.e., electricity generates magnetism. The terms such as magnetic field lines, magnetic flux, magnetic dipole moment etc., were discussed in the context of magnetic fields due to current flowing through a straight conductor, a circular coil, a solenoid and a toroid. These terms are closely related to magnetism, a complete topic in its own right. It is therefore imperative that we look into the details of magnetism afresh by combining its relation to electric current.

The science of magnetism grew when people observed that a certain ore could attract bits of iron and always pointed in a particular direction when freely suspended.

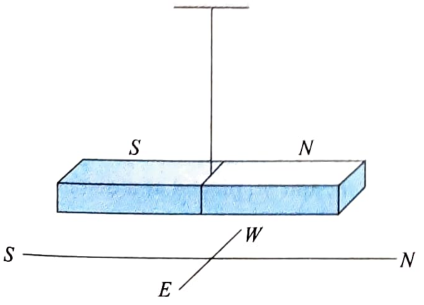
The property of attracting small pieces of iron and steel is referred to as **magnetism**. The mineral exercising this influence is called a **natural magnet**. Thus, a magnet is a substance which has attractive and directive properties. Natural magnet consists of a number of oxides of iron with Fe_3O_4 as its structural formula.

The natural magnets are weak and often irregular in shape. It is found that when natural magnets are rubbed against iron or steel bars, the same property of attraction is communicated to these pieces. They are, therefore, called **artificial magnets**. Though the magnets we commonly use are artificial magnets, which can be of desired shape and strength.

BAR MAGNET

A bar magnet is a rod shaped metal object which produces magnetic field in its surrounding at the two ends of it, there are two magnetic poles: north pole and south pole.

A bar magnet attracts iron fillings near the ends. A magnet when suspended freely with the help of a thread always rests in the **north-south** direction. The pole pointing towards north is called **north pole** and the one pointing towards south is called **south pole**.



PROPERTIES OF BAR MAGNET

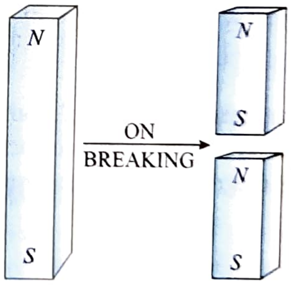
A bar magnet has the following properties:

(i) A magnet can attract small pieces of magnetic substances like iron, steel, cobalt, nickel, etc. The attraction is maximum at poles.

(ii) Like poles repel each other and unlike poles attract each other.

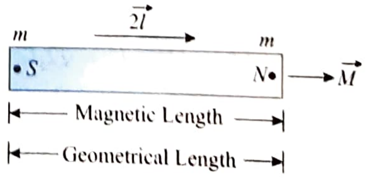
Confirm test of polarity (magnetism) is repulsion and not attraction. Attraction can also occur between a pole of a magnet and a piece of unmagnetized material.

(iii) Breaking a permanent magnet in half does not yield one north pole and one south pole. Instead, two new magnets, each with its own north and south pole result. Unlike electric charge, which exists as separate positive (proton) and negative (electron) charges, no separate magnetic monopoles (isolated north and south poles) exist. Scientists have carried out extensive searches for magnetic monopoles, but nothing has been found. The discussion of the source of magnetism in this chapter will help you understand why there are no magnetic monopoles.



(iv) The poles are situated near the ends of the magnet and not exactly at the ends. The distance between the two poles of a bar magnet is called the magnetic length of the magnet.

Magnetic length is a vector from S-pole of the magnet to its N-pole and is denoted by $2\vec{l}$ [figure].



Since the magnetic poles are situated within the magnet, the distance between its two magnetic poles called its magnetic length is always less than its geometric length.

A line joining north and south poles of a freely suspended magnet is called **magnetic axis**. A vertical plane passing through the magnetic axis (i.e., S-N of a freely suspended magnet) is called the **magnetic meridian**.

STRENGTH OF BAR MAGNET

The strength of a bar magnet is characterized by its magnetic dipole moment. If the pole strength of either pole of a bar magnet is m and the separation between its poles is $2l$, then the magnetic dipole moment of a bar magnet is given as

$$\mu = m(2l) \quad \text{...(i)}$$

The symbol used for magnetic dipole moment is either μ or M. Like electric dipole moment, magnetic dipole moment is also a vector quantity with direction taken from south-pole to north-pole. The unit used for measurement of magnetic dipole moment is 'ampere-meter square' or 'A-m^2'. Magnetic dipole moment of a bar magnet in general also called as 'magnetic moment' of the magnet.

ILLUSTRATION 3.1

A bar magnet of magnetic moment 5.0 Am^2 has poles 20 cm apart. Calculate the pole strength.

Sol. The magnetic moment $M = 5.0\ \text{Am}^2$

The separation between its poles

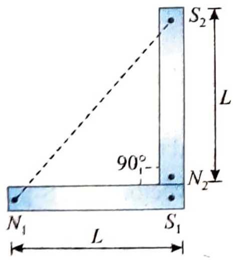
$$2l = 20\ \text{cm} = 20 \times 10^{-2}\ \text{m}$$

$$\text{Pole strength, } m = \frac{M}{2l} = \frac{5.0\,\text{Am}^2}{20 \times 10^{-2}\ \text{m}} = 25\ \text{Am}$$

ILLUSTRATION 3.2

Two identical thin bar magnets, each of length L and pole-strength m are placed at right angles to each other, with the N-pole of one touching the south pole of the other. Find the magnetic moment of the system.

Sol. When the magnets N_1S_1 and N_2S_2 each of pole strength m and length L are placed at right angles to each other (figure), the system behaves as a magnet, whose pole strength is m and length is N_1S_2.



$$\text{Now, } N_1S_2 = \sqrt{N_1S_1^2 + N_2S_2^2} = \sqrt{L^2 + L^2} = \sqrt{2}\,L$$

Therefore, magnetic moment of the system,

$$M = \text{Pole strength} \times N_1S_2 = m \times \sqrt{2}L = \sqrt{2}mL$$

MAGNETIC FIELD

Magnetic Field is the space around a magnet (or a conductor carrying current) in which its magnetic effect can be experienced. The SI unit of strength of magnetic field (also known as magnetic induction or magnetic induction field or magnetic flux density) is tesla (T). It is measured in terms of force on a charge moving inside the magnetic field or by using the concept of magnetic flux linked with a surface.

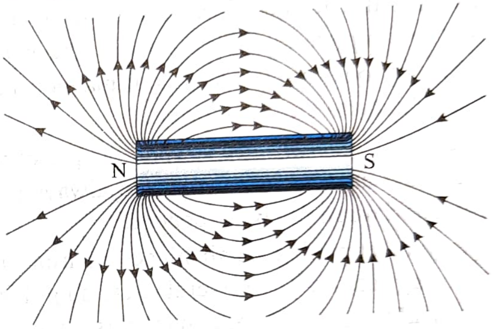
$$1 \text{ tesla (T)} = 1 \text{ newton ampere}^{-1}\text{metre}^{-1}\ (\text{N A}^{-1}\ \text{m}^{-1})$$
$$= 1 \text{ weber metre}^{-2}\ (\text{Wb m}^{-2})$$

Note:

1. Strength of magnetic field is measured in gauss (G).
   1 gauss (G) = 10^{-4} tesla (T)
2. The strength of magnetic field B is also known as magnetic field.

MAGNETIC FIELD LINES

Like an electric field, a magnetic field is represented using field lines. The magnetic field vector is always tangent to the **magnetic field lines**. The magnetic field lines from a permanent bar magnet are shown in figure.



(i) A magnetic field line is an imaginary curve, the tangent to which at any point P gives the direction of $\vec{B}$ at that point.

(ii) The magnetic field lines are closed continuous loops extending through the body of the magnet.

(iii) The number of field lines drawn per unit cross-sectional area is proportional to B. If the field is uniform, the field lines are evenly spaced.

(iv) Magnetic field lines never intersect each other.

(v) Externally, magnetic field lines appear to originate on north poles and terminate on south poles, but these field lines are actually closed loops that penetrate the magnet itself. This formation of loops is an important difference between electric and magnetic field lines

The direction of the magnetic field is established in terms of the direction in which a compass needle points. A compass needle, with a north pole and a south pole, will orient itself so that its north pole points in the direction of the magnetic field.

COULOMB'S LAW OF FORCE BETWEEN TWO MAGNETIC POLES

Isolated magnetic poles do not exist. But the idea of a magnetic pole is useful in certain analogies. We will discuss later how the idea of a magnetic dipole is a more acceptable concept.

The force of attraction or repulsion between two magnetic poles of strengths m_1 and m_2 placed at a distance r apart is given by

$$F \propto \frac{m_1 m_2}{r^2} \text{ or } F = k\frac{m_1 m_2}{r^2} \quad \text{...(i)}$$

where k is a constant of proportionality whose value depends on (i) the system of units selected and (ii) the medium in which the poles are placed.

In SI units: $k = \frac{\mu_0}{4\pi} = 10^{-7}$ Wb/Am

Equation (i) is of limited application as it applies to long, thin magnets with well separated poles.

ILLUSTRATION 3.3

Two equal magnetic poles placed 5 cm in air attract each other with a force of 14.4×10^{-4} N. How far from each other should they be placed so that the force of attraction will be 1.6×10^{-4} N?

Sol. Let m be the pole strength of each pole.

In first case: $F = 14.4 \times 10^{-4}$ N; $r = 5$ cm $= 0.05$ m

Now, $F = \frac{\mu_0}{4\pi} \cdot \frac{m \times m}{r^2}$

or $14.4 \times 10^{-4} = 10^{-7} \times \frac{m^2}{(0.05)^2}$

or $m = \pm 6$ Am

In second case: Let r' be the distance between the poles, when force of attraction becomes $F' = 1.6 \times 10^{-4}$ N.

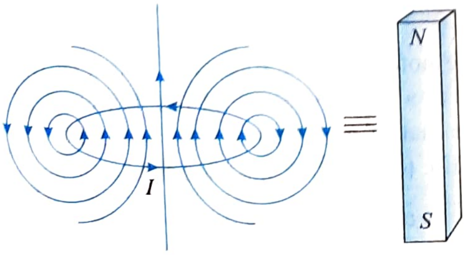
Now: $F' = \frac{\mu_0}{4\pi} \cdot \frac{m \times m}{r'^2}$

or $1.6 \times 10^{-4} = 10^{-7} \times \frac{6 \times 6}{r'^2}$

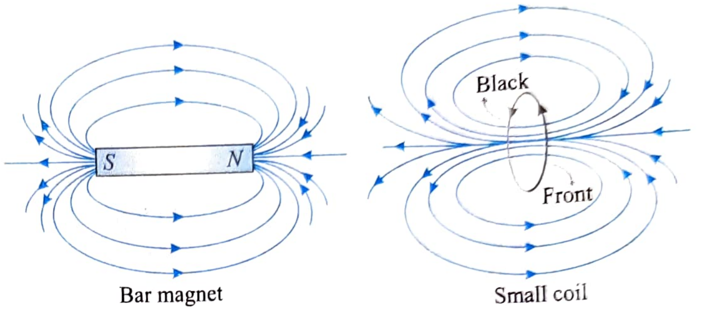
or $r' = 0.15$ m

A SMALL CURRENT-CARRYING COIL AS A MAGNETIC DIPOLE

Consider a circular coil-carrying current I. Suppose that the current flows in anticlockwise direction, when seen from above (figure). The magnetic field lines due to each elementary portion of the circular coil will be of the shape of circular loops near that portion and almost straight near the centre of the circular coil. From right hand thumb rule, it follows that the magnetic field lines seem to enter at the lower face of the coil and leave at its upper face. Thus, the lower face of the coil, through which magnetic field lines enter, behaves as the south pole; while the upper face, through which the field lines leave, behaves as the north pole. Hence, the current carrying circular coil behaves as a magnetic dipole.



The figure below shows the magnetic field produced by a bar magnet and that of a current carrying coil which looks similar except the length of bar magnet but if we look at the figure which shows the magnetic field in surrounding of a small magnetic dipole and that of a small current-carrying coil. Both the configurations of magnetic field lines look almost identical. Thus a very small current-carrying coil can be regarded as a magnetic dipole and all the results we have obtained for a magnetic dipole can be used for such small current-carrying loops with magnetic dipole moment of dipole replaced with the magnetic moment of the coil.

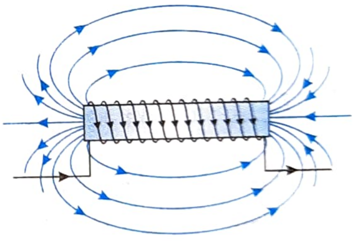


Bar magnet (a)

Small coil (b)

Figure (c) shows a tightly wound long solenoid and the magnetic induction produced by it by the magnetic field lines configuration. This magnetic field lines configuration is almost same as that produced by a bar magnet. Thus a current-carrying solenoid can be considered to be behaving like a bar magnet with magnetic moment given as

$$M = NiA$$



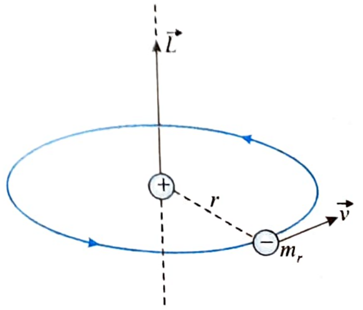
(c)

ATOM AS A MAGNETIC DIPOLE

In an atom, electrons revolve around the nucleus and as such the circular orbits of electrons may be considered as the small current loops. Due to this, an atom possesses magnetic dipole moment and hence behaves as a magnetic dipole.

Expression for magnetic moment: Suppose that an electron of mass m_e and charge e revolves in a circular orbit of radius r around the positive nucleus in anticlockwise direction, as seen in figure. The angular momentum of the electron due to its orbital motion is given by

$$L = m_e vr \quad \text{...(i)}$$



For the sense of orbital motion of electron shown in the figure, the angular momentum vector $\vec{L}$ acts along normal to the plane of the electron orbit and in upward direction.

Suppose that the period of orbital motion of the electron is T. Then, the electron crosses any point on its orbit after every T seconds or $1/T$ times in one second.

Therefore, orbital motion of electron is equivalent to a current,

$$I = e\left(\frac{1}{T}\right)$$

Now, the period of revolution of the electron is given by

$$T = \frac{2\pi r}{v}$$

Therefore, $I = e\left(\frac{1}{2\pi r/v}\right) = \frac{ev}{2\pi r}$

The area of the electron orbit (area of current loop), $A = \pi r^2$

Therefore, the magnetic dipole moment of the atom,

$$M = \frac{ev}{2\pi r} \times \pi r^2 = \frac{evr}{2}$$

For rotating body, the ratio of magnetic moment and angular momentum is a constant, which is equal to $\frac{q}{2m}$, where q is the charge and m is the mass of the body. Here is this case, electron is rotating around nucleus. Hence here we can write

$$M = \left(\frac{e}{2m_e}\right)L \qquad \text{...(ii)}$$

In vector notation,

$$\vec{M} = -\left(\frac{e}{2m_e}\right)\vec{L} \qquad \text{...(iii)}$$

The negative sign has been included for the reason that electron has negative charge. Equation (iii) tells that the magnetic dipole moment vector is directed in a direction opposite to that of angular momentum vector.

According to Bohr's theory, angular momentum of electron in a stationary orbitcan have only those values, which are integral multiples of $\frac{h}{2\pi}$ i.e.

$$L = n\frac{h}{2\pi}, \text{ where } n = 1, 2, 3 \qquad \text{...(iv)}$$

From Eqs. (ii) and (iv), we have

$$M = \left(\frac{e}{2m_e}\right)\frac{n\,h}{2\,\pi}$$

or $$M = n\left(\frac{eh}{4\pi\, m_e}\right) \qquad \text{...(iv)}$$

Equation (iv) gives the magnetic moment of an electron due to its orbital motion.

Bohr magneton: It is defined as the magnetic dipole moment associated with an atom due to orbital motion of an electron in the first orbit of hydrogen atom.

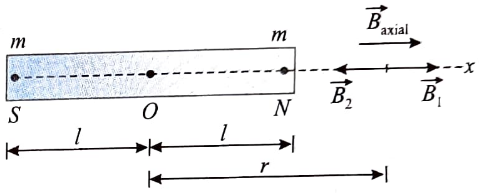
From Eq. (iv), it follows that $\frac{eh}{4\pi m_e}$ is the least value of the magnetic moment. This natural unit of magnetic moment is called Bohr magneton. It is denoted by μ_B. Its value is

$$\mu_B = \frac{(1.6\times10^{-19}\,\text{C})\times(6.62\times10^{-34}\,\text{J s})}{4\pi\times(9.1\times10^{-31}\,\text{kg})} = 9.27\times10^{-24}\ \text{Am}^2$$

We know that in an atom, in addition to orbital motion, an electron has got spin motion also. The electron possesses magnetic moment due to its spin motion also. The total magnetic moment of the electron is the vector sum of its magnetic moments due to orbital and spin motion.

MAGNETIC FIELD ON AXIAL LINE OF A BAR MAGNET

Consider a bar magnet NS, with pole strength m and length $2l$. P is a point located at a distance r from the center and on the axis of magnet. The two magnetic poles produce magnetic induction at point P in opposite directions as shown in figure.



The net magnetic field $\vec{B}_{\text{axial}}$ at point P is given as

$$\vec{B}_{\text{axial}} = \vec{B}_1 + \vec{B}_2$$

Now, $\left|\vec{B}_1\right| = \frac{\mu_0}{4\pi}\cdot\frac{m}{NP^2} = \frac{\mu_0}{4\pi}\cdot\frac{m}{(r-l)^2}$ (along PX)

and $\left|\vec{B}_2\right| = \frac{\mu_0}{4\pi}\cdot\frac{m}{SP^2} = \frac{\mu_0}{4\pi}\cdot\frac{m}{(r+l)^2}$ (along PS)

It follows that $\left|\vec{B}_1\right|$ is greater than $\left|\vec{B}_2\right|$.

Since $\vec{B}_1$ and $\vec{B}_2$ act along the same line but in opposite directions,

$$\therefore \quad \left|\vec{B}_{\text{axial}}\right| = \left|\vec{B}_1\right| - \left|\vec{B}_2\right| \qquad \text{(along } PX\text{)}$$

or $$B_{\text{axial}} = \frac{\mu_0}{4\pi}\cdot\frac{m}{(r-l)^2} - \frac{\mu_0}{4\pi}\cdot\frac{m}{(r+l)^2}$$

$$= \frac{\mu_0}{4\pi}\cdot m\left[\frac{(r+l)^2-(r-l)^2}{(r^2-l^2)^2}\right] = \frac{\mu_0}{4\pi}\cdot\frac{m(4rl)}{(r^2-l^2)^2}$$

Now, $m\,(2l) = M$, magnitude of the magnetic dipole moment of the magnet.

$$\therefore \quad B_{\text{axial}} = \frac{\mu_0}{4\pi}\cdot\frac{2Mr}{(r^2-l^2)^2} \qquad \text{(along } PX\text{)} \quad \text{...(i)}$$

It may be noted that the direction of magnetic field due to a bar magnet at a point on its axial line is from S-pole to N-pole i.e. same as that of magnetic dipole moment of the bar magnet. Therefore, Eq. (i) may be expressed as

$$\vec{B}_{\text{axial}} = \frac{\mu_0}{4\pi}\cdot\frac{2\vec{M}\,r}{(r^2-l^2)^2} \qquad \text{...(ii)}$$

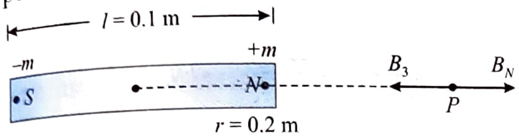
If bar magnet is of very small length. For a bar magnet of very small length, $l << r$, and hence in Eq. (ii), l^2 can be neglected in comparison to r^2. Therefore, for a very short bar magnet,

$$B_{\text{axial}} = \frac{\mu_0}{4\pi}\cdot\frac{2M}{r^3} \quad \text{(along } PX\text{)} \quad \text{...(iii)}$$

ILLUSTRATION 3.4

A bar magnet is 0.1m long and its pole strength is 12 Am. Find the magnetic induction at a point on its axis at a distance of 0.2 m from its centre.

Sol. Net magnetic induction due to bar magnet at point P is given by the vector sum of the magnetic inductions at P due to the two poles is given as



$$B_P = B_N - B_S$$

$$\Rightarrow \quad B_P = \frac{\mu_0}{4\pi}\cdot\frac{m}{(r-l/2)^2} - \frac{\mu_0}{4\pi}\cdot\frac{m}{(r+l/2)^2}$$

$$\Rightarrow \quad B_P = \frac{\mu_0}{4\pi}\left[\frac{2rl}{(r^2-l^2/4)^2}\right] = \frac{32\mu_0 mrl}{4\pi(4r^2-l^2)^2}$$

$$\Rightarrow \quad B_P = \frac{32\times10^{-7}\times12\times0.2}{(4(0.2)^2-(0.1)^2)^2}$$

$$\Rightarrow \quad B_P = 3.4\times10^{-4}\ \text{T}$$

MAGNETIC FIELD ON EQUATORIAL LINE OF A BAR MAGNET

Consider a bar magnet NS, whose each pole is of strength m. Let $2l$ be the magnetic length of the bar magnet and O be its centre. Let P be a point on the equitorial line of the magnet at a distance r from the centre of the magnet.

Let $\vec{B}_1$ and $\vec{B}_2$ be the magnetic fields at point P due to N-pole and S-pole of the bar magnet. Then, resultant magnetic field at point P due to the bar magnet is given by

$$\vec{B}_{\text{equi}} = \vec{B}_1 + \vec{B}_2$$

Now, $$|\vec{B}_1| = |\vec{B}_2| = \frac{\mu_0}{4\pi}\cdot\frac{m}{(r^2+l^2)} \quad \text{(along } NP\text{)}$$

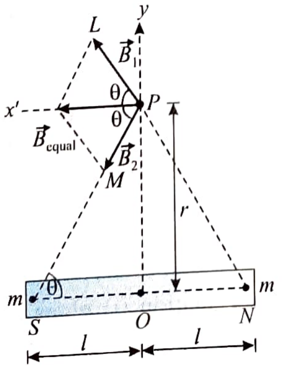
and $$|\vec{B}_2| = \frac{\mu_0}{4\pi}\cdot\frac{m}{SP^2} = \frac{\mu_0}{4\pi}\cdot\frac{m}{(r^2+l^2)} \quad \text{(along } PS\text{)}$$

The two magnetic poles produce magnetic field of equal magnitude at point P due to symmetry along the directions as shown in figure.

Therefore, resultant magnetic field at point P due to the bar magnet,

$$|\vec{B}_{\text{equi}}| = 2|\vec{B}_1|\cos\theta$$

As, $$|\vec{B}_1| = |\vec{B}_2| = \frac{\mu_0}{4\pi}\cdot\frac{m}{(r^2+l^2)}$$



or $$B_{\text{equi}} = \frac{\mu_0}{4\pi}\cdot\frac{2m}{(r^2+l^2)}\cos\theta$$

From right-angled ΔSOP,

$$\cos\theta = \frac{SO}{SP} = \frac{l}{(r^2+l^2)^{1/2}}$$

Hence, $$B_{\text{equi}} = \frac{\mu_0}{4\pi}\cdot\frac{2m}{(r^2+l^2)}\cdot\frac{l}{(r^2+l^2)^{1/2}}$$

or $$B_{\text{equi}} = \frac{\mu_0}{4\pi}\cdot\frac{M}{(r^2+l^2)^{3/2}} \quad \text{(along } PX'\text{)} \quad \text{...(iv)}$$

Here, $M = 2ml$ is magnetic dipole moment of the bar magnet.

It may be noted that direction of magnetic field due to a bar magnet on its equatorial line is from N-pole to S-pole of the magnet i.e. in a direction opposite to that of the magnetic dipole moment of the magnet. Therefore, Eq. (iv) may be expressed as

$$\vec{B}_{\text{equi}} = -\frac{\mu_0}{4\pi}\cdot\frac{\vec{M}}{(r^2+l^2)^{3/2}} \quad \text{...(v)}$$

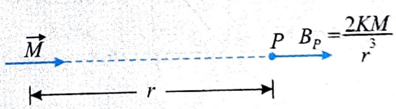
For a bar magnet of very small length, $l << r$ and hence in Eq. (v), l^2 can be neglected in comparison to r^2. Therefore, for a very short bar magnet,

$$B_{\text{equi}} = \frac{\mu_0}{4\pi}\cdot\frac{M}{r^3} \quad \text{(along } PX'\text{)} \quad \text{...(vi)}$$

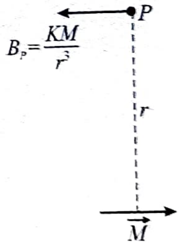
Important Points:

From Eqs. (iii) and (vi), it follows that magnetic field at a point at a certain distance on the equatorial line of a short bar magnet is just one-half of that at the same distance on its axial line.

Axial position **Equatorial position**



$$B_{\text{axial}} = \frac{\mu_0}{4\pi}\frac{2M}{r^3} = K\frac{2M}{r^3}$$



$$B_{\text{equ}} = \frac{\mu_0}{4\pi}\frac{M}{r^3} = K\frac{M}{r^3}$$

ILLUSTRATION 3.5

A bar magnet of length 0.1 m has a pole strength of 50 A m. Calculate the magnetic field at distance of 0.2 m from its center on **(a)** its axial line and **(b)** its equatorial line.

Sol. Here, $m = 50\,\text{Am}$; $r = 0.2\,\text{m}$; $2l = 0.1\,\text{m}$ or $l = 0.05\,\text{m}$

Therefore, $M = m(2l) = 50\times0.1 = 5\,\text{Am}^2$

(a) $$B_{\text{axial}} = \frac{\mu_0}{4\pi}\cdot\frac{2Mr}{(r^2-l^2)^2}$$

$$= \frac{10^{-7} \times 2 \times 5 \times 0.2}{(0.2^2 - 0.05^2)^2} = \frac{10^{-7} \times 2 \times 5 \times 0.2}{(0.0375)^2}$$

$$= 1.42 \times 10^{-4}\ \text{T}$$

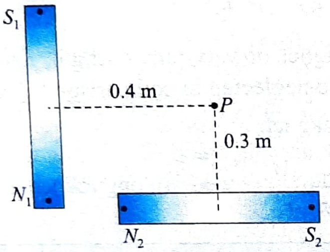
(b) $$B_{\text{equi}} = \frac{\mu_0}{4\pi} \cdot \frac{M}{(r^2 + l^2)^{3/2}}$$

$$= \frac{10^{-7} \times 5}{(0.2^2 + 0.05^2)^{3/2}} = \frac{10^{-7} \times 5}{(0.0425)^{3/2}} = \frac{5 \times 10^{-7}}{0.00876}$$

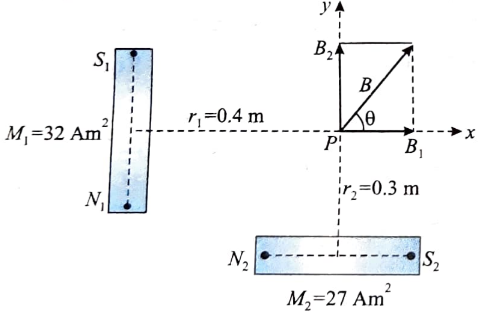
$$= 5.708 \times 10^{-5}\ \text{T}$$

ILLUSTRATION 3.6

Two short bar magnets N_1S_1 and N_2S_2 of magnetic moments 32 Am² and 27 Am² are placed on the table as shown in figure. Find the magnitude and direction of the magnetic fields produced by these magnets at point P located on equatorial lines of both the magnets at distances of 0.4 m and 0.3 m respectively from their centers.



Sol. As shown in figure, the point P lies on the equatorial line of both the magnets.



Let B_1 and B_2 be magnetic fields at point P due to the magnets N_1S_1 and N_2S_2 respectively. Then,

$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{M_1}{r_1^3} = \frac{10^{-7} \times 32}{(0.4)^3} = 0.5 \times 10^{-4}\ \text{T} \qquad \text{(along } PY\text{)}$$

and $$B_2 = \frac{\mu_0}{4\pi} \cdot \frac{M_2}{r_2^3} = \frac{10^{-7} \times 27}{(0.3)^3} = 10^{-4}\ \text{T} \qquad \text{(along } PX\text{)}$$

Therefore, resultant magnetic field at point P,

$$B = \sqrt{B_1^2 + B_2^2} = \sqrt{(0.5 \times 10^{-4})^2 + (10^{-4})^2} = 1.12 \times 10^{-4}\ \text{T}$$

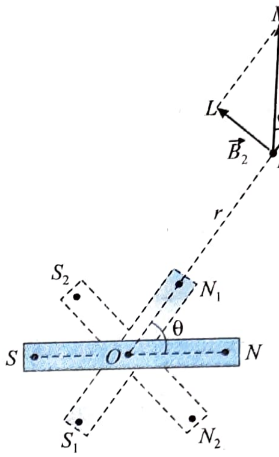
If the direction of B makes an angle θ with the X-axis (or N_1S_1), then

$$\tan\theta = \frac{B_2}{B_1} = \frac{10^{-4}}{0.5 \times 10^{-4}} = 2$$

$$\Rightarrow \quad \theta = \tan^{-1} 2$$

MAGNETIC FIELD AT ANY POINT DUE TO A SHORT MAGNETIC DIPOLE

In the given figure, a point P is at a distance r from a magnetic dipole NS of dipole moment M. The line joining point P to the center of dipole is making an angle θ with the axis of dipole.



The dipole moment $\vec{M}$ of the dipole (SN) can be resolved into two components:

(i) The component $M\cos\theta$ along OP and

(ii) The component $M\sin\theta$ along a direction perpendicular to OP.

Here we can consider the magnetic dipole NS to be equivalent to the combination of two magnetic dipoles i.e. one dipole N_IS_I having dipole moment $M\cos\theta$ and another magnetic dipole N_2S_2 (placed perpendicular to N_1S_1) having magnetic dipole moment $M\sin\theta$.

The point P lies on the axial line of dipole N_1S_1. Therefore, magnetic field intensity at point P due to the dipole N_1S_1,

$$|\vec{B}_1| = \frac{\mu_0}{4\pi} \cdot \frac{2M\cos\theta}{r^3} \text{ (along } PK\text{)}$$

Further, the point P lies on the equitorial line of dipole N_2S_2. Therefore, magnetic field intensity at point P due to the dipole N_2S_2,

$$|\vec{B}_2| = \frac{\mu_0}{4\pi} \cdot \frac{M\sin\theta}{r^3} \text{ (along } PL\text{, perpendicular to } PK\text{)}$$

The resultant magnetic field intensity at point P,

$$\vec{B} = \vec{B}_1 + \vec{B}_2$$

or $$B = \sqrt{|\vec{B}_1|^2 + |\vec{B}_2|^2}$$

$$= \sqrt{\left(\frac{\mu_0}{4\pi} \cdot \frac{2M\cos\theta}{r^3}\right)^2 + \left(\frac{\mu_0}{4\pi} \cdot \frac{M\sin\theta}{r^3}\right)^2}$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{M}{r^3} \sqrt{4\cos^2\theta + \sin^2\theta}$$

or $$B = \frac{\mu_0}{4\pi} \cdot \frac{M\sqrt{3\cos^2\theta + 1}}{r^3} \qquad \text{...(i)}$$

Equation (i) gives the magnitude of magnetic field intensity due to a short magnetic dipole at a distance r from its centre in a direction making an angle θ with the dipole.

To find the direction of magnetic field intensity due to the dipole, suppose that it makes an angle α with the direction of $\vec{B}_1$ i.e. with the line OK. Then, from the right-angled ΔPKM, we have

$$\tan\alpha = \frac{MK}{PK} = \frac{|\vec{B}_2|}{|\vec{B}_1|} = \frac{\mu_0}{4\pi}\cdot\frac{M\sin\theta}{r^3} \Big/ \frac{\mu_0}{4\pi}\cdot\frac{2M\cos\theta}{r^3}$$

or

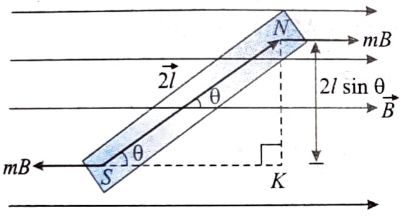
$$\tan\alpha = \frac{1}{2}\tan\theta \qquad \text{...(ii)}$$

Equation (ii) can be used to find the value of the angle α and hence the direction of magnetic field intensity due to the short magnetic dipole.

TORQUE ON A BAR MAGNET IN A MAGNETIC FIELD

The figure shows a magnetic dipole of dipole moment M placed in a uniform magnetic induction B. As both of its poles will experience equal and opposite forces as shown in figure, the net magnetic force on the dipole will be zero similar to an electric dipole placed in uniform electric field. Due to equal and opposite forces acting at different lines of action, a couple will be produced and the torque on magnetic dipole due to these couple forces can be given by its analogy with the case of electric dipole already studied in the chapter of electrostatics.

Consider a bar magnet having pole strength m and of length $2\vec{l}$ be placed in a uniform magnetic field of strength B making an angle θ with the direction of the magnetic field.



Force on N-pole of the magnet = mB (along the direction of magnetic field B)

Force on S-pole of the magnet = mB (opposite to the direction of magnetic field B)

As both of its poles will experience equal and opposite forces as shown in figure, the net magnetic force on the dipole will be zero similar to an electric dipole placed in uniform electric field.

But these two forces constitute a torque and it tends to rotate the magnet in the clockwise direction. The magnitude of the torque is given by

τ = Either force × perpendicular distance between the two forces

$= mB \times KN$

From the right angled triangle NKS, we have

$$KN = NS\sin\theta = 2l\sin\theta$$

$$\therefore \quad \tau = mB \times 2l\sin\theta = m(2l)B\sin\theta$$

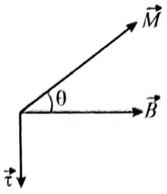
Since $m(2l) = M$, the magnetic dipole moment of the bar magnet, we have

$$\tau = MB\sin\theta \qquad \text{...(i)}$$

In vector notation,

$$\vec{\tau} = \vec{M}\times\vec{B} \qquad \text{...(ii)}$$

If the vectors $\vec{M}$ and $\vec{B}$ are in the plane of paper (figure), then torque vector $\vec{\tau}$, which acts in the direction of $\vec{M}\times\vec{B}$ will be perpendicular to the plane of the paper and in inward direction.



Unit of magnetic dipole moment: From Eq. (i), we have

$$M = \frac{\tau}{B\sin\theta}$$

Therefore, unit of $M = \frac{\text{newton meter}}{\text{tesla}} = \text{NmT}^{-1} = \text{JT}^{-1}$

Therefore, magnetic dipole moment may be measured as newton metre tesla^{-1} (NmT^{-1}) or as joule tesla^{-1} (JT^{-1}). It may be measured as ampere metre^2 (Am^2). Since tesla is weber metre^{-2}, the magnetic dipole moment is dimensionally equivalent to joule metre^2 weber^{-1} (J m^{2}Wb^{-1}).

ILLUSTRATION 3.7

A bar magnet has poles of strength 48 Am, which are 25 cm apart. **(a)** What is the magnetic moment of the magnet? **(b)** What torque is required to hold this magnet at an angle 30° with a uniform field of flux density 0.15 T?

Sol. Here, $m = 48$ Am; $2l = 25$ cm $= 0.25$ m

(a) Magnetic moment,

$$M = m\times 2l = 48\times 0.25 = 12\ \text{Am}^2$$

(b) Here, $\theta = 30°; B = 0.15$ T

Now, $\tau = MB\times\sin\theta = 12\times 0.15\times\sin 30° = 0.9$ N m

POTENTIAL ENERGY OF A BAR MAGNET PLACED IN A MAGNETIC FIELD

Let a bar magnet of dipole moment M be placed in a uniform magnetic field of strength B, such that the magnet makes an angle θ with the direction of the field.

Then, magnitude of the torque acting on the dipole is given by

$$\tau = MB\sin\theta$$

This torque tends to align the magnet along the direction of the field. If the magnet is rotated against the action of this torque, work has to be done. Suppose that the magnet is rotated through an infinitesimally small angle $d\theta$ under the action of the constant torque τ. Then, small amount of work done in rotating the magnet through an infinitesimally small angle $d\theta$,

$$dW = \tau\, d\theta = MB\sin\theta\, d\theta$$

If the magnet is rotated from initial position $\theta = \theta_1$ to the final position $\theta = \theta_2$, then the total work done is given by

$$W = \int_{\theta_1}^{\theta_2} MB\sin\theta\, d\theta = MB\int_{\theta_1}^{\theta_2}\sin\theta\, d\theta = MB\left|-\cos\theta\right|_{\theta_1}^{\theta_2}$$

$$= -MB(\cos\theta_2 - \cos\theta_1) = MB(\cos\theta_1 - \cos\theta_2)$$

The work done in rotating the magnet is stored inside the magnet as its potential energy (U). Thus, potential energy of the magnet inside the magnetic field,

$$U = MB(\cos\theta_1 - \cos\theta_2) \qquad \text{...(i)}$$

Suppose that the magnet is initially perpendicular to the direction of the magnetic field i.e. $\theta_1 = 90°$. Then, potential energy of the magnet in any position making angle θ with the direction of the field can be obtained by setting $\theta_1 = 90°$ and $\theta_2 = \theta$ in Eq. (i) i.e.

$$U = MB(\cos 90° - \cos\theta) \qquad \text{...(ii)}$$

or $$U = -MB\cos\theta$$

In vector notation, $U = -\vec{M}\cdot\vec{B}$ ...(iii)

Special cases: Let us find the potential energy of the dipole in following cases:

(i) When $\theta = 0°$, $U = -MB\cos 0° = -MB$ (Minimum)

(ii) When $\theta = 90°$, $U = -MB\cos 90° = 0$

(iii) When $\theta = 180°$, $U = -MB\cos 180° = MB$ (Maximum)

Thus, a magnetic dipole possesses minimum potential energy when $\vec{M}$ and $\vec{B}$ are like parallel and maximum potential energy when $\vec{M}$ and $\vec{B}$ are antiparallel.

ILLUSTRATION 3.8

Calculate the work done in rotating a bar magnet of magnetic moment 3 J T^{-1} through an angle of 60° from its position along a magnetic field of strength 0.34×10^{-4}T.

Sol. Here, $M = 3\text{ J T}^{-1}; B = 0.34\times10^{-4}\text{ T}; \theta_1 = 0°; \theta_2 = 60°$

Now, work done to rotate the magnet from the position θ_1 to θ_2 is given by

$$W = MB(\cos\theta_1 - \cos\theta_2)$$

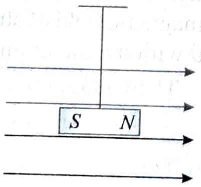
$$\therefore \quad W = 3\times0.34\times10^{-4}(\cos 0° - \cos 60°)$$

$$= 3\times0.34\times10^{-4}(1-0.5)$$

$$= 5.1\times10^{-5}\text{ J}$$

ILLUSTRATION 3.9

A magnetic dipole of magnetic moment M is suspended by a string in a uniform horizontal magnetic field as shown in figure. If in horizontal plane this dipole is slightly tilted and released, show that it will execute simple harmonic motion and find its oscillation period. Consider the dipole as a uniform rod of mass m and length l.



Sol. If dipole is tilted from its equilibrium position by a small angle θ, in horizontal plane, restoring torque on magnetic dipoles is given as

$$\tau_R = MB\sin\theta \simeq MB\theta$$

The angular acceleration of dipole is given as

$$\alpha = -\frac{\tau_A}{I}$$

$$\Rightarrow \quad \alpha = -\frac{MB}{(ml^2/12)}\theta$$

$$\Rightarrow \quad \alpha = -\frac{12MB}{ml^2}\theta \qquad \text{...(i)}$$

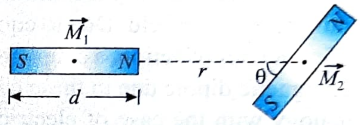
Equation (i) shows that restoring angular acceleration of the dipole is directly proportional to the angular displacement hence it will execute SHM. Comparing this with angular acceleration of standard angular SHM equation given as $\alpha = -\omega^2\theta$, we get

$$\omega = \sqrt{\frac{12MB}{ml^2}}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{ml^2}{12MB}}$$

ILLUSTRATION 3.10

Figure shows two small bar magnets having dipole moments M_1 and M_2 placed at separation r. Find the magnetic interaction energy of this system of dipoles for $d << r$.



Sol. Magnetic induction at the location of M_2 due to M_1 is given as

$$B_1 = \frac{\mu_0 M_1}{2\pi r^3}$$

Interaction energy of M_2 in the field of M_1 is given as

$$U = -\vec{M}_2\cdot\vec{B}_1$$

$$\Rightarrow \quad U = -M_2B_1\cos\theta = -\frac{\mu_0 M_1M_2}{2\pi r^3}\cos\theta$$

ILLUSTRATION 3.11

A magnetic dipole is free to rotate in a uniform magnetic field. For what orientation of the magnet w.r.t. the magnetic field **(a)** torque is maximum, **(b)** potential energy is maximum and **(c)** rate of change of torque with angle of orientation is maximum?

Sol.

(a) We know, $\tau = MB\sin\theta$

Therefore, torque on the dipole will be maximum, when $\sin\theta$ is maximum i.e. equal to 1 or $\theta = 90°$.

(b) Also $U = -MB\cos\theta$

The maximum value of the potential energy of the dipole is equal to MB. Therefore, potential energy of the dipole will be maximum, when cos θ is equal to -1. i.e. $\theta = 180°$.

Now, rate of change of torque with angle of orientation,

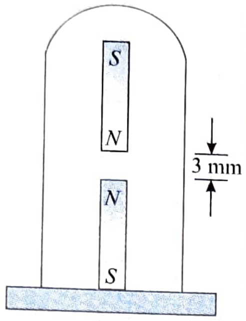
$$\frac{d\tau}{d\theta} = \frac{d}{d\theta}(MB\sin\theta) = MB\cos\theta$$

Therefore, $\frac{d\tau}{d\theta}$ will be maximum, when cos θ is maximum, i.e., equal to 1 or $\theta = 0°$.

CONCEPT APPLICATION EXERCISE 3.1

1. Two magnetic poles, one of which is four times stronger than the other, exert a force of 5 gf on each other, when placed at distance of 10 cm in air. Find the strength of each pole.
2. Two like magnetic poles of strength 25 Am and 64 Am are situated 1.0 m apart in air. At what point on the line, joining the two poles the magnetic field will be zero?
3. Two identical magnets with a length 10 cm and weight 50 gf each are arranged freely with their like poles facing in a vertical glass tube as shown in figure.

   The upper magnet hangs in the air above the lower one so that the distance between the nearest poles of the magnets is 3 mm. Determine the pole strength of the poles of these magnets.



4. The electron in the hydrogen atom is moving with a speed of 2.3×10^6 m s^{-1} in an orbit of radius 0.53 A. Calculate the magnetic moment of the revolving electron.
5. Two thin bar magnets of pole strengths 4 Am and 7 Am and lengths 0.21 m and 0.12 m respectively are placed at right angles to each other with the N-pole of the first touching the S-pole of the second. Find the magnetic moment of the system.
6. Two short bar magnets of magnetic moments 96 Am^2 and 81 Am^2 are placed along mutually perpendicular straight lines meeting at a point P. Find the magnitude and direction of magnetic field at point P, if it lies at distances of 0.4 m and 0.3 m respectively from the centres of the two magnets.
7. A bar magnet with poles 25 cm apart and pole strength 14.4 Am rests with its centre on a frictionless pivot. It is held in equilibrium at 60° to a uniform magnetic field of induction 0.25 T by applying a force F at right angle to its axis, 10 cm from its pivot. Calculate F. What will happen if the force is removed?
8. A small coil of radius 0.002 m is placed on the axis of a magnet of magnetic moment 10^5 JT^{-1} and length 0.1 m at a distance of 0.15 m from the centre of the magnet. The plane of the coil is perpendicular to the axis of the magnet. Find the force on the coil, when the current of 2.0 A is passed through it.

ANSWERS

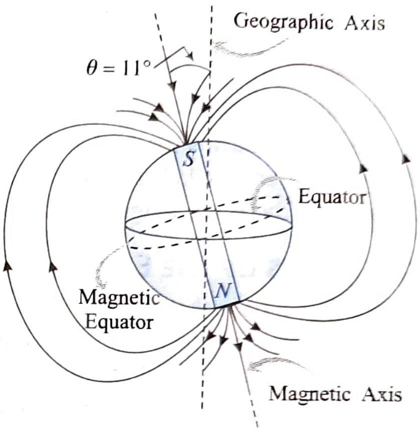
1. 35 Am, 140 Am
2. Either at 0.385 m from pole of 25 Am (towards 64 Am) or at 1.667 m from pole of 25 Am (on the other side of 64 Am)
3. 6.64 Am **4.** 9.75×10^{-24} Am^2 **5.** 1.19 Am^2
6. $3\sqrt{5}\times10^{-4}$ T (making an angle of $\tan^{-1}(2)$ with N_1S_1]
7. 6.24 N, the bar magnet will tend to align itself along the field
8. $F = 4.4\times10^{-3}$ N

EARTH'S MAGNETISM

The branch of physics that deals with the study of Earth's magnetic field is called terrestrial magnetism or geomagnetism.

Whenever a bar magnet is suspended freely (near the equator), its N-pole points towards the geographical north and S-pole towards the geographical south of the earth.

Earth's magnetic field is also called 'geomagnetic field'. The cause of Earth's magnetic field is the rotation of ionized particles in molten core of earth which constitutes the convection currents at the Earth's core and behave like a coil which produces magnetic field in surrounding and it extends from inner core to outer space. The configuration of Earth's magnetic field is shown in figure.



The axis of rotation of earth is called geographic axis and the points, where it passes through the surface of earth, are called geographic poles. The points N and S represent the geographic north and south poles. The great circle on the earth's surface perpendicular to the geographic axis is called equator.

A vertical plane passing through the geographic axis is called the geographic meridian.

The axis of the huge magnet assumed to be lying inside the earth is called magnetic axis of the earth and the points where the magnetic axis cuts the earth's surface are called earth's magnetic poles. The points N' and S' indicate the magnetic north and south of the earth. The great circle on the earth's surface perpendicular to the magnetic axis is called magnetic equator.

A vertical plane passing through the magnetic axis of the earth is called magnetic meridian.

The axis of Earth's magnetic field is slightly tilted from its axis of rotation (geographic axis) at about 11°. This angle changes with time very slowly due to some geophysical factors not in scope of this book. This axis along which Earth's magnetic coil is considered is called 'magnetic axis' of Earth.

The points on Earth surface where this magnetic axis meet are called magnetic poles. Due to the direction of convection current inside the core of Earth its magnetic induction comes out from the geographic south pole and it gets into the Earth at geographic north pole. Thus the magnetic poles of Earth are located near to the opposite geographic poles as shown in figure.

Cause of earth's magnetism: It may be pointed out that the source of earth's magnetism is not a large solid mass of magnetic material located inside the earth. It is guessed that earth's magnetism is due to molten charged metallic fluid. It gives rise to a current flowing inside the core of the earth. This core has a radius of about 3500 km. Up to a distance of about 3×10^4 km (≈ five earth's radii), the magnetic field is entirely due to earth's magnetism. At larger distances, the motion of the ions influence the earth's magnetic field. The motion of the ions is affected by a thin hot gas expelled by the sun, called solar wind.

Prof. Blackett studied earth's magnetism in detail and suggested that since every substance is made up of charged particles (protons and electrons). Hence, a substance rotating about an axis will become magnetized. The earth's magnetism is also due to the rotation of the earth about its own axis.

There is another theory about the earth's magnetism. The gases in the atmosphere are in the ionized states. The high energy radiation coming from the sun ionizes the atoms in the upper part of the atmosphere. The radioactivity of the atmosphere and cosmic rays also ionize the gases in the atmosphere. Hence, due to rotation of the earth, strong electric currents flow. Due to action of these currents, the earth is magnetized.

The third view regarding earth's magnetism is that the core of the earth is made of iron and nickel. Due to rotation, the earth behaves as a dynamo and becomes magnetized.

MAGNETIC ELEMENTS OF THE EARTH

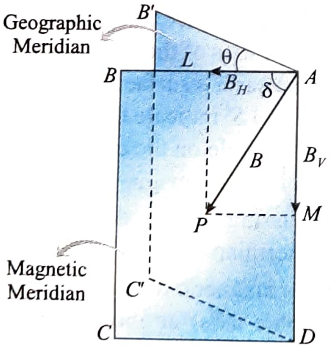
Those quantities which describe the magnetic field of the Earth at a particular place completely in magnitude as well as in direction are known as the magnetic elements at that place.

These are: (a) magnetic declination, (b) dip and (c) horizontal component of the Earth's magnetic field.

Magnetic Declination

Declination at a place is the angle between the geographic meridian and the magnetic meridian. It is denoted by θ.

If $ABCD$ and $AB'C'D$ represent magnetic and geographical meridians, then $\angle BAB' = \theta$ represents the magnetic declination.



Magnetic Inclination or Dip

Dip at a place is defined as the angle made by the direction of the earth's total magnetic field with the horizontal direction. It is denoted by δ.

Suppose that a small bar magnet is suspended freely at point A and it points in direction AP. Then, AP is the direction of the intensity of earth's total magnetic field (B). The line AP lies in magnetic meridian. Consider that AB is a horizontal line in the magnetic meridian. Then, $\angle BAP = \delta$ represents the dip at point A.

Horizontal Component (B_H)

The total intensity of the Earth's magnetic field at any point can be resolved into two rectangular components, one along the horizontal and the other along the vertical direction.

The component of the resultant intensity of the Earth's magnetic field in the horizontal direction is called its horizontal component and is denoted by B_H.

The component of the resultant intensity of the Earth's magnetic field in the vertical direction is called its vertical component and is denoted by B_V.

In figure, the resultant intensity, i.e., B along AP has been resolved into two rectangular components. Clearly, the horizontal component along AB, i.e.,

$$B_H = B\cos\delta \qquad \text{...(i)}$$

and the vertical component along AD, i.e.,

$$B_V = B\sin\delta \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), $\dfrac{B_V}{B_H} = \dfrac{B\sin\delta}{B\cos\delta} = \tan\delta$

and $B_H^2 + B_V^2 = B^2\cos^2\delta + B^2\sin^2\delta$

$$= B^2(\cos^2\delta + \sin^2\delta) = B^2 \qquad \text{...(iii)}$$

or $B = \sqrt{B_H^2 + B_V^2}$ ...(iv)

Important Points:

- At a place on equator, $B_H = B$ and as such $B\cos\delta = B$ or $\cos\delta = 1$ or $\delta = 0°$.
- Thus, at the equator; $\delta = 0°$, $B_H = B$ and $B_V = 0$.
- At the poles, $B_V = B$ and as such $B\sin\delta = B$ or $\sin\delta = 1$ or $\delta = 90°$. Thus, at the poles; $\delta = 90°$, $B_H = 0$, $B_V = B$.
- The lines drawn through different places having same declination are called **isogonic lines**. A line which passes through places having zero declination is called **agonic line**.
- The lines which pass through different places having same dip are called **isoclinic lines**. A line which passes through places having zero dip is called **aclinic line**.
- The lines drawn through places having the same value of B_H are called **isodynamic lines**.

ILLUSTRATION 3.12

The vertical component of earth's magnetic field at a place is $\sqrt{3}$ times the horizontal component. What is the value of angle of dip at this place?

Sol. Here, $B_V = \sqrt{3}\,B_H$

Now, $\tan\delta = \dfrac{B_V}{B_H} = \dfrac{\sqrt{3}\,B_H}{B_H} = \sqrt{3}$

or $\delta = 60°$

ILLUSTRATION 3.13

A compass needle whose magnetic moment is 60 Am² is pointing geographical north at a certain place whereas the horizontal component of the Earth's magnetic field is 40 μWb/m² and experiences a torque of 1.2 × 10^{-3} Nm. What is the declination of the place?

Sol. Here $m = 60\ \text{Am}^2$,

$$B_H = 40\ \mu\text{Wb/m}^2$$

$$= 40\times10^{-6}\ \text{Wb/m}^2, \tau = 1.2\times10^{-3}\ \text{N m}$$

As $\tau = mB\sin\theta$,

$$\sin\theta = \frac{\tau}{mB_H} = \frac{1.2\times10^{-3}\,\text{N m}}{(60\text{Am}^2)(40\times10^{-6}\ \text{Wb/m}^2)} = 0.5$$

Thus, $\theta = 30°$

Here θ = 30°, being the angle between geographic north and magnetic north, is the declination of the place as in stable equilibrium a compass needle points along magnetic north.

ILLUSTRATION 3.14

The horizontal and vertical components of Earth's field at a place are 0.22 G and 0.38 G respectively. Calculate the angle of dip and resultant intensity of Earth's field.

Sol. Here $B_H = 0.22$ G, $B_V = 0.38$ G.

As $$\tan\delta = \frac{B_V}{B_H} = \frac{0.38}{0.22} = 1.73,\ \delta = 60°$$

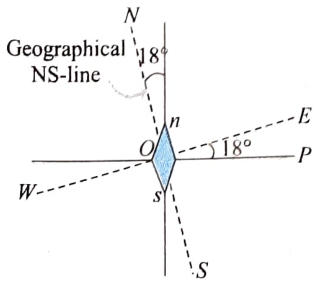
Resultant intensity,

$$B = \sqrt{B_H^2 + B_V^2} = \sqrt{(0.22)^2 + (0.38)^2} = 0.44\ \text{G}$$

ILLUSTRATION 3.15

A ship is sailing due east according to Mariner's compass. If the declination of that place is 18° east of north, what is the true direction of the ship?

Sol. The compass needle NS points along the magnetic NS-line and the ship is sailing along *OP* i.e. towards east (as indicated by compass needle). As the declination of that place is 18° east of north, the geographical NS line is as shown by dotted NS line (figure).



Thus, the true direction of ship is 18° south of east.

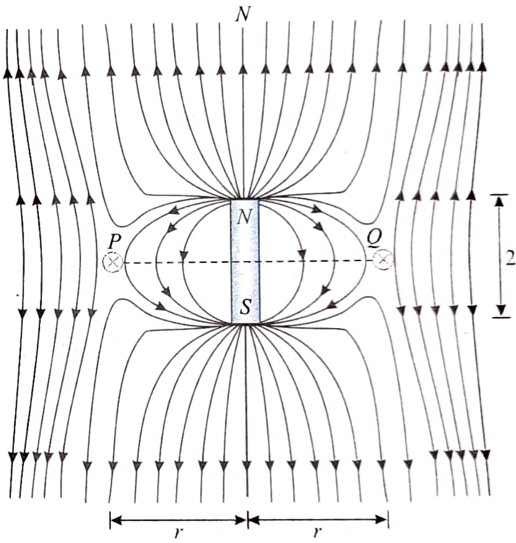
NEUTRAL POINT AND TO FIND MAGNETIC MOMENT OF A BAR

Neutral point: A neutral point in the magnetic field of a bar magnet is that point where the field due to the magnet is completely neutralized by the horizontal component of earth's magnetic field. Thus, at the neutral point, the magnetic field due to the bar magnet (B) is just equal and opposite to the horizontal component of earth's magnetic field (B_H) i.e.

$$B = B_H \qquad \text{...(i)}$$

To find magnetic moment of a bar magnet: The magnetic moment of a bar magnet can be found by plotting magnetic field of the magnet, locating the neutral points and then using Eq. (i).

(a) When magnet is placed with its north pole towards north of the earth: If we plot the magnetic field due to a small bar magnet by placing its north pole towards north of the earth, then plot of the magnetic field of the magnet (which is superposed by earth's magnetic field) will be as shown in figure.



The horizontal component of earth's magnetic field and the magnetic field due to the bar magnet cancel out at the two cross-marks on the equatorial line of the magnet (the normal bisector of the magnet). These points are called neutral points. Let r be distance of a neutral point from the center of the magnet. As the neutral point lies on the equatorial line of the magnet, the magnetic field due to the magnet at the neutral point is given by

$$B_{\text{equi}} = \frac{\mu_0}{4\pi}\cdot\frac{M}{r^3}$$

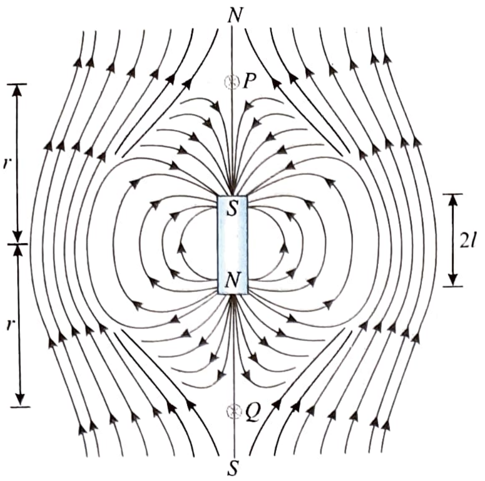
where M is magnetic dipole moment of the magnet.

Since at the neutral point, magnetic field due to the magnet is equal to B_H, we have

$$\frac{\mu_0}{4\pi}\cdot\frac{M}{r^3} = B_H \qquad \text{...(ii)}$$

Knowing r and B_H, the magnetic dipole moment of the bar magnet can be calculated from Eq. (ii).

(b) When magnet is placed with its south pole towards north of the earth: The plot of the magnetic field of a bar magnet in this case will be as shown in figure.



In this case, the neutral points are found to be on axial line of the magnet. If neutral point is at a distance r from the centre of the magnet, then magnetic field due to the magnet at the neutral point (point on the axial line of the magnet) is given by

$$B_{\text{axial}} = \frac{\mu_0}{4\pi} \cdot \frac{2M}{r^3}$$

Since at the neutral point, the magnetic field due to the magnet is equal to B_H, we have

$$\frac{\mu_0}{4\pi} \cdot \frac{2M}{r^3} = B_H \qquad \text{...(iii)}$$

Knowing r and B_H, Eq. (iii) can be used to find the magnetic dipole moment of the bar magnet.

Note. From the plot of magnetic field in the two cases i.e. when magnet is placed with its N-pole towards north of earth and when magnet is placed with its S-pole towards north of the earth, it follows that when magnet is turned through 180°, the neutral points turn through 90°.

ILLUSTRATION 3.16

How many neutral points on a horizontal board are there and why, when a magnet is held vertically on the board?

Sol. Let us consider a magnet NS placed vertically on a horizontal board as shown in figure. The neutral point on the horizontal board will be obtained, where the component of the magnetic field (B) due to the magnet in the plane of the board is neutralized, by the horizontal component of the earth's magnetic field (B_H). The directions of magnetic fields B and B_H have been shown at four points P_1, P_2, P_3 and P_4. It follows that the two fields can balance each other only at the point P_2. Therefore, when a magnet is placed vertically on a the horizontal board, only one neutral point will be obtained.

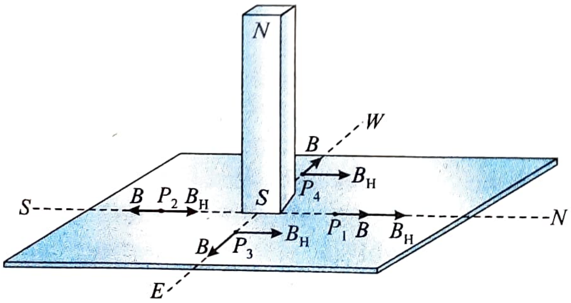


ILLUSTRATION 3.17

A bar magnet 30 cm long is placed in the magnetic meridian with its north pole pointing south. The neutral point is observed at a distance of 30 cm from its one end. Calculate the pole strength of the magnet. Given, horizontal component of earth's field = 0.34 G.

Sol. Here, $2l = 30$ cm or $l = 15$ cm $= 0.15$ m;

$r = 30$ cm $= 0.30$ m,

$B_H = 0.34\,G = 0.34 \times 10^{-4}$ T

When the magnet is placed with its north pole pointing south, neutral point is obtained on its axial line. Therefore, at the neutral point,

$$B_{\text{axial}} = B_H$$

or
$$\frac{\mu_0}{4\pi} \times \frac{2Mr}{(r^2 - l^2)^2} = B_H$$

or
$$M = \frac{4\pi}{\mu_0} \times \frac{B_H (r^2 - l^2)^2}{2r}$$

$$= \frac{1}{10^{-7}} \times \frac{0.34 \times 10^{-4} \times (0.30^2 - 0.15^2)^2}{2 \times 0.30}$$

$$= \frac{0.34 \times 10^{-4} \times (0.0675)^2}{10^{-7} \times 2 \times 0.30} = 2.582 \text{ Am}^2$$

The pole strength of the magnet,

$$m = \frac{M}{2l} = \frac{2.582}{0.30} = 8.607 \text{ Am}$$

ILLUSTRATION 3.18

A bar magnet is 10 cm long. Its magnetic moment is 0.4 Am². It is placed in magnetic meridian with its north pole towards north. If the neutral point is obtained at a distance of 10 cm from the center of the magnet, find the horizontal component of earth's magnetic field.

Sol. Here, $M = 0.4 \text{ Am}^2$; $r = 10$ cm $= 0.1$ m;

$2l = 10$ cm or $l = 5$ cm $= 0.05$ m

When the magnet is placed with its north pole towards north of earth, the neutral point is obtained on the equatorial line. If B_H is horizontal component of the earth's field, then at the neutral point

$$B_{\text{equi}} = B_H$$

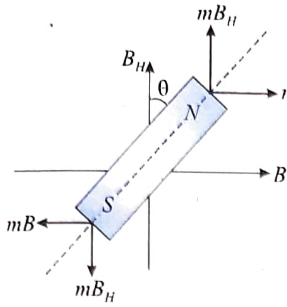
or
$$B_H = \frac{\mu_0}{4\pi} \times \frac{M}{(r^2 \times l^2)^{3/2}}$$

$$= \frac{10^{-7} \times 0.4}{(0.1^2 + 0.05^2)^{3/2}} = \frac{10^{-7} \times 0.4}{(0.0125)^{3/2}} = \frac{10^{-7} \times 0.4}{0.0014}$$

$$= 0.286 \times 10^{-4} \text{ T}$$

TANGENT LAW

When a small magnet is suspended in two uniform magnetic fields B and B_H which are at right angles to each other, the magnet comes to rest at an angle θ with respect to B_H.



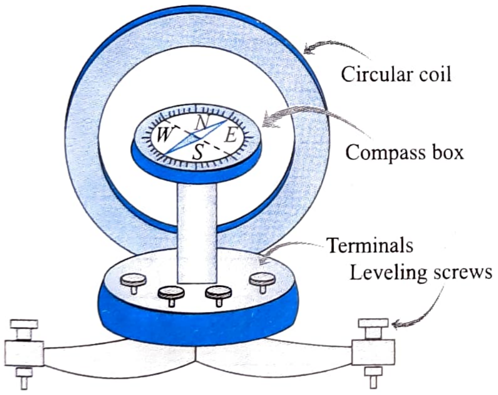
In equilibrium

$$MB_H \sin\theta = MB\sin(90^\circ - \theta)$$

$\Rightarrow$ $B = B_H \tan\theta$. This is called tangent law.

TANGENT GALVANOMETER

Tangent galvanometer is a device used to find horizontal component of Earth's magnetic induction. It consists of a circular coil (*C*) of insulated copper wire wound on a vertical circular frame made of nonmagnetic material as ebonite or wood. A small magnetic compass needle is pivoted at the center of the vertical circular frame. When the coil of the tangent galvanometer is kept in magnetic meridian and current passes through any of the coil then the needle at the center gets deflected and comes to an equilibrium position under the action of two perpendicular field: one due to horizontal component of earth and the other due to field ($\vec{B}$) set up by the coil due to current.



In equilibrium $B = B_H \tan\theta$

where $B = \dfrac{\mu_0 ni}{2r}$; n = number of turns, r = radius of coil, i = the current to be measured, θ = angle made by needle from the direction of B_H in equilibrium.

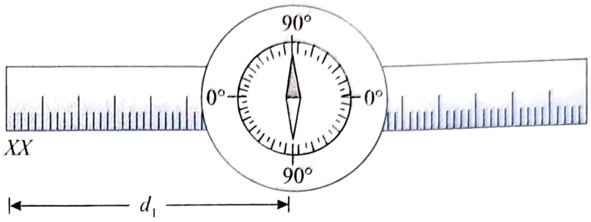
Hence $\dfrac{\mu_0 Ni}{2r} = B_H \tan\theta$

$\Rightarrow$ $i = k\tan\theta$ where $k = \dfrac{2rB_H}{\mu_0 N}$ is called reduction factor.

DEFLECTION MAGNETOMETER

Deflection magnetometer is an experimental setup used to measure the magnetic moment and pole strength of a bar magnet. It is also used to compare the pole strengths of two bar magnets. It's working is based on the principle of tangent law. It consists of a small compass needle, pivoted at the centre of a circular box. The box is kept in a wooden frame having two meter scale fitted on its two arms. Reading of a scale at any point directly gives the distance of that point from the centre of compass needle.

It consists of a small magnetic compass needle pivoted at the center of a scale as shown in figure. The compass is fixed at the scale in such a way that its circular scale can be rotated which is marked with 0°–90°–0°–90° in perpendicular direction, which is kept exactly perpendicular to the scale, as shown in figure.

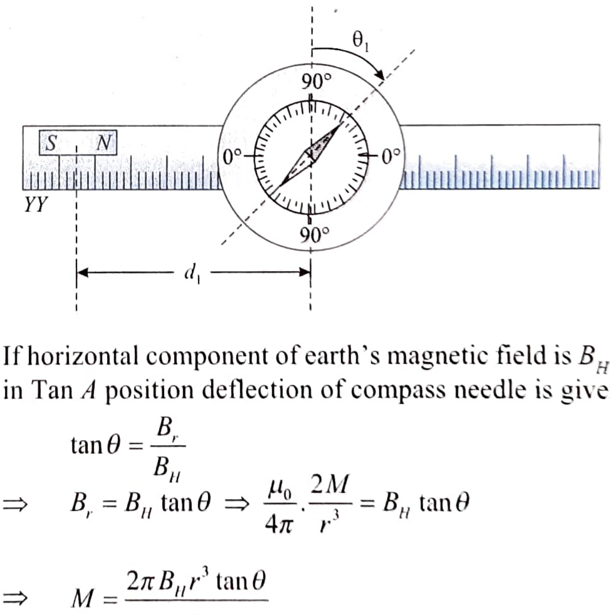


For measuring the magnetic moment of a bar magnet or a magnetic dipole, there are two positions of the magnetometer called 'Tan *A*' and 'Tan *B*' positions.

(1) Tan *A* position: In this position the magnetometer is set perpendicular to magnetic meridian. So that, magnetic field due to magnet, is in axial position and perpendicular to earth's field.

The magnetic dipole of which magnetic moment is to be measured is also placed along the scale line as shown at some distance *r* from the compass needle. The magnetic induction due to the magnetic dipole at the location of compass is given as

$$B_r = \frac{\mu_0}{4\pi}\cdot\frac{2M}{r^3}$$



If horizontal component of earth's magnetic field is B_H then in Tan *A* position deflection of compass needle is given as

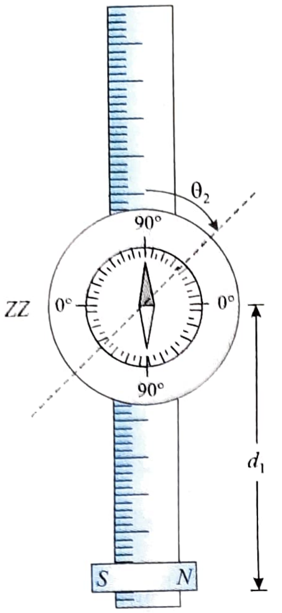
$$\tan\theta = \frac{B_r}{B_H} \qquad \text{...(i)}$$

$$\Rightarrow \quad B_r = B_H \tan\theta \Rightarrow \frac{\mu_0}{4\pi}\cdot\frac{2M}{r^3} = B_H \tan\theta$$

$$\Rightarrow \quad M = \frac{2\pi B_H r^3 \tan\theta}{\mu_0} \qquad \text{...(ii)}$$

(2) Tan *B* position: The arms of magnetometer are set in magnetic meridian, so that the magnetic field due to magnet is at it's equatorial position. At the same distance from the compass the magnetic dipole is now placed along east-west line perpendicular to the scale as shown in figure. At the location of compass needle the magnetic induction due to the magnetic dipole is given as

$$B_r = \frac{\mu_0}{4\pi}\cdot\frac{M}{r^3}$$



Now again the compass needle deflects under the influence of two mutually perpendicular magnetic fields. So its deflection is given as

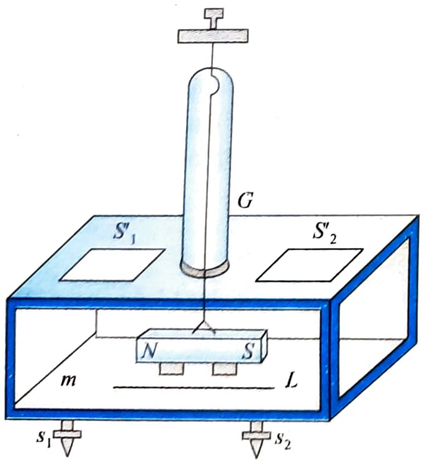
$$\tan\theta = \frac{B_r}{B_H} \quad \text{...(iii)}$$

$$\Rightarrow \quad B_r = B_H \tan\theta \Rightarrow \frac{\mu_0}{4\pi}\cdot\frac{M}{r^3} = B_H \tan\theta$$

$$\Rightarrow \quad M = \frac{4\pi B_H r^3 \tan\theta}{\mu_0} \quad \text{...(iv)}$$

VIBRATION MAGNETOMETER

Vibration magnetometer is used for comparison of magnetic moments and magnetic fields. This device works on the principle that whenever a freely suspended magnet in a uniform magnetic field is disturbed from it's equilibrium position, it starts vibrating about the mean position.



Time period of oscillation of experimental bar magnet (magnetic moment M) in earth's magnetic field (B_H) is given by the formula. $T = 2\pi\sqrt{\dfrac{I}{MB_H}}$; where, I = moment of inertia of short bar magnet $= \dfrac{wL^2}{12}$ (w = mass of bar magnet)

Determination of magnetic moment of a magnet: The experimental (given) magnet is put into vibration magnetometer and its time period T is determined. Now

$$T = 2\pi\sqrt{\frac{I}{MB_H}} \Rightarrow M = \frac{4\pi^2 I}{B_H . T^2}$$

Comparison of horizontal components of earth's magnetic field at two places

$$T = 2\pi\sqrt{\frac{I}{MB_H}}$$; since I and M of the magnet are constant,

So $$T^2 \propto \frac{1}{B_H} \Rightarrow \frac{(B_H)_1}{(B_H)_2} = \frac{T_2^2}{T_1^2}$$

Comparison of magnetic moment of two magnets of same size and mass

$$T = 2\pi\sqrt{\frac{I}{M.B_H}}$$; Here I and B_H are constants.

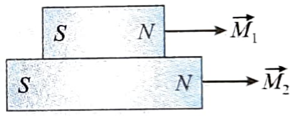
So $$M \propto \frac{1}{T^2} \Rightarrow \frac{M_1}{M_2} = \frac{T_2^2}{T_1^2}$$

Comparison of magnetic moments by sum and difference method

Sum position

Net magnetic moment $M_s = M_1 + M_2$

Net moment of inertia $I_s = I_1 + I_2$



Time period of oscillation of this pair in earth's magnetic field (B_H)

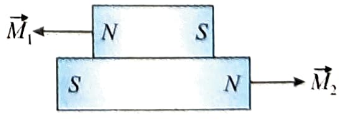
$$T_s = 2\pi\sqrt{\frac{I_s}{M_s B_H}} = 2\pi\sqrt{\frac{I_1 + I_2}{(M_1 + M_2)B_H}} \quad \text{...(i)}$$

Frequency $$f_s = \frac{1}{2\pi}\sqrt{\frac{(M_1 + M_2)B_H}{I_s}}$$

Difference position

Net magnetic moment, $M_d = (M_1 - M_2)$

Net moment of inertia $I_d = I_1 + I_2$



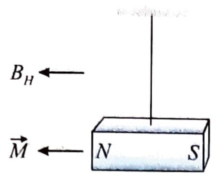
and $$T_d = 2\pi\sqrt{\frac{I_d}{M_d B_H}} = 2\pi\sqrt{\frac{I_1 + I_2}{(M_1 - M_2)B_H}} \quad \text{...(ii)}$$

and $$f_d = \frac{1}{2\pi}\sqrt{\frac{(M_1 + M_2)B_H}{(I_1 + I_2)}}$$

From Eqs. (i) and (ii), we get

$$\frac{T_s}{T_d} = \sqrt{\frac{M_1 - M_2}{M_1 + M_2}} \Rightarrow \frac{M_1}{M_2} = \frac{T_d^2 + T_s^2}{T_d^2 - T_s^2} = \frac{f_s^2 + f_d^2}{f_s^2 - f_d^2}$$

To find the ratio of magnetic field: Suppose it is required to find the ratio B/B_H where B is the field created by magnet and B_H is the horizontal component of earth's magnetic field.

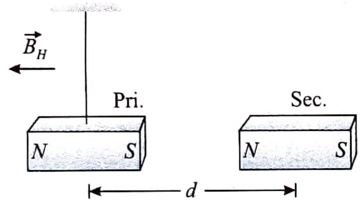


To determine B/B_H a primary (main) magnet is made to first oscillate in earth's magnetic field (B_H) alone and its time period of oscillation (T) is noted.

$$T = 2\pi\sqrt{\frac{I}{MB_H}}$$

and frequency, $f = \dfrac{1}{2\pi}\sqrt{\dfrac{MB_H}{I}}$

Now a secondary magnet is placed near the primary magnet. So primary magnet oscillate in a new field which is the resultant of B and B_H and now time period is noted again.



$$T' = 2\pi\sqrt{\frac{I}{M(B+B_H)}}$$

or $$f' = \frac{1}{2\pi}\sqrt{\frac{M(B+B_H)}{I}}$$

$$\Rightarrow \quad \frac{B}{B_H} = \left(\frac{f'}{f}\right)^2 - 1$$

ILLUSTRATION 3.19

A vibration magnetometer consists of two identical bar magnets placed one over the other, such that they are mutually perpendicular and bisect each other. The time period of oscillation in a horizontal magnetic field is 4 s. If one of the magnets is taken away, find the period of the other in the same field.

Sol. For combination of the two magnets: let M' be the net magnetic moment of the combination of the two magnets and I', the moment of inertia of the system. If T' is period of vibration, then

$$T' = 2\pi\sqrt{\frac{I'}{M'B}}$$

When two magnets, each of magnetic moment M and moment of inertia I are placed perpendicular to each other, the moment of inertia of the combination becomes

$$I' = 2I$$

and the magnetic moment of the combination is given by

$$M' = \sqrt{M^2 + M^2 + 2M\,M\cos 90^\circ} = \sqrt{2}\,M$$

Since $T' = 4$ s, we have

$$\therefore \quad 4 = 2\pi\sqrt{\frac{2I}{\sqrt{2}MB}} \qquad \text{...(i)}$$

When one magnet is taken away:

$$T = 2\pi\sqrt{\frac{I}{MB}} \qquad \text{...(ii)}$$

Dividing Eq. (ii) by (i), we have

$$\frac{T}{4} = 2\pi\sqrt{\frac{I}{M\,B}} \Big/ 2\pi\sqrt{\frac{2I}{\sqrt{2}\,M\,B}}$$

or $T = 4\times(2)^{1/4} = 4\times 1.189 = 4.76$ s

CONCEPT APPLICATION EXERCISE 3.2

1. The horizontal component of earth's magnetic field at a place is $\sqrt{3}$ times the vertical component. What is the value of angle of dip at this place?
2. The horizontal component of earth's magnetic field at a place is 0.3×10^{-4} T. If the angle of dip is 60°, what is the total earth's magnetic field?
3. At 52° from the magnetic meridian, a magnetic needle in a vertical plane makes an angle of 45° with the horizontal plane. Find the actual angle of dip at that place. [$\cos 52^\circ = 0.6157$]
4. A small bar magnet has a magnetic moment 5 Am². The neutral point is obtained on axial line, when it is placed in magnetic meridian with its north pole pointing south of earth and neutral point is obtained on equitorial line, when it is placed with its north pole pointing north of earth. If horizontal component of earth's field is 0.38 G, find the position of neutral points in two cases.
5. A short bar magnet is placed in a horizontal plane with its axis in the magnetic meridian. Null points are found on its equitorial line (i.e. its normal bisector) at 12.5 cm from the centre of the magnet. The earth's magnetic field at the place is 0.38 G and the angle of dip is zero.

   (a) What is the total magnetic field at points on the axis of the magnet located at the same distance (12.5 cm) as the null-points from the centre ?

   (b) Locate the null-points, when the bar is turned around by 180°.

   Assume that the length of the magnet is negligible as compared to the distance of the null-points from the centre of the magnet.
6. A bar magnet of length 8 cm and having a pole strength of 1 Am is placed vertically on a horizonal table with its N-pole on the table. A neutral point is found on the table at a distance of 6 cm south of the magnet. Calculate the earth's horizontal magnetic field.

ANSWERS

1. 30° **2.** 0.6×10^{-4} T **3.** $\tan^{-1} 0.6157$
4. 29.7 cm, 23.6 cm
5. (a) 144 G; (b) 15.7 cm **6.** 2.18×10^{-5} T

MAGNETIC PROPERTIES OF MATERIALS

The magnetic properties of a material are described in terms of following physical parameters.These properties are used to classify the magnetic substances and to understand their response to the magnetic fields:

INTENSITY OF MAGNETIZATION

It is defined as the magnetic moment developed per unit volume, when a magnetic specimen is subjected to magnetizing field. It is denoted by I.

If a magnetic specimen of volume V acquires magnetic dipole moment M due to the magnetizing field, then

$$I = \frac{M}{V} \qquad \text{...(i)}$$

Since unit of magnetic moment is ampere metre², the unit of intensity of magnetization is

$$[I] = \frac{\text{ampere metre}^2}{\text{metre}^3} = \text{ampere metre}^{-1} (\text{Am}^{-1})$$

i.e. intensity of magnetization and the magnetic intensity have the same units. The intensity of magnetization is sometimes called simply magnetization.

If the magnetic specimen has area of cross-section a and length $2l$, then its volume,

$$V = a \times 2l$$

Further, if m is pole strength developed, then magnetic moment of the specimen,

$$M = m \times 2l$$

Therefore, Eq. (i) gives

$$I = \frac{m \times 2l}{a \times 2l} = \frac{m}{a} \qquad \text{...(ii)}$$

Hence, intensity of magnetization may also be defined as the pole strength developed per unit area of cross-section of the specimen.

Relation Between Intensity of Magnetization and Magnetic Field or Induction of an Ideal Solenoid

If a solenoid with n turns per unit length is carrying a current i, magnetic moment of the solenoid due to each turn of the solenoid $= i_M A$.

Total magnetic moment of the material due to all the turns (N) of the solenoid $= N i_M A$

Here, A is the area of cross-section of the solenoid and l is its length.

$$\text{Clearly, } M = \frac{\text{magnetic moment of the material}}{\text{volume of the material (i.e., solenoid)}}$$

$$\text{or} \qquad M = \frac{N i_M A}{Al} = n i_M \qquad (\text{here } N/l = n)$$

If B_M is the additional magnetic induction (or field) set up by the material of the core.

Then, $B_M = \mu_0 n i_M$

Hence, $B_M = \mu_0 M$ ...(iii)

MAGNETIC INTENSITY

Consider that inside vacuum, a magnetic field B_0 exists. Then, magnetic intensity is given by the relation

$$H = \frac{B_0}{\mu_0}, \qquad \text{...(iv)}$$

where $\mu_0 = 4\pi \times 10^{-7}$ tesla metre ampere^{-1} is absolute permeability of vacuum. Since unit of magnetic field is tesla, the unit of magnetic intensity is

$$[H] = \frac{\text{tesla}}{\text{tesla metre ampere}^{-1}} = \text{ampere meter}^{-1} (\text{A m}^{-1})$$

Magnetic intensity is also known as H-field or magnetic field strength. The unit of magnetic intensity i.e. Am^{-1} is also equivalent to N m^{-2} T^{-1}or N Wb^{-1}or J m^{-3} T^{-1} or J m^{-1}Wb^{-1}.

The older unit of magnetic intensity is oersted. It is related to gauss (the older unit of magnetic field) as given below:

$$1 \text{ oersted} = \frac{1 \text{gauss}}{\mu_0}$$

$$\text{Therefore, } 1 \text{ oersted} = \frac{10^{-4}\,\text{T}}{4\pi \times 10^{-7}\,\text{T m A}^{-1}} \simeq 80 \text{ Am}^{-1}$$

Important Point:

A solenoid with n turns per unit length is carrying a current i. Clearly, magnetic field (or induction) inside the empty solenoid, i.e., $B_0 = \mu_0 n i$.

Then magnetic intensity is given by the relation

$$H = \frac{B_0}{\mu_0} = \frac{\mu_0 n i}{\mu_0} = ni \qquad \text{...(v)}$$

Thus product of n and i, i.e., ni is called the magnetizing field intensity, H.

TOTAL MAGNETIC FIELD OR INDUCTION

Consider that a soft iron bar (ferromagnetic material) is placed in a uniform magnetic field B_0 set up in vacuum. Let us consider a solenoid with n turns per unit length, carrying a current i. Clearly, magnetic field (or induction) inside the empty solenoid, i.e.,

$$B_0 = \mu_0 n i_0 = \mu_0 H \qquad \text{...(vi)}$$

If we fill the solenoid by a magnetic material as a core, the total or resultant magnetic field, i.e.,

$$B = B_0 + B_M \qquad \text{...(vii)}$$

Here, B_M is the additional magnetic induction (or field) set up by the material of the core. Further,

$$B_M = \mu_0 n i_M = \mu_0 M \qquad \text{...(viii)}$$

where i_M is the magnetization current (or Amperean current) arising on account of the magnetization of the material.

It is to be noted that B_M is produced by small circulating currents inside the magnetic material due to orbiting and spining electrons in the atoms.

From Eqs. (vi), (vii) and (viii), we get

$$B = \mu_0 ni + \mu_0 ni_M = \mu_0(H + M) \quad \text{...(ix)}$$

The magnetic induction is defined as the number of magnetic lines of induction (magnetic field lines inside the material) crossing per unit area normally through the magnetic substance. It is denoted by B.

Magnetic induction is also known as magnetic flux density or simply magnetic field. The SI unit of magnetic induction is tesla (T) or weber metre^{-2} (Wb m^{-2}). These units of magnetic induction are equivalent to N m^{-1} A^{-1} or J A^{-1} m^{-2}.

The older unit of magnetic induction is **guass (G)**.

1 tesla = 10^4 gauss (G)

Important Point:

Equation (ix) can be written as $H = \frac{B}{\mu_0} - M$

Since $\vec{B}$ and $\vec{M}$ are vectors, $\vec{H}$ is also a vector. Hence, the magnetic intensity vector.

Thus, $\vec{H} = \frac{\vec{B}}{\mu_0} - \overline{M}$

MAGNETIC SUSCEPTIBILITY (χ_m)

For a large class of isotropic materials, $M \propto H$

or $\quad M = \chi_m H \quad$ ...(x)

where χ_m is a dimensionless quantity and is called the magnetic susceptibility.

MAGNETIC PERMEABILITY (μ)

Total magnetic field or induction can be given by the relation,

$$B = \mu_0(H + M)$$

Hence we can write, $B = \mu_0(H + \chi_m H) = \mu_0(1 + \chi_m)H$

or $\quad B = \mu H \quad$ ...(xi)

The constant μ is called the absolute magnetic permeability, where

$$\mu = \mu_0(1 + \chi_m) \quad \text{...(xii)}$$

Further, $\mu/\mu_0 = \mu_r$ is called the relative permeability of the material.

From Eq. (xii), $\mu_r = 1 + \chi_m \quad$ ...(xiii)

Important Points:

- $\frac{B}{B_0} = \frac{\mu H}{\mu_0 H} = \frac{\mu}{\mu_0} = \mu_r$
- The unit of μ is the same as that of μ_0, i.e., Wb/Am. μ_r is a pure ratio and has no unit.

MAGNETIC FLUX

Magnetic flux through a surface is defined as the number of magnetic field lines passing normally through the surface. It is denoted by (ϕ).

Consider that a surface area $\Delta\vec{S}$ is held in a magnetic field of induction B. Then, magnetic flux passing normally through the surface is given by

$$\phi = \vec{B} \cdot \Delta\vec{S}$$

The SI unit of magnetic flux is weber (Wb).

ILLUSTRATION 3.20

The absolute magnetic permeabilityis measured to be 0.12 T A^{-1} m. Find its relative permeability and susceptibility.

Sol. Here $\mu = 0.12 \text{ TA}^{-1}\text{ m}$

We know $\mu_0 = 4\pi \times 10^{-7} \text{ TA}^{-1}\text{ m}$

$$\therefore \quad \mu_r = \frac{\mu}{\mu_0} = \frac{0.12}{4\pi \times 10^{-7}} = 9.55 \times 10^4$$

Also, $\chi_m = \mu_r - 1 = 9.55 \times 10^4 - 1 = 9.55 \times 10^4$

ILLUSTRATION 3.21

A magnetizing field of 1500 A/m produces a magnetic flux of 2.4 × 10^{-5} Wb in a bar of iron of cross-section 0.5 cm^2. Calculate permeability and susceptibility of iron bar used.

Sol. Here, $H = 1500$ A/m, $\phi_B = 2.4 \times 10^{-5}$ Wb

$$A = 0.5 \text{ cm}^2 = 0.5 \times 10^{-4} \text{ m}^2$$

Magnetic field, $B = \frac{\phi_B}{A} = \frac{2.4 \times 10^{-5} \text{ Wb}}{0.5 \times 10^{-4} \text{ m}^2}$

$$= 0.48 \text{ Wb/m}^2$$

Permeability, $\mu = \frac{B}{H} = \frac{0.4 \text{ Wb/m}^2}{1500 \text{ A/m}} = 3.2 \times 10^{-4} \frac{\text{Tm}}{\text{A}}$

Susceptibility, $\chi_m = \frac{\mu}{\mu_0} - 1 = \left(\frac{3.2 \times 10^{-4}}{4\pi \times 10^{-7}}\right) - 1$

$$= 255 - 1 = 254$$

CLASSIFICATION OF MAGNETIC MATERIALS

On the basis of mutual interactions or behavior of various materials in an external magnetic field, the materials are classified into three major categories, namely (a) diamagnetic (b) paramagnetic and (c) ferromagnetic

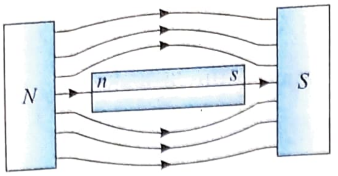
DIAMAGNETIC SUBSTANCES

The substances which are feebly magnetized in a direction opposite to that of magnetizing field in which these are placed are called diamagnetic substances.

For example; bismuth, antimony, copper, gold, quartz, mercury, water, alcohol, air, hydrogen, etc., are all diamagneticsubstances. Some other properties of diamagnetic substancesare:

- When a diamagnetic substance is placed inside an external magnetic field, the magnetic field inside the diamagnetic is found to be slightly less than the external magnetic field.

- When a diamagnetic material is placed inside a magnetic field, the magnetic field lines become slightly less dense in the diamagnetic material



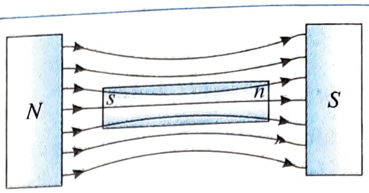
- When a diamagnetic substance is placed in a magnetic field, a small magnetization occurs opposite to the direction of the field. As a result, the magnetic field lines become less dense inside the diamagnetic material.
- It is observed that when a diamagnetic sample is placed inside a nonuniform magnetic field, it tends to move from stronger part to the weaker part of the magnetic field. It may be pointed out that the diamagnetic effects are too feeble to be detected, unless the applied magnetic field is strong.
- The behavior of a diamagnetic substance is independent of temperature.
- A diamagnetic substance has the nature similar to that of a dielectric having non-polar atoms.
- For a diamagnetic substance, the intensity of magnetization (*I*) has a small negative value. It is because, on being placed in a magnetic field, a diamagnetic substance gets slightly magnetized in a direction opposite to that of the applied field.
- The relative permeability of diamagnetic substance is less than unity.
- As $\mu_r = 1 + \chi_m$ and $\mu_r < 1, \chi_m$ is negative, i.e., the susceptibility of diamagnetic is negative.
- Susceptibility of diamagnetic does not change withtemperature. *Bi* at low temperatures is an exception to thisgeneral property.

PARAMAGNETIC SUBSTANCES

The substances which are feebly magnetized in the direction of the magnetizing field in which these are placed are called paramagnetic substances.

For example, aluminum, platinum, chromium, manganese, copper sulphate, crown glass, solutions of salts of iron and nickel and oxygen are all paramagnetic substances. Some of the other properties of paramagnetic substance are:

- When a paramagnetic substance is placed inside an external magnetic field, the magnetic field inside the paramagnetic is found to be a slightly greater than the external magnetic field. In contrast to the behavior of diamagnetic,
- A paramagnetic substance tends to move from weaker part of the magnetic field to stronger part, when placed in a non-uniform magnetic field.
- In contrast to diamagnetic, the behavior of a paramagnetic is temperature dependent. Also, the paramagnetic effects are perceptible only with a strong magnetic field. The nature of a paramagnetic is similar to that of a dielectric having polar atoms.
- When a paramagnetic material is placed inside a magnetic field, the magnetic field lines become slightly more dense in the paramagnetic material



- When a paramagnetic substance is placed in a magnetic field, a small magnetization occurs in the direction of the field. As a result, the magnetic field lines become more dense inside the paramagnetic substance.
- The relative permeability of paramagnetic substances is greater than unity.
- As $\mu_r = 1 + \chi_m$ and $\mu_r > 1$, χ_m is positive. Hence, the susceptibility of paramagnetics is positive.
- Susceptibility of paramagnetic varies inversely as the temperature of the substance, i.e.,

  $$\chi_m \propto 1/T \text{ or } \chi_m = C/T \qquad \text{...(i)}$$

  where C = Curie constant depending upon the nature of the substance and T = absolute temperature.

  Equation (i) is called the **Curie's law** which was experimentally established by the French physicist and chemist, Pierre Curie.

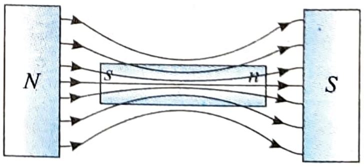
FERROMAGNETIC SUBSTANCES

The substances which are strongly magnetized in the direction of the magnetizing field in which they are placed are called ferromagnetic substances.

For example, iron, cobalt, nickel, gadolinium and a number of alloys (Alnico, Nipennag) are ferromagnetic in nature.

The ferromagnetic materials exhibit the properties of paramagnetic substances to a much higher degree. For example:

- When a ferromagnetic substance is placed inside a magnetic field, the field inside the ferromagnetic substance gets greatly enhanced. As a result, when a ferromagnetic is placed in a non-uniform magnetic field, it quickly moves from weaker part to stronger part of the magnetic field. In other words, the ferromagnetic effects are perceptible even in the presence of a weak magnetic field.
- When a ferromagnetic material is placed inside a magnetic field, the magnetic field lines becomes highly dense in the ferromagnetic substance as shown in figure.



- When a ferromagnetic material is placed in a magnetic field, a large magnetization occurs in the direction of magnetic field.
- For a ferromagnetic substance, the intensity of magnetization (*I*) has a large positive value. It is because, on being placed in a magnetic field, a ferromagnetic substance gets strongly magnetized in the direction of the field.
- Permeability of ferromagnetic materials is very large, of the order of hundreds and thousands.

- Similarly, their susceptibility is also very large. That is why these can be magnetized easily and strongly.
- The ferromagnetic behavior of a substance becomes temperature dependent. With the rise of temperature, susceptibility of ferromagnetic materials decreases. At a certain temperature, ferromagnetics pass over to paramagnetics. This transition temperature is called **Curie temperature or Curie point (T_c)**. For example, Curie temperature for iron is 770°C and 365°C for nickel and only 70°C for permalloy (an alloy of 70% Fe and 30% Ni).
- At a temperature above the Curie point, a ferromagnetic becomes an ordinary paramagnetic whose magnetic susceptibility obeys the **Curie-Weiss law**.

According to Curie-Weiss law, at temperatures above Curie temperature the magnetic susceptibility of ferromagnetic materials is inversely proportional to $(T - T_c)$

i.e., $\chi \propto \frac{1}{T - T_c} \Rightarrow \chi = \frac{C}{(T - T_c)}$

Here T_c = Curie temperature.

χ-T curve is shown (for Curie-Weiss Law).

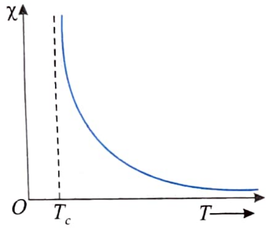


ILLUSTRATION 3.22

For a magnetizing field of intensity 2×10^3 Am^{-1}, aluminum at 280 K acquires intensity of magnetization of 4.8×10^{-2} Am^{-1}. Find the susceptibility of aluminum at 280 K. If the temperature of the aluminum is raised to 320 K, what will be its susceptibility and intensity of magnetization?

Sol. Here, $H = 2 \times 10^3$ Am^{-1};

$I = 4.8 \times 10^{-2}\,\text{Am}^{-1}$ and $T = 280$ K

Now, $\chi_m = \frac{I}{H} = \frac{4.8 \times 10^{-2}}{2 \times 10^3} = 2.4 \times 10^{-5}$

Aluminum is a paramagnetic substance and hence obeys Curies' law. If χ_m' is susceptibility of Aluminum at temperature $T'(= 320\text{ K})$, then

$$\frac{\chi_m'}{\chi_m} = \frac{T}{T'} \text{ or } \chi_m' = \chi_m \times \frac{T}{T'}$$

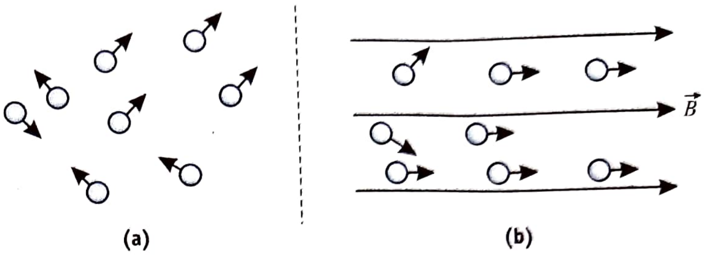
or $\chi_m' = 2.4 \times 10^{-5} \times \frac{280}{320} = 2.1 \times 10^{-5}$

If I' is intensity of magnetization of aluminum at 320 K, then

$$I' = \chi_m' H = 2.1 \times 10^{-5} \times 2 \times 10^3 = 4.2 \times 10^{-2}\,\text{Am}^{-1}$$

PARAMAGNETISM ON BASIS OF ELECTRON THEORY

An atom of a paramagnetic material possesses resultant magnetic moment as the magnetic moments of the electrons in such an atom do not exactly cancel. Each atom, therefore, behaves like a tiny magnet. In the absence of any external magnetic field, the resultant magnetic moment due to all theatoms is zero as these moments are randomly oriented as shown in Fig. (a). When an external magnetics field is applied to a paramagnetic material, the magnetic moments of its atoms tend to align with the direction of the magnetic field, giving rise to a small net magnetic moment. However, thermal motion interferes with the aligning process and as such does not permit a complete alignment as shown in Fig. (b).



(a) (b)

Obviously, the paramagnetic behavior decreases with rise in temperature. On the contrary, when a paramagnetic salt is cooled, the magnetic moments of the atoms of the salt tend to align with the magnetic field intensity, thereby increasing its magnetization.

The magnetic susceptibility of paramagnetic materials is about a hundred times higher than that for diamagnetic substances.

FERROMAGNETISM

Ferromagnetic materials possess a very large resultant magnetic moment. The fundamentals of the theory of ferro-magnetism were given by *Y.* Frenkel and Werner Heisenberg. This theory is called the **domain theory**. According to this theory:

(i) The spin magnetic moments of the electrons are responsible for the magnetic properties of ferromagnetics.

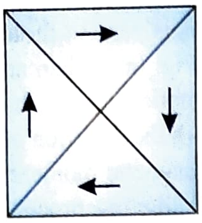
(ii) Under certain definite forces, these spin magneticmoments are lined up parallel to one another. This resultsin setting up of regions of spontaneous magnetization,which are called **domains**.

(iii) In each domain, the atoms have magnetic moments whichare aligned strictly in a single direction.

(iv) A very strong magnetic field appears inside a domain andas a result of this, it is magnetised to saturation. A domain contains from 10^{21} to 10^{17} atoms and has a dimension of the order of 10^{-8} to 10^{-12} m^3.

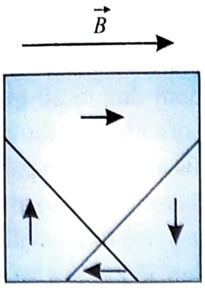
(v) Magnetic moments of different domains have different directions in the absence of external magnetic field and as a result of this, the resultant magnetic moment is zero as shown in Fig. (a).

(vi) In the presence of an external magnetic field ($\vec{B}$), the domains suffer two effects: (a) those domains oriented favourably with respect to the magnetic field grow at the expense of those oriented less favorably [Fig. (b)] due to reorientation effect of the magnetic field (b) the magnetisation of the domains tends to align in the direction of the field [Fig. (c)] and the piece of matter becomes a magnet.



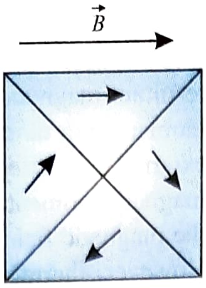
Unmagnetised substance

(a)



Magnetisation by domain growth

(b)



Magnetisation by domain orientation

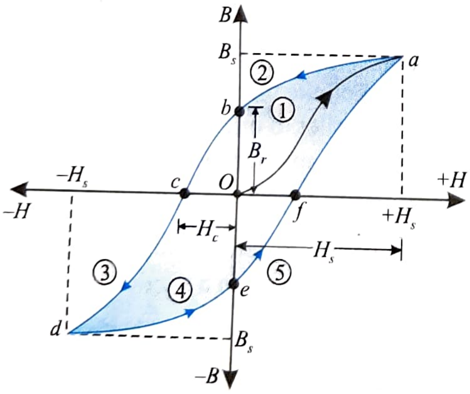
(c)

HYSTERESIS

A specimen of ferromagnetic material is placed in a magnetizing field whose strength and direction can be changed. Suppose that the specimen is unmagnetized initially. When the magnetizing field (H) is increased, the intensity of magnetization (I) of the material of the specimen also increases. It is found that when the magnetizing field is made zero, the intensity of magnetization does not become zero but still has some finite value. It becomes zero only, when magnetizing field is increased in reverse direction. In other words, intensity of magnetization does not become zero on making magnetizing field zero but does so a little late and this effect is called hysteresis.

Hysteresis is the phenomenon shown by ferromagnetic substances where by the magnetic field induction (B) of the medium depends not only on the existing magnetising field (H) but also on the previous state of the substance.

The relationship between B and H in a ferromagnetic substance is not linear and is quite complex. In order to study hysteresis, let us take a ferromagnetic substance through a cycle of magnetization.



(i) When H changes from 0 to $+H_S$, B changes along $O\ a$ (path 1)

(ii) When H changes from $+H_S$ to 0, B changes along ab (path 2)

(iii) When H changes from 0 to $-H_s$, B changes along bcd (path 3)

(iv) When H changes from $-H_s$ to 0, B changes along de (path 4)

(v) When H changes from 0 to $+H_S$, B changes along efa (path 5)

The graph $abcdefa$ between B and H is called the hysteresis loop or the hysteresis curve as shown in figure.

SALIENT FEATURES OF HYSTERESIS LOOP

(a) When $H = 0$ (after having increased from 0 to H_s), $B_r = B$. Here, B_r is B called the residual magnetism, retentivity or remanence. Thus, retentivity is the property by virtue of which the magnetism (I) remains in a material even on the removal of magnetizing field.

(b) When magnetic field H is reversed, magnetization decreases and for a particular value of H. When $H = H_c$, $B = 0$. This value of H is called the coercivity of the specimen. Thus, coercivity of a material is the measure of the magnetizing field (H) required to destroy the residual magnetism.

Magnetic hard substance (steel) → High coercivity

Magnetic soft substance (soft iron) → Low coercivity

(c) The process of taking a ferromagnetic through a cycle of magnetization results in loss of energy. This is called hysteresis loss and it appears in the form of heat.

(d) Area of the B-H loop is equal to the energy loss per cycle per unit volume since area of B-H loop $= B \times H$

$$= \mathrm{T} \times (\mathrm{A/m}) \left(\frac{\mathrm{N}}{\mathrm{Am}}\right)\left(\frac{\mathrm{A}}{\mathrm{m}}\right) = \frac{\mathrm{Nm}}{\mathrm{m}^3} = \mathrm{J/m}^3.$$

(e) The maximum value of B is called the saturation field (B_s) and $B_s \sim 10^3 B_0$, where $B_0 = \mu_0 nI$. The value of H corresponding to B_s is the saturation value of magnetising field (H_s).

(f) A similar curve is obtained if we plot a graph between M and H.

(g) Unit of area of M-H loop = unit of $M \times H$

$$= \left(\frac{\mathrm{A}}{\mathrm{m}}\right)\left(\frac{\mathrm{A}}{\mathrm{m}}\right) = \frac{\mathrm{A}^2}{\mathrm{m}^2}$$

Further, $\mu_0 \times$ area of M-H loop

$$= \mu_0 \times \frac{\mathrm{A}^2}{\mathrm{m}^2} = \left(\frac{\mathrm{Wb}}{\mathrm{Am}}\right)\frac{\mathrm{A}^2}{\mathrm{m}^2} = \frac{\mathrm{TA}}{\mathrm{m}}$$

= area of B-H loop.

Area of B-H loop $= \mu_0 \times$ area of M-H loop.

(h) By studying the hysteresis loops of various magnetic materials, we can study the difference in their magnetic properties.

SOFT AND HARD MAGNETIC MATERIALS

The shape of the hysteresis loop is a characteristic of a ferromagnetic substance. It gives the idea about many important magnetic properties of the substance.

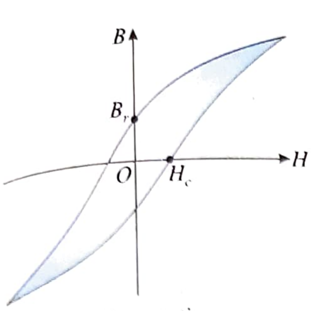
(a) Soft magnetic materials: They have:

(i) low coercivity (ii) lower retentivity (iii) small hysteresis loss, i.e., small hysteresis loop [Fig. (a)]. These materials are used in:

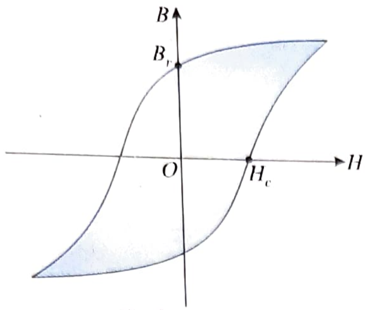
(a) electromagnets (b) cores of transformers, motors and generators.

(b) Hard magnetic materials: They have:

(i) high coercivity (ii) high retentivity (iii) large hysteresis loss, i.e., large hysteresis loop [Fig. (b)]. These materials are used in making: (a) permanent magnets, (b) magnetic tapes, (c) magnetic core computer memories.



Soft magnetic material



Hard magnetic material

Examples of hard magnetic materials are: Steel, Alnico (Al, Ni, Co), Alcomax, Ticonal (Ti, Co, Ni, Al).

It is caused by charged particles from the Sun that reach the Earth's upper atmosphere and set up currents that change the planet's overall magnetic field.

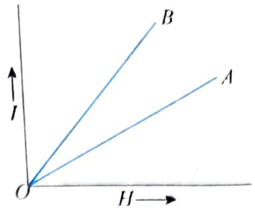
The following conclusions can be drawn from the study of the hysteresis loops of soft iron and steel:

1. The area of hysteresis loop for soft iron is much smaller than that for steel. Therefore, loss of energy per unit volume in case of soft iron will be very small as compared to that in case of steel, when they are taken over a complete cycle of magnetisation and demagnetisation.
2. Soft iron acquires maximum intensity of magnetisation for comparatively much lesser value of magnetising field than in case of steel. In other words, soft iron is much strongly magnetised (or more susceptible to magnetism) than steel.
3. The retentivity of soft iron is greater than that of steel. On removing magnetising field, quite a large amount of magnetisation is retained by soft iron.
4. The coercivity of steel is much larger than that of soft iron. Therefore, the residual magnetism in steel cannot be destroyed that easily as in case of soft iron.

The choice of a particular ferromagnetic material for a practical application is decided from the magnetic properties such as retentivity, coercivity and area of the hysteresis loop.

ILLUSTRATION 3.23

The figure shows the variation of intensity of magnetization (I) versus the applied magnetic field intensity (H) for two magnetic materials A and B.



(a) Identify the materials A and B.

(b) Why does the material B has a larger susceptibility than A for a given field at constant temperature?

Sol.

(a) Paramagnetic materials have a small positive susceptibility (I/H), while ferromagnetic materials have a much larger positive susceptibility. Therefore, material A is paramagnetic and material B is ferromagnetic.

(b) For a given magnetic field intensity, intensity of magnetisation in case of ferromagnetic materials is very large.

ILLUSTRATION 3.24

A bar magnet has coercivity 4×10^3 Am^{-1}. It is desired to demagnetize it by inserting it inside a solenoid 12 cm long and having 60 turns. What current should be sent through the solenoid?

Sol. The coercivity of 4×10^3 Am^{-1} of the bar magnet implies that magnetic intensity $H = 4 \times 10^3$ Am^{-1} (in opposite direction) is required to demagnetize it.

Here, number of turns per unit length of the solenoid.

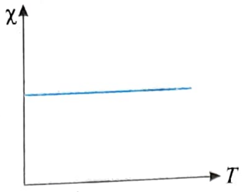
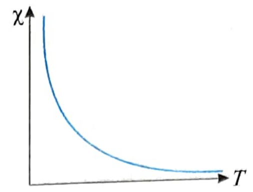
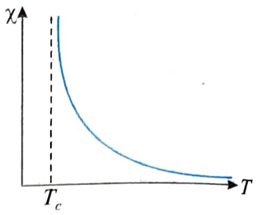
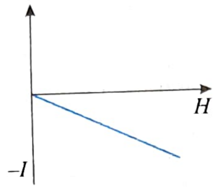
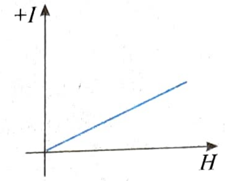
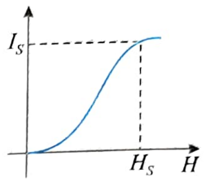
$$n = \frac{60}{12} = 5 \text{ turns cm}^{-1} = 500 \text{ turns m}^{-1}$$

Now, $B = \mu_0 n I$

or $$I = \frac{B}{\mu_0 n} = \frac{H}{n} = \frac{40 \times 10^3}{500} = 8 \text{ A}$$

Comparative Study of Magnetic Materials

Property	Diamagnetic substances	Paramagnetic substances	Ferromagnetic substances
Cause of magnetism	Orbital motion of electrons	Spin motion of electrons	Formation of domains
Explanation of magnetism	On the basis of orbital motion of electrons	On the basis of spin and orbital motion of electrons	On the basis of domains formed
Value of magnetic induction B	$B < B_0$ (where B_0 is the magnetic induction in vacuum)	$B > B_0$	$B >> B_0$
Magnetic susceptibility (χ)	Low and negative $\lvert\chi\rvert \approx 1$	Low but positive $\chi \approx 1$	Positive and high $\chi \approx 10^2$

Dependence of χ on temperature	Does not depend on temperature (except *Bi* at low temperature) 	On cooling, these get converted to ferromagnetic materials at Curie temperature 	These get converted into paramagnetic materials at Curie temperature 
Relative permeability (μ_r)	$\mu_r < 1$	$\mu_r > 1$	$\mu_r >> 1$, $\mu_r = 10^2$
Intensity of magnetisation (I)	I is in a direction opposite to that of H and its value is very low	I is in the direction of H but value is low	I is in the direction of H and value is very high.
I-H curves			
Magnetic moment (M)	Very low (≈ 0)	Very low	Very high
Examples	Cu, Ag, Au, Zn, Bi, Sb, NaCl, H_2O air and diamond etc.	Al, Mn, Pt, Na, $CuCl_2$, O_2 and crown glass	Fe, Co, Ni, Cd, Fe_3O_4, etc.

CONCEPT APPLICATION EXERCISE 3.3

1. A magnet weighs 75 g and its magnetic moment is 2×10^{-4} Am^2. If the density of the material of the magnet is 7.5×10^3 kg m^{-3}, calculate the intensity of magnetization.
2. The magnetic induction and magnetizing field in a sample of magnetic material are 1.0 Wb m^{-2} and 2×10^3 A m^{-1} respectively. Find **(a)** magnetic permeability **(b)** relative permeability of the material, **(c)** magnetic susceptibility and **(d)** intensity of magnetization. Given that $\mu_0 = 4\pi \times 10^{-7}$ T A^{-1} m.
3. The magnetizing field of 1600 A m^{-1} produces a magnetic flux of 2.4×10^{-5} weber in a bar of iron of cross-section 0.2 cm^2. Calculate relative permeability, intensity of magnetization and susceptibility of the bar.
4. The core of a toroid having 3,000 turns has inner and outer radii of 11 cm and 12 cm respectively. The magnetic field in the core for a current of 0.70 A is 2.5 T. What is the relative permeability of the core?
5. A Rowland ring has 10^3 turns per unit length. On passing a current of 2 A, magnetic induction is measured to be 1.5 Wb m^{-2}. Calculate **(a)** relative permeability of the core, **(b)** magnetic susceptibility, **(c)** magnetizing field intensity and **(d)** magnetization.
6. The susceptibility of magnesium at 300 K is 1.2×10^{-5}. At what temperature will the susceptibility equal to 144×10^{-5}?
7. The hysteresis loss for a specimen of iron weighing 12 kg is equivalent to 300 J m^{-3} cycle^{-1}. Find the loss of energy per hour at 50 cycles s^{-1}. Given, density of iron = 7,500 kg m^{-3}.
8. Assume that each iron atom has a permanent magnetic moment equal to 2 Bohr magneton (1 Bohr magneton = 9.27×10^{-24} A m^2). The number density of atoms in iron is 8.52×10^{28} m^{-3}. Find the maximum value of **(a)** intensity of magnetization and **(b)** magnetic induction in an iron bar.

ANSWERS

1. 20 Am^{-1}

2. (a) 5×10^{-4} T mA^{-1}; (b) 397.9; (c) 396.9; (d) 7.94×10^5 Am^{-1}

3. 596.8, 9.534×10^5 Am^{-1}, 595.8

4. 684.5

5. (a) 596.8; (b) 595.8; (c) 2,000 Am^{-1}; (d) 1.192×10^6 Am^{-1}

6. 250 K

7. 8.64×10^4 J

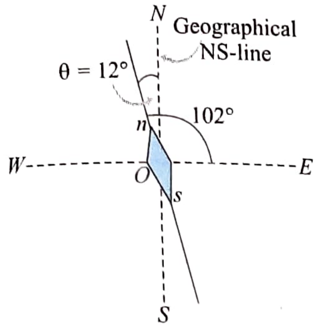
8. (a) 1.58×10^6 Am^{-1} (b) 1.985 T

Solved Examples

EXAMPLE 3.1

The declination at a place is 12° west of north. Find in what direction a ship should steer so that it reaches a place due east?

Sol. The dotted *N-S* line represents the geographical *N-S* line and the compass needle *ns* points 12° west of the geographical north (figure).

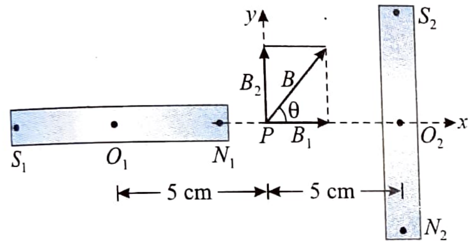


Therefore, the ship would reach a place due east, if it stears 12° + 90° i.e. 102° with the direction of the compass needle.

EXAMPLE 3.2

Two identical short bar magnets each of magnetic moment 12.5 Am^2 are placed at a separation of 10 cm between their centres such that their axes are perpendicular to each other. Find the magnetic field at a point midway between the two magnets.

Sol. Suppose that the two identical magnets N_1S_1 and N_2S_2 are placed as shown in figure. Let P be the point midway between the two magnets.



Here, $M_1 = M_2 = 12.5\ \text{Am}^2$

and $O_1P = O_2P = 5\ \text{cm} = 0.05\ \text{m}$

The point P lies on axial line of magnet N_1S_1. If magnetic field due to the magnet N_1S_1 at point P is B_1, then

$$B_1 = \frac{\mu_0}{4\pi}\cdot\frac{M_2}{(O_1P)^3} = \frac{10^{-7}\times 2\times 12.5}{(0.05)^3} = 0.02\,\text{T (along } PX)$$

The point P lies on the equatorial line of the magnet N_2S_2. If B_2 is magnetic field due to the magnet N_2S_2 at point P, then

$$B_2 = \frac{\mu_0}{4\pi}\cdot\frac{M_2}{(O_2P)^3} = \frac{10^{-7}\times 12.5}{(0.05)^3} = 0.01\ \text{T (along } PY)$$

The resultant magnetic field at point P,

$$B = \sqrt{B_1^2 + B_2^2} = \sqrt{(0.02)^2 + (0.01)^2} = 2.236\times 10^{-2}\ \text{T}$$

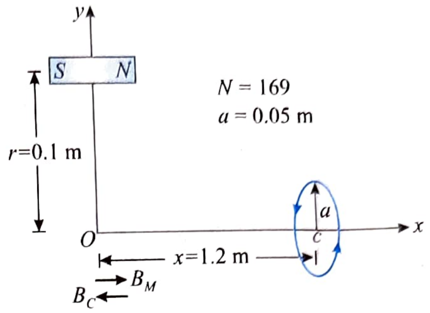
If θ is the angle, which the resultant field makes with the X-axis, then

$$\tan\theta = \frac{B_2}{B_1} = \frac{0.01}{0.02} = 0.5 \text{ or } \theta = \tan^{-1}\left(\frac{1}{2}\right)$$

EXAMPLE 3.3

A small magnetic dipole of magnetic moment $\pi\times 10^{-3}$ A-m^2 is placed on the y-axis at a distance of 0.1m from the origin with its axis parallel to x-axis. A coil having 169 turns and radius 0.05 m is placed on the x-axis at a distance of 0.12 m from the origin with the axis of the coil coinciding with x-axis. Find the magnitude and direction of the current in the coil for a compass needle placed at the origin, to point in the north-south direction.

Sol. The situation described in question is shown in figure.



The magnetic induction at O due to magnetic dipole moment M is given as

$$B_M = \frac{\mu_0}{4\pi}\frac{M}{r^3}\ \text{N/A-m} \qquad \text{...(i)}$$

The direction of above magnetic induction is along positive x-direction. The magnetic induction at O due to the current carrying coil of radius a and having N turns is given as

$$B_C = \frac{\mu_0 Nia^2}{2(a^2+x^2)^{3/2}} \qquad \text{...(ii)}$$

The direction of above magnetic induction is along negative or positive X-direction depending upon the direction of current in the coil. As the deflection of the compass needle at O has to remain north-south that means resultant field at O other than Earth's magnetic field should be zero or in this case B_M should be equal and opposite to B_C so as to nullify it. Thus, we have

$$\frac{\mu_0}{4\pi}\cdot\frac{M}{r^3} = \frac{\mu_0 Nia^2}{2(a^2+x^2)^{3/2}}$$

$$\Rightarrow \quad i = \frac{2M(a^2+x^2)^{3/2}}{4\pi r^3\times Na^2} \qquad \text{...(iii)}$$

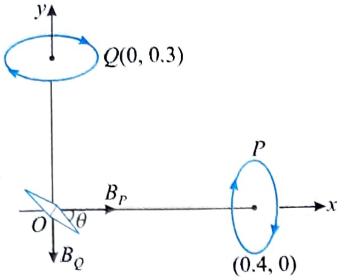
$$\Rightarrow \quad i = \frac{2\times(\pi\times 10^{-3})[(0.05)^2+(0.12)^2]^{3/2}}{4\pi\times(0.1)^3\times 169\times(0.05)^2}$$

$$\Rightarrow \quad i = 2{,}6\,\text{mA}$$

For the field B_C to be along negative X-direction, the current in the coil should be in anticlockwise direction.

EXAMPLE 3.4

The centers of two similar coils P and Q having same number of turns are located at the coordinates (0.4, 0) and (0, 0.3) such that the plane of coils are perpendicular to X and Y axes respectively. The areas of cross sections of coils P and Q are in the ratio 4 : 3. Coil P has 16A current in clockwise direction and coil Q has $9\sqrt{3}$ A current in anti-clockwise direction as seen from the origin. A small compass needle is placed at the origin. Find the deflection of the needle, assuming the Earth's magnetic field negligible and the radii of the coils very small compared to their distance from the origin.



Sol. As the radius of coils is considered to be very small, we can consider these coils as magnetic dipoles. So the magnetic induction due to coil at a distance x along its axis is given as

$$B = \frac{2KM}{x^3} = \frac{\mu_0 NiA}{2\pi x^3}$$

Magnetic induction at O due to coil P is given as

$$B_P = \frac{\mu_0 N i_P A_P}{2\pi x^3} \qquad \text{(along } OX\text{)}$$

Similarly the magnetic induction due to coil Q at O is given as

$$B_Q = \frac{\mu_0 N i_Q A_Q}{2\pi y^3} \qquad \text{(along } YO\text{)}$$

If the compass needle makes an angle θ with X-axis, then it is along the direction of net magnetic induction at point O which is given as

$$\tan\theta = \frac{B_Q}{B_P} = \left(\frac{i_Q}{i_P}\right)\left(\frac{A_Q}{A_P}\right)\left(\frac{x^3}{y^3}\right)$$

$$\Rightarrow \quad \tan\theta = \left(\frac{9\sqrt{3}}{16}\right)\left(\frac{3}{4}\right)\left(\frac{0.4}{0.3}\right)^3 = \sqrt{3}$$

$$\Rightarrow \quad \theta = \tan^{-1}\sqrt{3} = 60°$$

EXAMPLE 3.5

A small magnet A makes 10 vibrations in 90 seconds in earth's field. When another magnet B of short length is placed 0.1 m due south of the direction of earth's field, the magnet A makes 10 vibrations in 45 seconds. Calculate the magnetic moment of magnet B. Given that $B_H = 0.34 \times 10^{-4}$ T.

Sol. **In earth's magnetic field:** Suppose that I_1 and M_1 are are the values of moment of inertia and magnetic moment of the magnet A. Its time period of vibration in the earth's magnetic field is given by

$$T_1 = 2\pi\sqrt{\frac{I_1}{M_1 B_H}} \qquad \text{...(i)}$$

In combined magnetic field of magnet B and B_H: Suppose that the time period of the magnet A becomes T_2, when the magnet B is placed due south. Since the magnet A lies on the axial line of the magnet B, the magnet A vibrates in the magnetic field,

$$B_H = B_H + B_{\text{axial}}$$

Here, B_{axial} is magnetic field on the axial line of the magnet B at the point, where the magnet A vibrates.

$$\therefore \quad T_2 = 2\pi\sqrt{\frac{I_1}{M_1 B_H'}} \qquad \text{...(ii)}$$

Dividing Eq. (i) by (ii), we have

$$\frac{T_1}{T_2} = \sqrt{\frac{B_H'}{B_H}}$$

$$\text{or} \quad B_H' = B_H \frac{T_1^2}{T_2^2}$$

$$\text{or} \quad B_H + B_{\text{axial}} = B_H \frac{T_1^2}{T_2^2}$$

$$\text{or} \quad B_{\text{axial}} = B_H\left(\frac{T_1^2}{T_2^2} - 1\right)$$

$$\text{Here,} \quad T_1 = \frac{90}{10} = 9 \text{ s}; \; T_2 = \frac{45}{10} = 4.5 \text{ s}$$

$$\text{and} \quad B_H = 0.34 \times 10^{-4} \text{ T}$$

$$\therefore \quad B_{\text{axial}} = 0.34 \times 10^{-4}\left(\frac{9^2}{4.5^2} - 1\right) = 1.02 \times 10^{-4} \text{ T}$$

If M_2 is the magnetic moment of the magnet B (a short bar magnet), then

$$B_{\text{axial}} = \frac{\mu_0}{4\pi} \cdot \frac{2M_2}{r^3}$$

$$\therefore \quad \frac{\mu_0}{4\pi} \cdot \frac{2M_2}{r^3} = 1.02 \times 10^{-4}$$

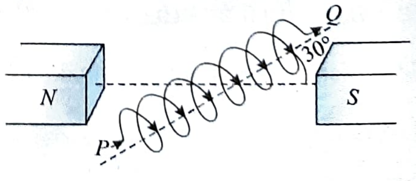
$$\text{Here,} \quad \frac{\mu_0}{4\pi} = 10^{-7} \text{ TA}^{-1}\text{m and } r = 0.1 \text{ m}$$

$$\therefore \quad 10^{-7} \times \frac{2M_2}{(0.1)^3} = 1.02 \times 10^{-4}$$

$$\text{or} \quad M_2 = \frac{1.02 \times 10^{-4} \times (0.1)^3}{2 \times 10^{-7}} = 0.51 \text{ A m}^2$$

EXAMPLE 3.6

A closely wound solenoid of 1000 turns and area of cross-section 2.0 × 10^{-4} m^2 carries a current of 20 A. It is placed with its horizontal axis at 30° with the direction of a uniform horizontal magnetic field of 0.16 T as shown in figure.



(a) What is the torque experienced by the solenoid due to the field?

(b) If the solenoid is free to turn about the vertical direction, specify its orientations of stable and unstable equilibrium. What is the amount of work needed to displace the solenoid from its stable orientation to its unstable orientation?

Sol. Here, number of turns in the solenoid = 1000;

$I = 2.0\,\text{A};\ A = 2.0\times10^{-4}\,\text{m}^2;$

The magnetic moment of the solenoid,

$$M = (IA)\times\text{Number of turns}$$
$$= 2.0\times2.0\times10^{-4}\times1000$$
$$= 0.4\,\text{Am}^2$$

(a) Now, $\tau = MB\sin\theta$

Here, $B = 0.16$ T, $\theta = 30°$

$\therefore\quad \tau = 0.4\times0.6\times\sin30° = 0.032$ Nm

(b) The solenoid will be in stable equilibrium, when $\vec{M}$ is parallel to $\vec{B}(\theta = 0°)$ and it will be in unstable equilibrium, when $\vec{M}$ is antiparallel to $\vec{B}(\theta = 180°)$.

Now, $W = MB(\cos\theta_1 - \cos\theta_2)$

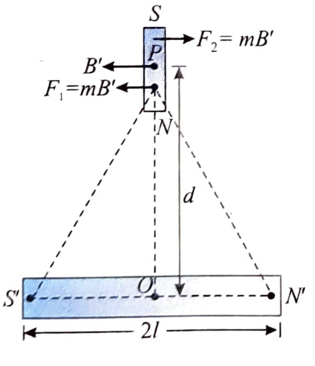
Therefore, work done to displace the solenoid from stable orientation to unstable orientation,

$$W = MB(\cos0° - \cos180°)$$
$$= 2MB = 2\times0.4\times0.16 = 0.128\text{ J}$$

EXAMPLE 3.7

A small magnet of magnetic moment M is placed at broad-side on position of a magnet of magnetic moment M', length $2l'$ in such a way that the axis of former coincides with the perpendicular bisector of the latter. The separation between their centres is d. Calculate the nature of interactions (force or torque) among them. What is its limiting value, when d becomes very large?

Sol. The small magnet NS of magnetic moment M and pole strength m (say) is placed at the point P on the broadside on position of the magnet $N'S'$ of magnetic moment M' and length $2l'$ as shown in figure.



The distance between the centres of two magnets,

$$OP = d$$

The magnetic field B' due to the magnet $N'S'$ at point P (a point on the equatorial line of $N'S'$),

$$B' = \frac{\mu_0}{4\pi}\cdot\frac{M'}{(d^2 + l'^2)^{3/2}}$$

Let F_1 and F_2 be the forces experienced by the north and south-pole of the small magnet NS due to the magnetic field B'. As the magnet NS is small, force on the two poles will be same in magnitude, say F. Therefore,

$$F = F_1 = F_2 = mB'$$

The forces on the two poles of the magnet NS are equal in magnitude but opposite in direction. Therefore, magnet NS is acted upon by a torque given by

$$\tau = F\times NS = mB'\times NS$$

Since $m\times NS = M$, magnetic moment of the magnet NS;

Hence, $\tau = MB'$

Substituting for B', we have $\tau = \dfrac{\mu_0}{4\pi}\cdot\dfrac{MM'}{(d^2 + l'^2)^{3/2}}$

Therefore, between the two magnets, the nature of the interaction is torque (net force on the magnet NS is zero). When d becomes very large i.e. if $d >> l$, then

$$d^2 + l'^2 \approx d^2$$

Hence, the limiting value of torque, $\tau = \dfrac{\mu_0}{4\pi}\cdot\dfrac{M\,M'}{d^3}$

EXAMPLE 3.8

A circular coil of radius 0157 m has 50 turns. It is placed such that its axis is in magnetic meridian. A dip needle is supported at the center of the coil with its axis of rotation horizontal and in the plane of the coil. The angle of dip is 30°, when a current flow through the coil. The angle of dip becomes 60° on reversing the current. Find the current in the coil assuming that the magnetic field due to the coil is smaller than the horizontal component of earth's magnetic field (horizontal component of earth's field = 3×10^{-5} T).

Sol. The horizontal component of the earth's magnetic field B_H is aided by the magnetic field B' produced by the coil, when the current is passed in one direction and is opposed by the magnetic field B' on reversing the direction of the current. It follows that

When $\delta = 30°, \tan\delta = \dfrac{B_V}{B_H - B'}$

and when $d = 60°,\ \tan\delta = \dfrac{B_V}{B_H - B'}\quad (B' < B_H)$

Here, $B_H = 3\times10^{-5}T$

In first case: Here, δ = 30°

$$\therefore\qquad \tan30° = \frac{B_V}{B_H + B'} \Rightarrow \frac{B_V}{B_H + B'} = \frac{1}{\sqrt3} \qquad\text{...(i)}$$

In second case: Here, $\delta = 60°$

$$\therefore\qquad \tan60° = \frac{B_V}{B_H - B'} \Rightarrow \frac{B_V}{B_H - B'} = \sqrt3 \qquad\text{....(ii)}$$

Dividing Eq. (ii) by (i), we have

$$\frac{B_V}{B_H - B'}\times\frac{B_H + B'}{B_V} = \sqrt3\times\frac{\sqrt3}{1}$$

or $$\frac{B_H + B'}{B_H - B'} = \frac{3}{1}$$

or $$\frac{(B_H + B') - (B_H - B')}{(B_H + B') + (B_H - B')} = \frac{3-1}{3+1}$$

or $$B' = \frac{1}{2} B_H = \frac{1}{2} \times 3 \times 10^{-5} = 1.5 \times 10^{-5}\ \text{T}$$

Now, the magnetic field at the centre of a circular coil,

$$B' = \frac{\mu_0}{4\pi} \cdot \frac{2\pi nI}{a}$$

$$\therefore \quad \frac{\mu_0}{4\pi} \cdot \frac{2\pi nI}{a} = 1.5 \times 10^{-5}$$

Here, $$10^{-7} \times \frac{2\pi \times 50 \times I}{0.157} = 1.5 \times 10^{-5}$$

or $$I = \left(\frac{1.5 \times 10^{-5} \times 0.157}{10^{-7} \times 2\pi \times 50} \right) 50 = 0.075\ \text{A}$$

EXAMPLE 3.9

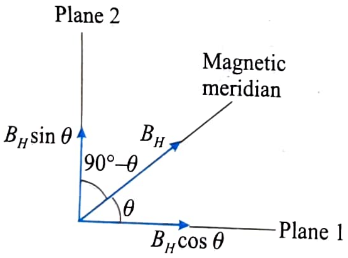
If δ_1 and δ_2 be the angles of dip observed in two planes at right angles to each other and δ is the true angle of dip, prove that

$$\cot^2 \delta_1 + \cot^2 \delta_2 = \cot^2 \delta$$

Sol. Let B_H and B_V be the horizontal and vertical components of the earth's magnetic field. As δ is true angle of dip,

$$\tan \delta = \frac{B_V}{B_H} \quad \text{...(i)}$$

The planes 1 and 2 are at right angles to each other. If the plane 1 makes an angle θ with the magnetic meridian, then the plane 2 will make an angle (90° – θ) with the magnetic meridian as shown in figure.



Whereas the vertical component of earth's magnetic field remains unaffected in the two planes, the effective horizontal components of earth's magnetic field in the planes 1 and 2 are $B_H \cos\theta$ and $B_H \sin\theta$ respectively. Therefore, the angle of dip δ_1 in the plane 1 is given by

$$\tan \delta_1 = \frac{B_V}{B_H \cos\theta} \text{ or } \cos\theta = \frac{B_V}{B_H \tan \delta_1}$$

Using Eq. (i), we have

$$\cos\theta = \frac{\tan\delta}{\tan\delta_1} \quad \text{...(ii)}$$

Further, the angle of dip δ_2 in the plane 2 is given by

$$\tan \delta_2 = \frac{B_V}{B_H \sin\theta}$$

Again using Eq. (i), we obtain

$$\sin\theta = \frac{\tan\delta}{\tan\delta_2} \quad \text{...(iii)}$$

Squaring and adding Eqs. (ii) and (iii), we have

$$\frac{\tan^2\delta}{\tan^2\delta_1} + \frac{\tan^2\delta}{\tan^2\delta_2} = \cos^2\theta + \sin^2\theta = 1$$

or $$\frac{1}{\tan^2\delta_1} + \frac{1}{\tan^2\delta_2} = \frac{1}{\tan^2\delta}$$

or $$\cot^2\delta_1 + \cot^2\delta_2 = \cot^2\delta$$

Exercises

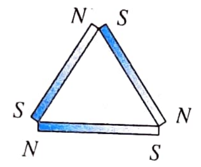
Single Correct Answer Type

1. A magnet of pole strength m is divided into four equal parts so that the length and breadth of each part is half that of the original magnet. Then pole strength of each of the magnet is
(1) m (2) $m/4$
(3) $m/2$ (4) $4m$

2. A magnetised wire of moment M is bent into an arc of a circle subtending an angle 60° at the centre, then the new magnetic moment is
(1) $(2M/\pi)$ (2) (M/π)
(3) $(3\sqrt{2}\,M/\pi)$ (4) $(3M/\pi)$

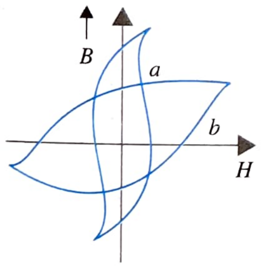
3. A thin bar magnet of length $2L$ is bent at the mid-point so that the angle between them is 60°. The new length of the magnet is
(1) $\sqrt{2}\,L$ (2) $\sqrt{3}\,L$
(3) $2L$ (4) L

4. Three identical bar magnets, each of magnetic moment M, are placed in the form of an equilateral triangle with north pole of one touching the south pole of the other (see figure). The net magnetic moment of the system is



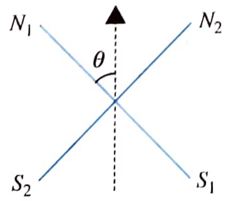
(1) zero (2) $3M$
(3) $3M/2$ (4) $M\sqrt{3}$

5. The B–H curves (a) and (b) shown in the figure are associated with



(1) a diamagnetic and a paramagnetic substance, respectively
(2) a paramagnetic and a ferromagnetic substance, respectively
(3) soft iron and steel, respectively
(4) steel and soft iron, respectively

6. Two magnets of equal mass are joined at right angles to each other. Magnet N_1S_1 has a magnetic moment $\sqrt{3}$ times that of N_2S_2. This arrangement is pivoted so that it is free to rotate in a horizontal plane. When in equilibrium, θ is



(1) 0° (2) 30°
(3) 45° (4) 60°

7. When a bar magnet is placed at 90° to a uniform magnetic field, it is acted upon by a couple which is maximum. For the couple to half the maximum value, the magnet should be inclined to the magnetic field at an angle of
(1) 45° (2) 30°
(3) 15° (4) 0°

8. The pole strength of a bar magnet is 48 Am and the distance between its poles is 25 cm. The moment of the couple by which it can be placed at an angle of 30° with the uniform magnetic field of flux density 0.15 N A^{-1} m^{-1} will be
(1) 12 Nm (2) 18 Nm
(3) 0.9 Nm (4) None of the above

9. The distance of two points on the axis of a magnet from its centre are 10 cm. The ratio of magnetic fields at these points is 12.5:1. The length of the magnet is
(1) 5 cm (2) 25 cm
(3) 10 cm (4) 20 cm.

10. Two identical thin bar magnets each of length l and pole strength m are placed at right angles to each other with north pole of one touching south pole of other, the magnetic moment of the system is
(1) ml (2) $2ml$
(3) $\sqrt{2}\,ml$ (4) $ml/2$

11. The work done to turn a magnet by 60° from its equilibrium position is W in a uniform field. The torque required to hold it in that position will be
(1) $\frac{W}{2}$ (2) $\frac{W}{\sqrt{3}}$
(3) $\frac{\sqrt{3}}{2}W$ (4) $W\sqrt{3}$

12. A compass needle of magnetic moment 60 Am^2 pointing geographical north at a place where the horizontal component of earth field is 40 μWbm^{-2} experiences a torque of 1.2×10^{-3} Nm.
(1) 15° (2) 30°
(3) 45° (4) 60°

13. In the above question, if the coil is in the magnetic field of 1.2 T, the field being in the plane of coil, then the torque acting on it is
(1) 1.42 Nm (2) 2.84 Nm
(3) zero Nm (4) 0.71 Nm

14. The force between two short bar magnets with magnetic moments M_1 and M_2 whose centres are r metre apart is 0.8 N when their axes are in the same line. If the separation is increased to $2r$, then force between them is reduced to
(1) 4.0 N (2) 2.0 N
(3) 1.0 N (4) 0.5 N

15. A magnet is parallel to uniform magnetic field. If it is rotated by 60°, the work done is 0.8 J. How much work is done in moving it 30° further?
(1) 0.8 ergs (2) 0.4 J
(3) 8 J (4) 0.8×10^7 ergs

16. In a vibration magnetometer, the time period of a bar magnet oscillating in horizontal component of earth's magnetic field is 2 s. When a magnet is brought near and parallel to it, the time period reduces to 1 s. The ratio *H*/*F* of the horizontal component H_a and the field *F* due to magnet will be

(1) 3 (2) 1/3

(3) $\sqrt{3}$ (4) $1/\sqrt{3}$

17. The magnetic needle of an oscillation magnetometer makes 0 oscillations per minute under the action of earth's magnetic field only. When a bar magnet is placed at some distance along the axis of the needle, it makes 14 oscillations per minute. If the bar magnet be turned so that its poles interchange the positions, the new frequency of the needle is

(1) 2 oscillations per minute

(2) 12 oscillations per minute

(3) 20 oscillations per minute

(4) 69.7 oscillations per minute

18. A magnetic needle of magnetic moment 60 Am^2 is directed towards the geographical north at a place. It experiences a torque of 1.2×10^{-3} Nm. If the earth's horizontal component of that place is 40 μWbm^{-2} then the angle of decimation at that plane will be

(1) 30° (2) 45°

(3) 60° (4) 90°

19. Two bar magnets of the same mass, same length and breadth but having magnetic moments *M* and 2*M* are joined together pole for pole and suspended by a string. The time period of assembly in a magnetic field of strength *H* is 3 seconds. If now the polarity of one of the magnets is reversed and the combination is again made to oscillate in the same field, the time of oscillation is

(1) $\sqrt{3}$ (2) $3\sqrt{3}$

(3) 3 (4) 6

20. A magnetic needle vibrates in a vertical plane parallel to the magnetic meridian about the horizontal axis. Its frequency is *n*. If the plane of oscillation is turned about a vertical axis by 90°, the frequency of oscillation will be .

(1) *n* (2) zero

(3) $<n$ (4) $>n$

21. A magnet needle makes 4 vibrations per second at a place where $H = 3.5 \times 10^{-5}$ T. What is the value of *H* at place, where the same needle makes 3 vibrations per second?

(1) 1.98×10^{-5} T (2) 3.5×10^{-5} T

(3) 4.0×10^{-5} T (4) 3.96×10^{-5} T

22. Two bar magnets when placed with their similar poles together makes 20 vib/min. When one of them is reversed the number of vibrations become 15 vib/min. What is the ratio of their dipole moment?

(1) 7 : 45 (2) 25 : 7

(3) 1 : 4 (4) 4 : 1

23. A magnet oscillates in earth's field with a time period *T*. If its mass is quadrupled, then its new time period will be

(1) 4*T*

(2) 2*T*

(3) *T*/2

(4) unaffected but motion is not SHM

24. A magnet is cut into four equal parts by cutting it parallel to its length. What will be the time period of each part, if the time period of the original magnet in the same field is T_0?

(1) $\dfrac{T_0}{\sqrt{2}}$ (2) $\dfrac{T_0}{2}$

(3) $\dfrac{T_0}{4}$ (4) $4T_0$

25. A magnet is suspended in such a way that it oscillates in the horizontal plane. It makes 20 oscillations per minute at a plane where dip angle is 30° and 15 oscillations per minute at a place where dip angle is 60°. The ratio of earth's magnetic field at two places is

(1) $3\sqrt{3} : 8$ (2) $16 : 9\sqrt{3}$

(3) 4 : 9 (4) $2\sqrt{2} : 3$

26. Which of the following relation is correct in magnetism?

(1) $I^2 = V^2 + H^2$ (2) $I = V + H$

(3) $V = I^2 + H^2$ (4) $V^2 = I + H^2$

27. At a certain place, the horizontal component B_0 and the vertical component V_0 of the earth's magnetic field are equal in magnitude. The total intensity at the place will be

(1) B_0 (2) B_0^2

(3) $2B_0$ (4) $\sqrt{2}B_0$

28. A dip circle lies initially in the magnetic meridian. If it is now rotated through angle *θ* in the horizontal plane, then tangent of the angle of dip is changed in the ratio:

(1) 1 : cos *θ* (2) cos *θ* : 1

(3) 1 : sin *θ* (4) sin *θ* : 1

29. A current carrying coil is placed with its axis perpendicular to N-S direction. Let horizontal component of earth's magnetic field be H_0 and magnetic field inside the loop is *H*. If a magnet is suspended inside the loop, it makes angle *θ* with *H*. Then $\theta =$

(1) $\tan^{-1}\left(\dfrac{H_0}{H}\right)$ (2) $\tan^{-1}\left(\dfrac{H}{H_0}\right)$

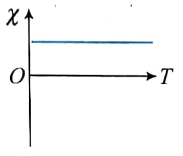
(3) $\operatorname{cosec}^{-1}\left(\dfrac{H}{H_0}\right)$ (4) $\cot^{-1}\left(\dfrac{H_0}{H}\right)$

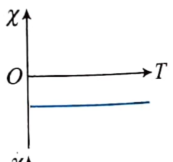
30. If a magnet is suspended at an angle 30° to the magnetic meridian, it makes an angle of 45° with the horizontal. The real dip is

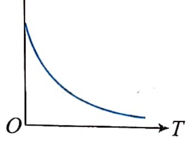
(1) $\tan^{-1}(\sqrt{3}/2)$ (2) $\tan^{-1}(\sqrt{3})$

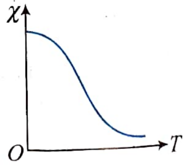
(3) $\tan^{-1}(\sqrt{3/2})$ (4) $\tan^{-1}(2/\sqrt{3})$

31. The variation of magnetic susceptibility (*χ*) with temperature for a diamagnetic substance is best represented by

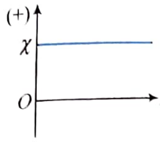
(1)


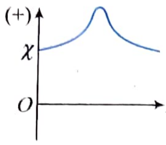
(2)


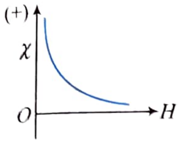
(3)


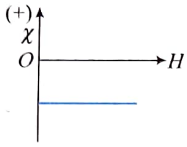
(4)


32. The variation of magnetic susceptibility (χ) with magnetising field for a paramagnetic substance is

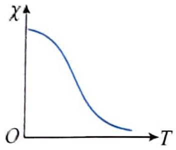
(1)


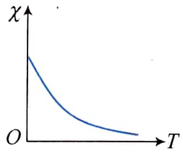
(2)


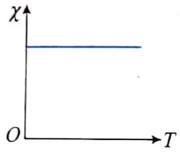
(3)


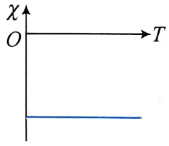
(4)


33. The variation of magnetic susceptibility (χ) with absolute temperature T for a ferromagnetic material is

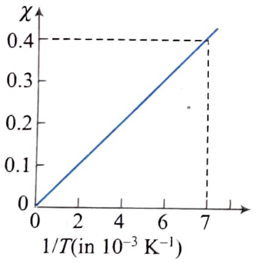
(1)


(2)


(3)


(4)


34. The χ–$1/T$ graph for an alloy of paramagnetic nature is shown in the figure. The curie constant is, then



(1) 57 K
(2) 2.8×10^{-3} K
(3) 570 K
(4) 17.5×10^{-3} K

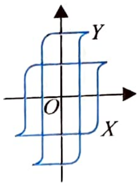
35. Which of the following is true?
(1) Diamagnetism is temperature dependent
(2) Paramagnetic is temperature dependent
(3) Paramagnetic is temperature independent
(4) None of these

36. If a magnetic substance is kept in a magnetic field, then which of the following is thrown out?
(1) Paramagnetic
(2) Ferromagnetic
(3) Diamagnetic
(4) Antiferromagnetic

37. A paramagnetic material is kept in a magnetic field. The field is increased till the magnetisation becomes constant. If the temperature is now decreased, the magnetisation
(1) will increase
(2) decreases
(3) remain constant
(4) may increase or decrease

38. The susceptibility of a ferromagnetic material is K at 27°C. At what temperature will its susceptibility be $0.5\,K$?
(1) 54°C
(2) 327°C
(3) 600°C
(4) 237°C

39. Two ferromagnetic materials X and Y have hysteresis curves of the shapes shown (see figure)

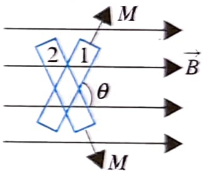


(1) X and Y are suitable for making a permanent magnet as well as an electromagnet.
(2) X is more suitable for making a permanent magnet while Y is more suitable for making an electromagnet.
(3) X is more suitable for making an electromagnet while Y is more suitable for making a permanent magnet.
(4) neither X nor Y is suitable for making either a permanent magnet or an electromagnet.

40. A bar magnet is suspended horizontally by a torsionless wire in the magnetic meridian. In order to deflect the magnet through 30° from the magnetic meridian, the upper end of the wire has to be rotated by 180°. Now this magnet is replaced by another magnet. If it has to be deflected by 30° from the magnetic meridian, upper end of the wire has to be rotated by 270°. Ratio of the magnetic moments of two magnets is

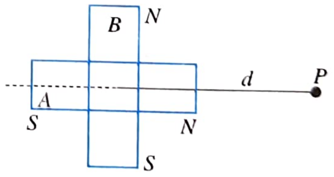
(1) $\dfrac{8}{5}$
(2) $\dfrac{7}{6}$
(3) $\dfrac{6}{7}$
(4) $\dfrac{5}{8}$

41. M and $M\sqrt{3}$ are the magnetic dipole moments of the two magnets which are joined to form a cross figure. The inclination of the system with the field, if their combination is suspended freely in a uniform external magnetic field B is



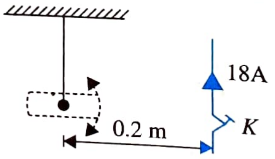
(1) $\theta = 30°$
(2) $\theta = 45°$
(3) $\theta = 60°$
(4) $\theta = 15°$

42. The magnetic induction at P, for the arrangement shown in the figure, when two similar short magnets of magnetic moment M are joined at the middle so that they are mutually perpendicular will be



(1) $\dfrac{\mu_0}{4\pi}\dfrac{M\sqrt{3}}{d^3}$
(2) $\dfrac{\mu_0}{4\pi}\dfrac{3M}{d^3}$
(3) $\dfrac{\mu_0 M\sqrt{5}}{4\pi d^3}$
(4) $\dfrac{\mu_0 2M}{4\pi d^3}$

43. Figure shows a short magnet executing small oscillations in a vibration magnetometer in earth's magnetic field having horizontal component 24 μT. The period of oscillation is 0.1 s. When the key K is closed, an upward current of 18 A is established as shown. The new time period is



(1) 0.1 s
(2) 0.2 s
(3) 0.3 s
(4) 0.4 s

44. A bar of mass M is suspended by two wires. Assume that a uniform magnetic field B is directed into the page. When the current through the bar is I, then the tension in each supporting wire is

(1) $\frac{Mg}{2}$ (2) $2BIL$

(3) $Mg - BIL$ (4) $\frac{Mg - BIL}{2}$

45. A coil in the shape of an equilateral triangle of side 0.02 m is suspended from its vertex such that it is hanging in a vertical plane between the pole pieces of permanent magnet producing a uniform field of 5×10^{-2} T. If a current of 0.1 A is passed through the coil, what is the couple acting?

(1) $5\sqrt{3}\times10^{-7}$ N-m (2) $5\sqrt{3}\times10^{-10}$ N-m

(3) $\frac{\sqrt{3}}{5}\times10^{-7}$ N-m (4) None of these

46. A vibration magnetometer consists of two identical bar magnets placed one over the other such that they are mutually perpendicular and bisect each other. The time period of combination is 4 s. If one of the magnets is removed, find the period of other

(1) 5 s (2) 3.36 s

(3) 4.36 s (4) 5.36 s

47. A short bar magnet of magnetic moment 255 JT^{-1} is placed with its axis perpendicular to earth's field direction. At what distance from the centre of the magnet, the resultant field is inclined at 45° with earth's field, $H = 0.4 \times 10^{-4}$ T.

(1) 5 m (2) 0.5 m

(3) 2.5 m (4) 0.25 m

48. A magnetic dipole is under the effect of two magnetic fields inclined at 75° to each other. One of the fields has a magnitude of 1.5×10^{-2} T. The magnets come to stable position at an angle of 30° with the direction of the above field. The magnitude of other field is

(1) $\frac{1.5}{2\sqrt{2}}\times10^{-2}$T (2) $\frac{1.5}{\sqrt{2}}\times10^{-2}$T

(3) $1.5\times\sqrt{2}\times10^{-2}$T (4) 1.5×10^{-2}T

49. The dip at a place is δ. For measuring it, the axis of the dip needle is perpendicular to the magnetic meridian. If the axis of the dip needle makes angle θ with the magnetic meridian, the apparent dip will be given tan δ_1 which is equal to

(1) tan δ cos θ (2) tan δ sec θ

(3) tan δ sin θ (4) tan δ cosec θ

50. The dipole moment of each molecule of a paramagnetic gas is 1.5×10^{-23} A × m^2. The temperature of gas is 27°C and the number of molecules per unit volume in it is 2×10^{26} m^{-3}. The maximum possible intensity of magnetisation in the gas will be

(1) 3×10^3 A/m (2) 4×10^{-3} A/m

(3) 5×10^5 A/m (4) 6×10^{-4} A/m

51. Each atom of an iron bar (5 cm × 1 cm × 1 cm) has a magnetic moment 1.8×10^{-23} A m^2, knowing that the density of iron is 7.78×10^3 kg^{-3} m, atomic weight is 56 and Avogadro's number is 6.02×10^{23}, the magnetic moment of bar in the state of magnetic saturation will be

(1) 4.75 Am^2 (2) 5.74 Am^2

(3) 7.54 Am^2 (4) 75.4 Am^2

52. An iron rod of volume 10^{-4} m^3 and relative permeability 1000 is placed inside a long solenoid wound with 5 turns/cm. If a current of 0.5 A is passed through the solenoid, then the magnetic moment of the rod is

(1) 10 Am^2 (2) 15 Am^2

(3) 20 Am^2 (4) 25 Am^2

53. A bar magnet has coercivity 4×10^3 Am^{-1}. It is desired to demagnetise it by inserting it inside a solenoid 12 cm long and having 60 turns. The current that should be sent through the solenoid is

(1) 2 A (2) 4 A

(3) 6 A (4) 8 A

54. The magnetic moment produced in a substance of 1 g is 6×10^{-7} ampere-metre^2. If its density is 5 g/cm^3, then the intensity of magnetisation in A/m will be

(1) 8.3×10^6 (2) 3.0

(3) 1.2×10^{-7} (4) 3×10^{-6}

55. The area of hysteresis loop of a material is equivalent to 250 joule. When 10 kg material is magnetised by an alternating field of 50 Hz then energy lost in one hour will be if the density of material is 7.5 g/cm^3

(1) 6×10^4 J (2) 6×10^4 erg

(3) 3×10^2 J (4) 3×10^2 erg

56. The magnet of vibration magnetometer is heated so as to reduce its magnetic moment by 36%. By doing this the periodic time of the magnetometer will

(1) increases by 36% (2) increases by 25%

(3) decreases by 25% (4) decreases by 64%

57. A magnetic needle lying parallel to a magnetic field requires W units of work to turn it through 60°. The torque required to maintain the needle in this position will be

(1) $\sqrt{3}W$ (2) W

(3) $\frac{\sqrt{3}}{2}W$ (4) $2W$

58. Two similar bar magnets P and Q, each of magnetic moment M, are taken. If P is cut along its axial line and Q is cut along its equatorial line, all the four pieces obtained have

(1) equal pole strength (2) magnetic moment $M/4$

(3) magnetic moment $M/2$ (4) magnetic moment M

59. A bar magnet is held perpendicular to a uniform magnetic field. If the couple acting on the magnet is to be halved by rotating it, then the angle by which it is to be rotated is

(1) 30° (2) 45°

(3) 60° (4) 90°

60. A current carrying coil is placed with its axis perpendicular to N-S direction. Let horizontal component of earth's magnetic field be H_0 and magnetic field inside the loop is H. If a magnet is suspended inside the loop, it makes angle θ with H. Then $\theta =$

(1) $\tan^{-1}\left(\frac{H_0}{H}\right)$ (2) $\tan^{-1}\left(\frac{H}{H_0}\right)$

(3) $\text{cosec}^{-1}\left(\frac{H}{H_0}\right)$ (4) $\cot^{-1}\left(\frac{H_0}{H}\right)$

61. A small rod of bismuth is suspended freely between the poles of a strong electromagnet. It is found to arrange itself at right angles to the magnetic field. This observation establishes that bismuth is

(1) diamagnetic (2) paramagnetic
(3) ferri-magnetic (4) antiferro-magnetic

62. Two identical magnetic dipoles of magnetic moments 1.0 A-m^2 each, placed at a separation of 2 m with their axis perpendicular to each other. The resultant magnetic field at a point midway between the dipoles is

(1) 5×10^{-7} T (2) $\sqrt{5} \times 10^{-7}$ T
(3) 10^{-7} T (4) None of these

63. Two identical short bar magnets, each having magnetic moment M, are placed a distance of $2d$ apart with axes perpendicular to each other in a horizontal plane. The magnetic induction at a point midway between them is

(1) $\frac{\mu_0}{4\pi}(\sqrt{2})\frac{M}{d^3}$ (2) $\frac{\mu_0}{4\pi}(\sqrt{3})\frac{M}{d^3}$

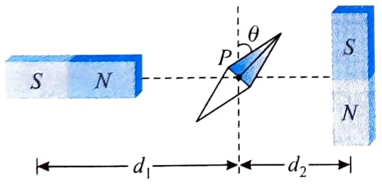
(3) $\left(\frac{2\mu_0}{\pi}\right)\frac{M}{d^3}$ (4) $\frac{\mu_0}{4\pi}(\sqrt{5})\frac{M}{d^3}$

64. A short bar magnet with its north pole facing north forms a neutral point at P in the horizontal plane. If the magnet is rotated by 90° in the horizontal plane, the net magnetic induction at P is (Horizontal component of earth's magnetic field = B_H)

(1) 0 (2) $2\,B_H$

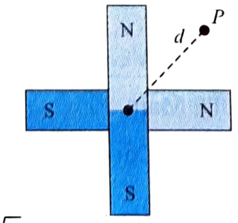
(3) $\frac{\sqrt{5}}{2}B_H$ (4) $\sqrt{5}\,B_H$

65. Two magnets A and B are identical and these are arranged as shown in the figure. Their length is negligible in comparison to the separation between them. A magnetic needle is placed between the magnets at point P which gets deflected through an angle θ under the influence of magnets. The ratio of distance d_1 and d_2 will be



(1) $(2 \tan \theta)^{1/3}$ (2) $(2 \tan \theta)^{-1/3}$
(3) $(2 \cot \theta)^{1/3}$ (4) $(2 \cot \theta)^{-1/3}$

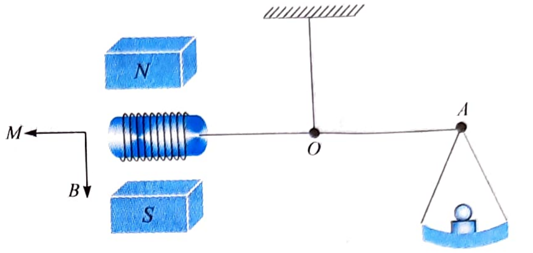
66. Two short magnets of equal dipole moments M are fastened perpendicularly at their centre (see figure). The magnitude of the magnetic field at a distance d from the centre on the bisector of the right angle is



(1) $\frac{\mu_0}{4\pi}\frac{M}{d^3}$ (2) $\frac{\mu_0}{4\pi}\frac{M\sqrt{2}}{d^3}$

(3) $\frac{\mu_0}{4\pi}\frac{2\sqrt{2}M}{d^3}$ (4) $\frac{\mu_0}{4\pi}\frac{2M}{d^3}$

67. A small coil C with $N = 200$ turns is mounted on one end of a balance beam and introduced between the poles of an electromagnet as shown in the figure. The cross-sectional area of coil is $A = 1.0$ cm^2, length of arm OA of the balance beam is $l = 30$ cm. When there is no current in the coil the balance is in equilibrium. On passing a current $I = 22$ mA through the coil the equilibrium is restored by putting the additional counter weight of mass $\Delta m = 60$ *mg* on the balance pan. Find the magnetic induction at the spot where coil is located.



(1) 0.4 T (2) 0.3 T
(3) 0.2 T (4) 0.1 T

68. If ϕ_1 and ϕ_2 be the angles of dip observed in two vertical planes at right angles to each other and ϕ be the true angle of dip, then

(1) $\cos^2 \phi = \cos^2 \phi_1 + \cos^2 \phi_2$
(2) $\sec^2 \phi = \sec^2 \phi_1 + \sec^2 \phi_2$
(3) $\tan^2 \phi = \tan^2 \phi_1 + \tan^2 \phi_2$
(4) $\cot^2 \phi = \cot^2 \phi_1 + \cot^2 \phi_2$

69. A dip needle lies initially in the magnetic meridian when it shows an angle of dip θ at a place. The dip circle is rotated through an angle x in the horizontal plane and then it shows an angle of dip θ'. Then $\frac{\tan \theta'}{\tan \theta}$ is

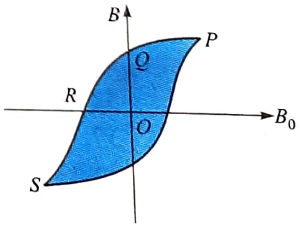
(1) $\frac{1}{\cos x}$ (2) $\frac{1}{\sin x}$

(3) $\frac{1}{\tan x}$ (4) $\cos x$

70. A dip circle is adjusted so that its needle moves freely in the magnetic meridian. In this position, the angle of dip is 40°. Now the dip circle is rotated so that the plane in which the needle moves makes an angle of 30° with the magnetic meridian. In this position the needle will dip by an angle

(1) 40° (2) 30°
(3) More than 40° (4) Less than 40°

71. The figure illustrates how B, the flux density, inside a sample of unmagnetised ferromagnetic material varies with B_0, the magnetic flux density in which the sample is kept. For the sample to be suitable for making a permanent magnet



(1) OQ should be large, OR should be small
(2) OQ and OR should both be large
(3) OQ should be small and OR should be large
(4) OQ and OR should both be small

Archives

JEE MAIN

Single Correct Answer Type

1. This question has Statement I and Statement II. Of the four choices given after the statements, choose the one that best describes the two statements.

   Statement I: Higher the range, greater is the resistance of ammeter.

   Statement II: To increase the range of ammeter, additional shunt needs to be used across it.

   (1) Statement I is true, statement II is true, statement II is not the correct explanation of statement I.
   (2) Statement I is true but statement II is false.
   (3) Statement I is false but statement II is true.
   (4) Statement I is true but statement II is true, statement II is the correct explanation of statement I

   (JEE Main 2013)

2. Two short bar magnets of length 1 cm each have magnetic moments 1.20 Am^2 and 1.00 Am^2, respectively. They are placed on a horizontal table parallel to each other with their N poles pointing towards the south. They have a common magnetic equator and are separated by a distance of 20.0 cm. The value of the resultant horizontal magnetic induction at the mid-point O of the line joining their centres is close to (horizontal component of earth's magnetic induction is 3.6×10^{-5} Wb/m^2)

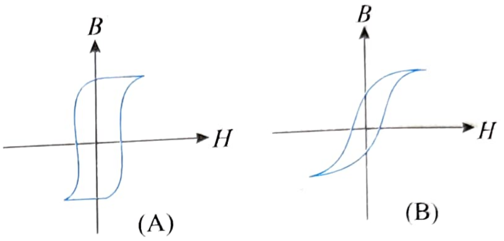
   (1) 2.56×10^{-4} Wb/m^2 (2) 3.50×10^{-4} Wb/m^2
   (3) 5.80×10^{-4} Wb/m^2 (4) 3.60×10^{-5} Wb/m^2

   (JEE Main 2013)

3. The coercivity of a small magnet where the ferromagnet gets demagnetized is 3×10^3 Am^{-1}. The current required to be passed in a solenoid of length 10 cm and number of turns 100, so that the magnet gets demagnetized when inside the solenoid, is:

   (1) 3 A (2) 6 A
   (3) 30 mA (4) 60 mA **(JEE Main 2014)**

4. Hysteresis loops for two magnetic materials A and B are given below:



   These materials are used to make magnets for electric generators, transformer core and electromagnet core. Then it is proper to use

   (1) A for electric generators and transformers
   (2) A for electromagnets and B for electric generators
   (3) A for transformers and B for electric generators
   (4) B for electromagnets and transformers

   (JEE Main 2016)

5. A magnetic needle of magnetic moment 6.7×10^{-2} Am^2 and moment of inertia 7.5×10^{-6} kg m^2 is performing simple harmonic oscillations in a magnetic field of 0.01 T. Time taken for 10 complete oscillations is

   (1) 6.98 s (2) 8.76 s
   (3) 6.65 s (4) 8.89 s **(JEE Main 2017)**

Answers Key

EXERCISES

Single Correct Answer Type

1. (3) 2. (4) 3. (4) 4. (1) 5. (3)
6. (2) 7. (2) 8. (3) 9. (3) 10. (3)
11. (4) 12. (2) 13. (1) 14. (4) 15. (4)
16. (2) 17. (1) 18. (1) 19. (2) 20. (3)
21. (1) 22. (2) 23. (2) 24. (1) 25. (2)
26. (1) 27. (4) 28. (1) 29. (1) 30. (1)
31. (2) 32. (1) 33. (1) 34. (1) 35. (2)
36. (3) 37. (3) 38. (2) 39. (2) 40. (4)
41. (3) 42. (3) 43. (2) 44. (4) 45. (1)
46. (2) 47. (2) 48. (2) 49. (2) 50. (1)
51. (3) 52. (4) 53. (4) 54. (2) 55. (1)
56. (2) 57. (1) 58. (3) 59. (3) 60. (1)
61. (1) 62. (2) 63. (4) 64. (4) 65. (3)
66. (3) 67. (1) 68. (4) 69. (1) 70. (3)
71. (2)

ARCHIVES

JEE Main

Single Correct Answer Type

1. (3) 2. (1) 3. (1) 4. (4) 5. (3)

4

Electromagnetic Induction

INTRODUCTION

From previous chapters, we know that current produces magnetic field. Is reverse possible i.e., can magnetic field produce electric current? The answer is "yes". It is found that currents were induced in closed coils when subjected to changing magnetic fields.

The phenomenon in which electric current is generated by changing magnetic fields is known as electromagnetic induction. The current so produced is known as **induced current**.

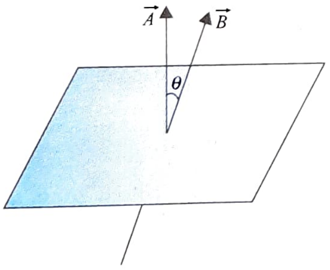
If current is produced in the circuit, this must be due to some emf produced in the circuit. This emf produced as a result of change in magnetic field is known as **induced emf**.

The phenomenon of electromagnetic induction (EMI) is the basis of the working of power generators, dynamos, transformers, etc.

MAGNETIC FLUX

Magnetic flux through any surface held in a magnetic field is measured as the total number of magnetic field lines crossing the surface. It is a scalar quantity.

Consider a plane area A placed in a magnetic field B as shown in figure. Area vector $\vec{A}$ makes an angle θ with the direction of magnetic field.



Now the magnetic flux passing through the area is defined as

$$\phi = \vec{B} \cdot \vec{A} \qquad \text{[for uniform } \vec{B}\text{]}$$

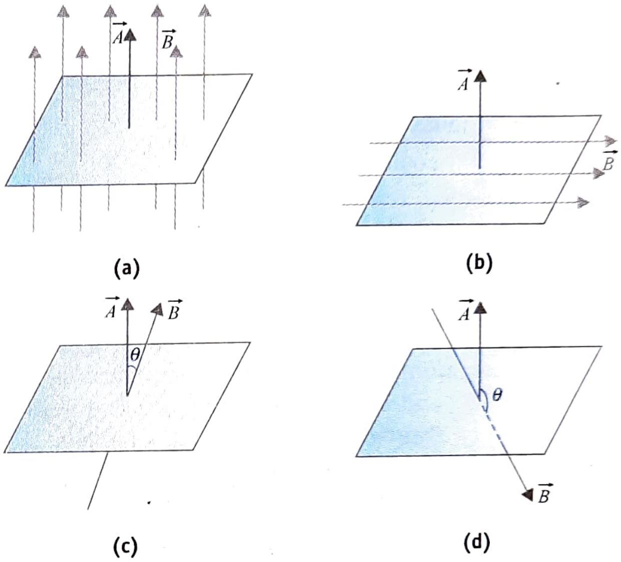
or $\quad \phi = BA\cos\theta = (B\cos\theta)\,A = B_{\perp} A$

where $B_{\perp} = B\cos\theta$ is the component of the magnetic field B perpendicular to the face of the area. Direction of area vector is normal to the face of the area.

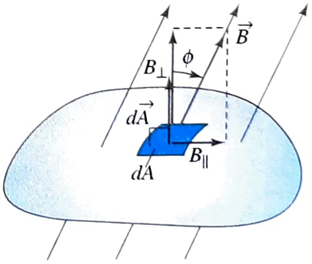
Special cases:

1. if $\theta = 0°$, then $\phi = BA\cos 0° \Rightarrow \phi = BA$ (maximum flux)
2. if $\theta = 90°$, then $\phi = BA\cos 90° \Rightarrow \phi = 0$
3. if $\theta < 90°$, then $\cos\theta > 0 \Rightarrow \phi > 0$ (positive flux)
4. if $\theta > 90°$, then $\cos\theta < 0 \Rightarrow \phi < 0$ (negative flux)

If the surface is not plane, we can divide any surface into elements of area dA (as shown in figure). For each element we determine $B_{\perp}$, the component of normal to the surface at the position of that element, as shown. From figure, $B_{\perp} = B\cos\phi$, where ϕ is the angle between the direction of B and a line perpendicular to the surface. In general, this component varies from point to point on the surface.



(a) (b) (c) (d)



We define the magnetic flux $d\Phi_B$ through the area element dA as

$$d\phi = B_{\perp}\, dA$$

The total magnetic flux through the surface is the sum of the contributions from the individual area elements:

$$\phi = \int B_{\perp}\, dA = \int B\cos\phi\, dA = \int \vec{B} \cdot d\vec{A}$$

(magnetic flux through a surface)

The unit of magnetic flux is tesla-meter^2 which is called weber (Wb) in honour of Wilhelm Weber. 1 Wb = 1 T m^2. Clearly, B can be measured in Wb m^{-2}. 1 Wb m^{-2} = 1 T. Sometimes it is referred to as flux density.

FLUX LINKAGE

Flux linkage is the linking of the magnetic field with the conductors of a coil when the magnetic field passes through the loops of the coil, expressed as a value.The flux linkage of a coil is simply an alternative term for total flux. If a coil has more than one turn, then the flux through the whole coil is the sum of the fluxes through the individual turns. If the magnetic field is uniform, the flux through one turn is $\phi = BA\cos\theta$ If the coil has N turns, the total flux linkage $\phi = NBA\cos\theta$

Magnetic lines of force are imaginary, magnetic flux is a real scalar physical quantity with dimensions

$$[\phi] = B \times \text{area} = \left[\frac{F}{IL}\right][L^2] \;\because B = \frac{F}{IL\sin\theta}$$

$$[\because F = BIL\sin\theta]$$

$$\therefore \quad [\phi] = \left[\frac{MLT^2}{AL}\right][L^2] = [ML^2T^{-2}A^{-1}]$$

SI unit of magnetic flux:

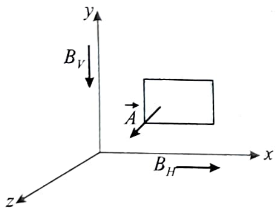
$\because$ $[ML^2T^{-2}]$ corresponds to energy

$$\frac{\text{joule}}{\text{ampere}} = \frac{\text{joule} \times \text{second}}{\text{coulomb}} = \text{weber(Wb)}$$

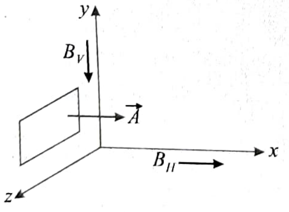
or $\quad$ $\text{T-m}^2 (\text{as tasla} = \text{Wb/m}^2)\left[\text{ampere} = \frac{\text{coulomb}}{\text{second}}\right]$

ILLUSTRATION 4.1

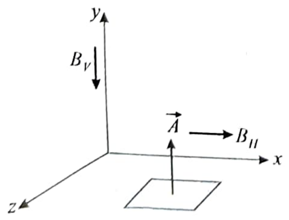
At a given place, horizontal and vertical components of earth's magnetic field B_H and B_V are along positive x and negative y axes respectively as shown in figure. What is the total flux of earth's magnetic field associated with an area A, if the area A is in **(a)** x-y plane **(b)** y-z plane and **(c)** z-x plane?



(a)



(b)



(c)

Sol. The magnetic field in given place, $\vec{B} = B_H(\hat{i}) - B_V(\hat{j})$ which is constant.

The flux associated with a closed surface in a constant magnetic field

$$\phi = \vec{B}.\vec{A} = BA\cos\theta$$

(a) In case (a), the area is in x-y plane $\vec{A} = A\hat{k}$,

$$\phi_{xy} = (B_H\hat{i} - B_V\hat{j}).(A\hat{k}) = 0$$

(b) In case (b), the area is in y-z plane $\vec{A} = A\hat{i}$,

$$\phi_{yz} = (B_H\hat{i} - B_V\hat{j}).(A\hat{i}) = B_H A$$

(c) In case (b), the area is in z-x plane $\vec{A} = A\hat{j}$,

$$\phi_{zx} = (B_H\hat{i} - B_V\hat{j}).(A\hat{j}) = -B_V A$$

ILLUSTRATION 4.2

A long solenoid with radius 2.0 cm carries a current of 5.0 A. The solenoid is 1.0 m long and is composed of 1000 turns of wire. Assuming ideal solenoid model, calculate the flux linked with a circular surface if

(a) it has a radius of 1 cm and is perpendicular to the axis of the solenoid:

(i) inside, **(ii)** outside.

(b) it has a radius of 3 cm and is perpendicular to the axis of the solenoid with its center lying on the axis of the solenoid.

(c) it has a radius greater than 2 cm and axis of the solenoid subtends an angle of 60° with the normal to the area (the center of the circular surface being on the axis of the solenoid).

(d) the plane of the circular area is parallel to the axis of the solenoid.

Sol. For an ideal solenoid magnetic field outside the solenoid is zero and inside the solenoid is constant.

Here $B_{\text{out}} = 0$ and $B_{\text{in}} = \mu_0 ni$,

n = number of turns per unit length $= \frac{N}{l} = \frac{1000}{1.0} = 1000$ turn/m

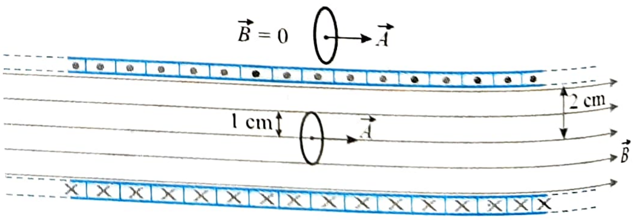
Hence $B_{\text{in}} = \mu_0 ni = (4\pi \times 10^{-7}) \times 1000 \times 5 = 2\pi \times 10^{-3}$ T

The magnetic flux in case of constant field is given by

$$\phi = \vec{B} \cdot \vec{A} = BA\cos\theta \qquad \text{...(i)}$$

θ is the angle between $\vec{B}$ and $\vec{A}$.

(a)



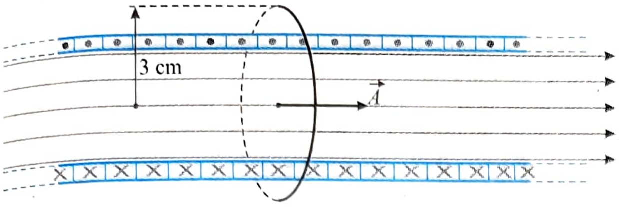
(i) In this case angle between $\vec{A}$ and $\vec{B}$ is zero.

$$\phi = BA\cos\theta = (2\pi \times 10^{-3}) \times \pi \times \left(\frac{1}{100}\right)^2 \times \cos 0°$$

$$= 2\pi^2 \times 10^{-7} \text{ Wb}$$

(ii) The magnetic field outside the solenoid is zero, hence the flux linked with a circular ring should be zero.

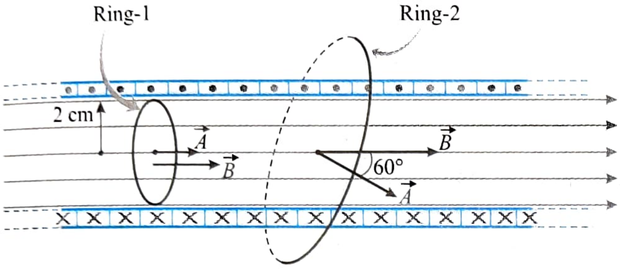
(b) No flux should be linked with the area outside the solenoid. The flux associated should be with the area common to solenoid and ring.



Hence $\phi = BA\cos\theta = (2\pi \times 10^{-3}) \times \pi \times \left(\frac{2}{100}\right)^2 \times \cos 0°$

$\phi = 8\pi^2 \times 10^{-7}$ Wb

(c) In this case the angle between $\vec{B}$ and $\vec{A}$ is 60°.



The flux is proportional to number of magnetic field lines. The number of magnetic field lines crossing in ring-1 and ring-2 are same. Hence the flux passing through ring-2 is equal to the flux associated with ring-1.

Hence $\phi = 2\pi \times 10^{-3} \times \pi \times \left(\frac{2}{100}\right)^2 \times \cos 0° = 8\pi^2 \times 10^{-7}$ Wb

(d) In this case the angle between $\vec{A}$ and $\vec{B}$ is 90°. Hence the flux associated with the ring should be zero.

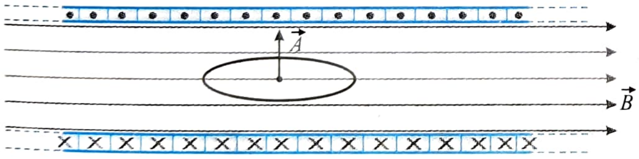
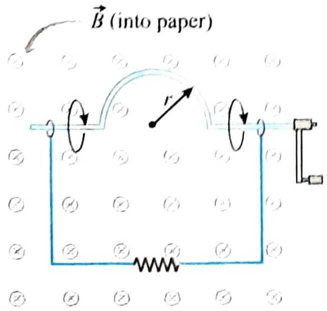


ILLUSTRATION 4.3

A closed loop of wire having shape as shown in the figure, the top part of the wire is bent into a semicircle of radius r. The normal to the plane of the loop is parallel to a constant magnetic field ($\theta = 0°$) of magnitude B. What is the change in the magnetic flux ($\Delta\Phi$) that passes through the loop, when starting with the position shown in the drawing, the semicircle is rotated through half a revolution?



Sol. As the handle turns, the angle between the normal to the loop and the direction of the magnetic field changes. At any given moment, the flux Φ that passes through the loop is given by $\Phi = BA\cos\theta$, where B is the magnitude of the magnetic field and A is the area of the loop.

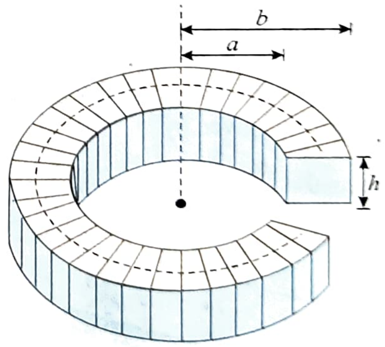
Here we can consider of the loop as being divided into a rectangular portion and a semicircular portion. Initially let A_0 be the area of the loop which is equal to the rectangular area A_{rec} plus the area $A_{\text{semi}} = \frac{1}{2}\pi r^2$ of the semicircle, where r is the radius of the semicircle: $A_0 = A_{\text{rec}} + \frac{1}{2}\pi r^2$ After half a revolution, the semicircle is once again within the plane of the loop, but now as a reduction of the area of the rectangular portion. Therefore, the final area A of the loop is equal to the area of the rectangular portion minus the area of the semicircle: $A = A_{\text{rec}} - \frac{1}{2}\pi r^2$. The angle between the normal to the loop and the direction of the magnetic field both directed into the page i.e., angle between both the vectors is zero degree ($\theta = 0°$).

$$\Delta\Phi = \Phi - \Phi_0 = BA\cos\theta - BA_0\cos\theta = B\cos\theta(A - A_0)$$

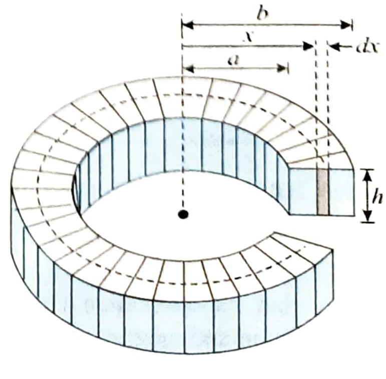
$$= B\cos 0^0\left[\left(A_{\text{rec}} - \frac{1}{2}\pi r^2\right) - \left(A_{\text{rec}} + \frac{1}{2}\pi r^2\right)\right] = -\pi r^2 B$$

ILLUSTRATION 4.4

Figure shows a toroidal solenoid whose cross-section is rectangular in shape. Find the magnetic flux through this cross-section if the current through the toroidal winding is I, total number of turns in winding is N, the inside and outside radii of the toroid are a and b respectively and the height of toroid is equal to h.



Sol. To find the magnetic flux through the cross section shown in figure, we consider an elemental strip of width dx at a distance x from the axis of toroid as shown in figure.



The magnetic field at a distance x from the axis of the toroid is given as

$$B = \mu_0 \left(\frac{N}{2\pi x} \right) I = \frac{\mu_0 NI}{2\pi x}$$

Magnetic flux through an elemental strip of radial thickness dx and height h as shown in figure is given as

$$d\phi = \vec{B} \cdot \vec{dA} = BdA$$

$$\Rightarrow \quad d\phi = \frac{\mu_0 NI}{2\pi x} = hdx$$

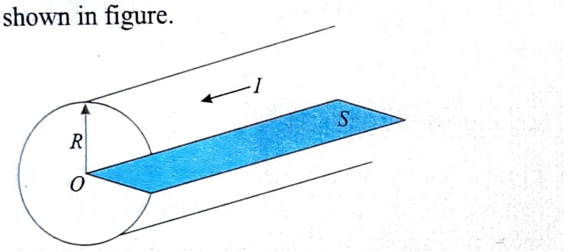
Total magnetic flux through the cross section of the toroid is given by integrating the above elemental flux within limits from a to b which is given as

$$\phi = \int_a^b \frac{\mu_0 NIh}{2\pi x} dx = \frac{\mu_0 NIh}{2\pi x} \int_a^b \frac{1}{x} dx = \frac{\mu_0 NIh}{2\pi x} [\ln x]_a^b$$

$$\phi = \frac{\mu_0 NIh}{2\pi x} [\ln b - \ln a] = \frac{\mu_0 NIh}{2\pi} \ln \left(\frac{b}{a} \right)$$

ILLUSTRATION 4.5

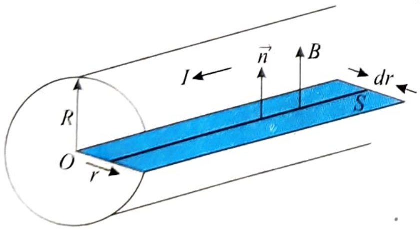
A long copper wire carries a current of I ampere. Calculate the magnetic flux per meter of the wire for a plane surface S inside the wire as shown in figure.



Sol. Consider an element of width dr at a distance r from the axis of the wire. The field due to the current I in the wire at the position of element will be $B = \dfrac{\mu_0 Ir}{2\pi R^2}$.

And as its direction is perpendicular to the plane surface, flux linked with the element is given by

$$d\phi = B\, ds \cos\theta = \frac{\mu_0}{2\pi} \frac{I}{R^2} r \ (1\, dr) \cos \theta$$



So, the flux linked with the plane surface is given by

$$\phi = \frac{\mu_0}{2\pi} \frac{I}{R^2} \int_0^R r\, dr = \frac{\mu_0 I}{4\pi}$$

WORK DONE IN CHANGING ORIENTATION OF A CURRENT CARRYING COIL IN MAGNETIC FIELD

When a current carrying coil is displaced in a uniform magnetic field in such a way that its orientation changes with angle between magnetic induction and magnetic moment of coil from θ_1 to θ_2 then the potential energy of the coil in magnetic field in initial and final state are given as

To change the orientation of coil externally we need to apply a torque against the magnetic torque of equal magnitude thus work done in changing the orientation in above situation can also be given as

$$W = \int dW = \int \tau d\theta \Rightarrow W = \int_{\theta_1}^{\theta_2} MB \sin\theta . d\theta$$

$$\Rightarrow \quad W = MB[-\cos\theta]_{\theta_1}^{\theta_2}$$

$$W = MB(\cos\theta_1 - \cos\theta_2) \quad \text{...(i)}$$

Above expression of work can be rewritten as

$$W = IAB(\cos\theta_1 - \cos\theta_2) = I(BA\cos\theta_1 - BA\cos\theta_2)$$

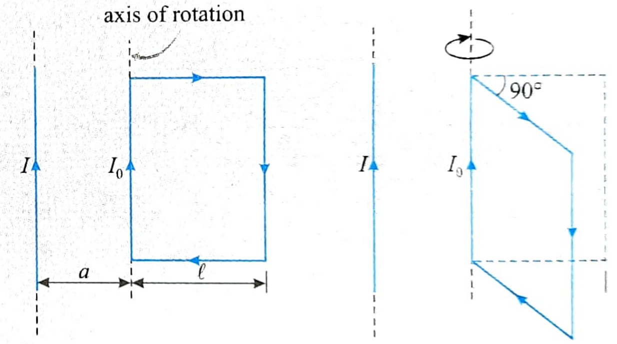
$$\Rightarrow \quad W = I(\phi_{\text{initial}} - \phi_{\text{final}})$$

$$\Rightarrow \quad W = -I(\phi_{\text{final}} - \phi_{\text{initial}})$$

$$\Rightarrow \quad W = -I\Delta\phi \quad \text{...(ii)}$$

ILLUSTRATION 4.6

A square frame carrying current I_0 is located in the same plane as a long straight wire carrying currents I [Fig. (a)]. The frame side has a length l. The axis of the frame is parallel to the wire as shown. Find



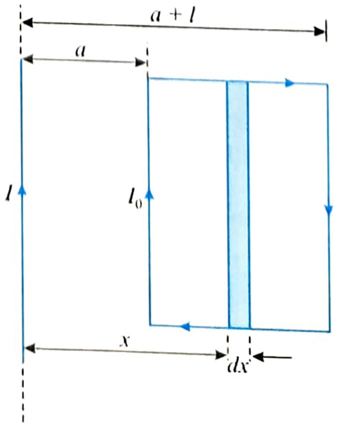
(a) (b)

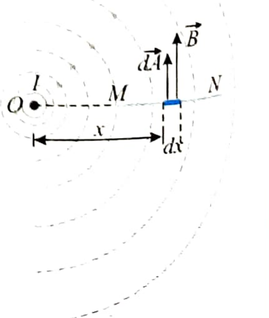
(a) the magnetic flux passing through the frame.

(b) the mechanical work to be performed to the frame through 90° about its axis [Fig. (b)].

Sol.

(a) Let us consider an elemental strip of thickness dx, at a distance x from the wire as shown. As this strip is parallel to the wire the magnitude field should be constant on this strip.





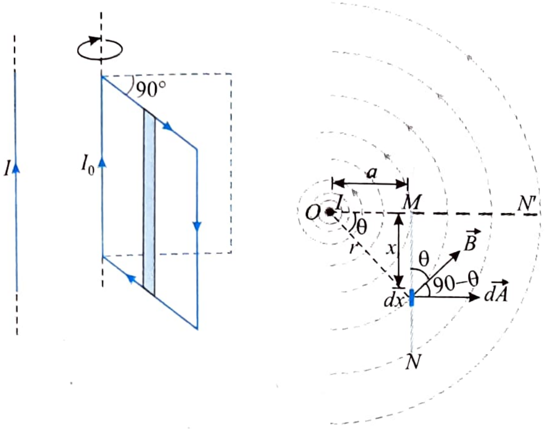
The magnetic flux through the elemental strip

$$d\phi_1 = \vec{B}\cdot d\vec{A} = B\cdot dA\cos 0^\circ \Rightarrow d\phi_1 = \frac{\mu_0 I}{2\pi x}(l\,dx)$$

Total magnetic flux through the coil

$$\phi_1 = \int d\phi = \frac{\mu_0 Il}{2\pi}\int_a^{a+l}\frac{dx}{x} = \frac{\mu_0 Il}{2\pi}\ln\frac{(a+l)}{a} \qquad \ldots\text{(i)}$$

(b) Now the coil is rotated by 90°.



In this situation the magnetic field vector will not be normal to the plane of the coil at every point. Let us again consider an elemental strip of the coil as shown in figure. In this situation the flux passing through the strip is

$$d\phi_2 = \vec{B}\cdot d\vec{A} = \left(\frac{\mu_0 I}{2\pi r}\right)(l\,dx)\cdot\cos(90^\circ - \theta)$$

$$\Rightarrow \quad d\phi_2 = \frac{\mu_0 Il}{2\pi\sqrt{a^2+x^2}}\,\frac{x}{\sqrt{a^2+x^2}}\cdot dx$$

For calculating total flux through the coil we need to integrate above equation

$$\phi_2 = \int d\phi_2 = \frac{\mu_0 Il}{2\pi}\int_a^l \frac{x\,dx}{(a^2+x^2)}$$

Let $a^2 + x^2 = t \Rightarrow 2x\,dx = dt$

where $x = 0, t = a^2$ and when $x = l, t = a^2 + t^2$

Changing the limits in above equation we get

$$\phi_2 = \frac{\mu_0 Il}{4\pi}\int_{a^2}^{a^2+l^2}\frac{dt}{t} = \frac{\mu_0 Il}{4\pi}[\ln t]_{a^2}^{a^2+l^2}$$

$$\Rightarrow \quad \phi_2 = \frac{\mu_0 Il}{4\pi}\ln\frac{(a^2+l^2)}{a^2} \qquad \ldots\text{(ii)}$$

The work done in rotation of the coil is given as

$$W = -I_0\Delta\phi = -I_0(\phi_2 - \phi_1) \qquad \ldots\text{(iii)}$$

Substituting the values of ϕ_1 and ϕ_2 from (i) and (ii), and substituting in (iii) we get

$$W = -I_0\left[\frac{\mu_0 Il}{4\pi}\ln\frac{(a^2+l^2)}{a^2} - \frac{\mu_0 Il}{2\pi}\ln\frac{(a+l)}{a}\right]$$

$$= \frac{\mu_0 II_0 l}{4\pi}\left[\ln\frac{(a^2+l^2)}{a^2} - 2\ln\frac{(a+l)}{a}\right]$$

$$\Rightarrow \quad W = \frac{\mu_0 II_0 l}{4\pi}\left[\ln\frac{(a^2+l^2)}{(a+l)^2}\right]$$

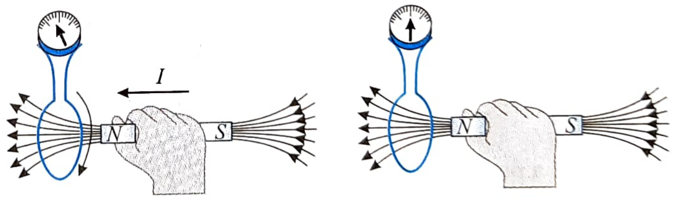
FARADAY'S LAW OF ELECTROMAGNETIC INDUCTION

British scientist Michael Faraday based on various experiment demonstrated that a changing magnetic field could generate a potential difference in a conductor, strong enough to produce an electric current. This discovery is of basic importance to all the electrical and magnetic devices we use every day, from computers to cell phones, from television to credit cards, from the tiniest batteries to the largest electrical power grid.

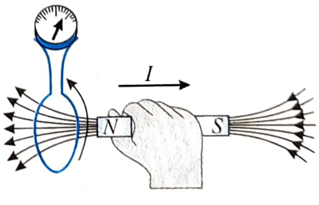
Let us examine two simple experiments to prepare for our discussion of Faraday's law of induction.

First experiment: Figure shows a conducting loop connected to a sensitive ammeter. Because there is no battery or other source of emf included, there is no current in the circuit. However, if we move a bar magnet toward the loop, a current suddenly appears in the circuitas shown in Fig. (a). The current disappears when the magnet stops as shown in Fig. (b). If we then move the magnet away, a current again suddenly appears, but now in the opposite direction as shown in Fig. (c). If we experimented for a while, we would discover the following:

1. A current appears in the loop only if there is relative motion between the loop and the magnet (one must move relative to the other); the current disappears when the relative motion between them ceases.
2. Faster motion of the magnet produces a greater current.
3. If moving the magnet's north pole toward the loop causes, say, clockwise current, then moving the north pole away causes counter clockwise current. Moving the south pole toward or away from the loop also causes currents, but in the reversed directions.



(a) (b)

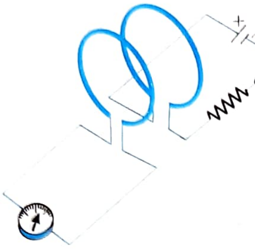


(c)

The current appeared in the loop is called an **induced current**; the work done per unit charge to produce that current (to move the conduction electrons that constitute the current) is called an **induced emf**; and the process of producing the current and emf is called **induction**.

Second Experiment: In this experiment two conducting loops are placed close to each other but not touching as shown in figure. If we close switch S, to turn on a current in the right-hand loop, the meter suddenly and briefly registers a current—

an induced current—in the left-hand loop. If we then open the switch, another sudden and brief induced current appears in the left-hand loop, but in the opposite direction. We get an induced current (and thus an induced emf) only when the current in the right-hand loop is changing (either turning on or turning off) and not when it is constant (even if it is large).



In both the experiments discussed above have one thing in common: in each case, an emf is induced in a loop when the magnetic flux through the loop changes with time. In general, this emf is directly proportional to the time rate of change of the magnetic flux through the loop. This statement can be written mathematically as **Faraday's law of induction:**

$$\varepsilon = -\frac{d\Phi_B}{dt} \quad \text{...(i)}$$

If a coil consists of N loops with the same area and Φ_B is the magnetic flux through one loop, an emf is induced in every loop. The loops are in series, so their emfs add; therefore, the total induced emf in the coil is given by

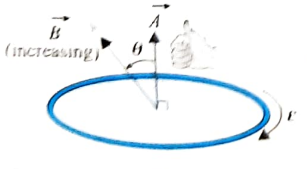
$$\varepsilon = -N\frac{d\Phi_B}{dt} \quad \text{...(ii)}$$

As such, Faraday's law in itself is complete to tell the magnitude and polarity of induced emf. But Lenz's rule is commonly used to determine the polarity of induced emf or direction of induced current.

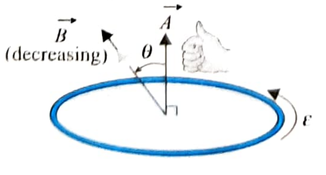
DIRECTION OF INDUCED EMF

We can find the direction of an induced emf or current by using $\varepsilon = -d\Phi_B/dt$ together with some simple sign rules. Here is the procedure:

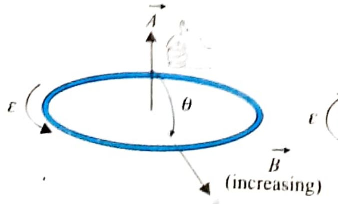
1. Define a positive direction for the area vector $\vec{A}$.
2. From the directions of $\vec{A}$ and the magnetic field $\vec{B}$, determine the sign of the magnetic flux Φ_B and its rate of change $d\Phi_B/dt$. Figure shows several examples.



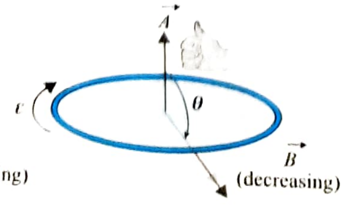
(a)



(b)



(c)



(d)

For Fig. (a):

1. $\vec{A}$ is upward so anticlockwise direction is positive.
2. $\phi = BA\cos\theta$

$$\varepsilon = -\frac{d\phi}{dt} = -\frac{d}{dt}(BA\cos\theta) \Rightarrow \varepsilon = -A\cos\theta\frac{dB}{dt} \quad \text{...(i)}$$

Here θ is acute, so cos θ is positive. B is increasing, so dB/dt is positive. Then we find that ε is negative. So ε is clockwise as shown.

For Fig. (b):

Here B is decreasing, so dB/dt is negative. Then from Eq. (i), ε is positive. So ε is anticlockwise as shown.

For Fig. (c):

Here θ is obtuse, so cos θ is negative. B is increasing, so dB/dt is positive. Then from Eq. (i), ε is positive. So ε is anticlockwise as shown.

For Fig. (d):

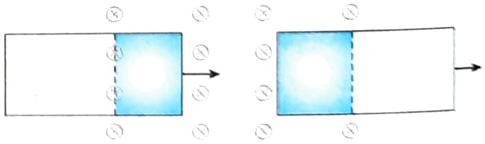
Here θ is obtuse, so cos θ is negative, B is decreasing so dB/dt is negative. Then from Eq. (i), ε is negative. So ε is clockwise as shown.

Important Points:

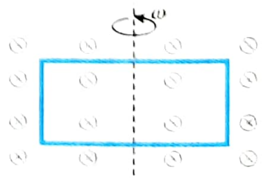
The induced emf is produced only when there is a change in magnetic flux passing through a loop. The flux passing through the loop is given by $\phi = BA\cos\theta$.

Thus, flux can be changed in several ways:

- The magnitude of $\vec{B}$ can change with time. In the problems if magnetic field is given as a function of time, it implies that the magnetic field is changing. Thus. $B = B(t)$.
- The area enclosed by the loop can change with time.
- This can be done by pulling a loop inside (or outside) a magnetic field. By doing so, the area enclosed by the loop (hatched area) can be changed.



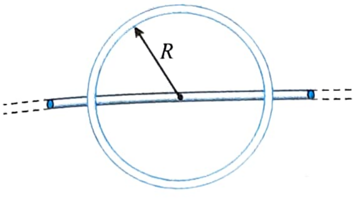
- The angle θ between $\vec{B}$ and the normal to the loop can change with time. This can be done by rotating a loop in a magnetic field.



- Any combination of the above can occur.

ILLUSTRATION 4.7

In figure shown below, a wire forms a closed circular loop, of radius R = 1.0 m and resistance 5.0 Ω. The circle is centered on a long straight wire; at time t = 0, the current in the long straight wire is 10.0 A rightward. Thereafter, the current changes according to $i = 10.0\text{ A} + (0.5\text{ A/s}^2)t^2$. (The straight wire is insulated; so there is no electrical contact between it and the wire of the loop.) What is the magnitude of the current induced in the loop at times $t > 0$?



Sol. The magnetic field due to long straight current wire is out of the page in the upper half of the circle and is into the page in the lower half of the circle, producing zero net flux, at any time. There is no induced current in the circle.

ILLUSTRATION 4.8

A coil is placed in a constant and uniform magnetic field of induction 10^{-3} Wb/m^2 acting normal to the plane of the coil. If the radius of a coil decreases steadily at the rate of 10^{-2} m/s, what will be the radius of the coil when the induced e.m.f. in the 1 μV.

Sol. According to Faraday's law of induction, the magnitude of induced emf in the coil,

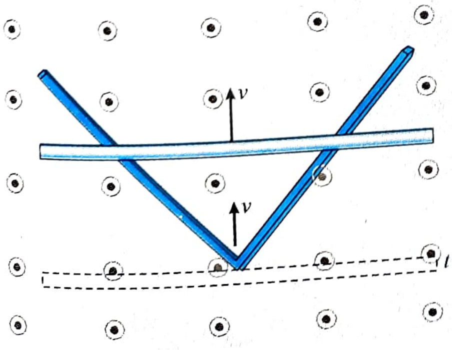
$$|\varepsilon| = \frac{d\Phi_B}{dt} = \frac{d(BA)}{dt} = \frac{Bd(\pi r^2)}{dt} = 2B\pi r\frac{dr}{dt}$$

Hence the radius of coil,

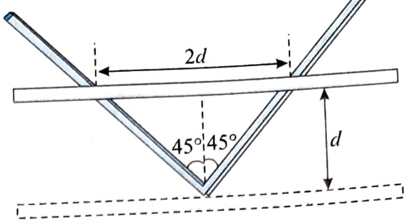
$$r = \frac{e}{2B\pi\left(\frac{dr}{dt}\right)} = \frac{1\times10^{-6}}{2\times10^{-3}\times\pi\times10^{-2}} = \frac{1}{20\pi}\text{ m} = \frac{5}{\pi}\text{ cm}$$

ILLUSTRATION 4.9

A structure formed from two straight conducting rods forming a right angle, is placed in a uniform magneticfield with $B = 0.40$ T is directed out of the page. A straight conducting bar in contact with the structure, starts at the vertex at time $t = 0$ and moves with a constant velocity of 5.0 m/s along them. Calculate the emf around the triangle at that time $t = 4.0$ s.



Sol. The perpendicular length of the triangular area enclosed by the structure and bar is the same as the distance travelled by it in time t: $d = vt$, where $v = 5.0$ m/s.



Here the "base" of that triangle (the distance between the intersection points of the bar with the structure) is $2d$. Thus, the area of the triangle is

$$A = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(2vt)(vt) = v^2t^2.$$

Since the field is a uniform $B = 0.350$ T, then the magnitude of the flux (in SI units) is

$$\Phi_B = BA = (0.40)(5.0)^2t^2 = 10.0t^2.$$

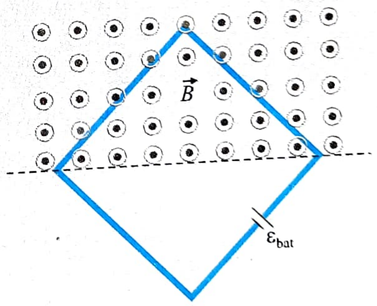
The magnitude of the emf is the (absolute value of) Faraday's law:

$$\varepsilon = \frac{d\Phi_B}{dt} = 10.0\frac{dt^2}{dt} = 20.0t$$

At $t = 4.0$ s, this yields $\varepsilon = 80.0$ V.

ILLUSTRATION 4.10

A square wire loop with side $L = 1.0$ m sides is perpendicular to a uniform magnetic field, with half the area of the loop in the field as shown in figure. The resistance of the loop is 35 Ω and the loop contains an ideal battery with emf $\varepsilon = 6.0$ V. If the magnitude of the field varies with time according to $B = 5.0 - 2.0t$, with B in teslas and t in seconds, what are **(a)** the net emf in the circuit and **(b)** the direction of the (net) current around the loop?



Sol.

(a) Let magnetic field at any time be B. Let upward direction of area vector as positive, the magnetic flux associated with the circuit is

$$\Phi_B = \frac{BL^2}{2}$$ and the induced emf is

$$\varepsilon_i = -\frac{d\Phi_B}{dt} = -\frac{L^2}{2}\frac{dB}{dt}.$$

As, $B = 5.0 - 2.0t$, hence,, $\frac{dB}{dt} = -2.0T/s$

It means the induced emf $\varepsilon_i = -\frac{(1.0)^2}{2}(-2.0) = 1.0$ V.

As ε_i is positive, so the induced emf is counter clockwise around the circuit, in the same direction as the emf of the battery. The total emf is

$$\varepsilon_{\text{total}} = \varepsilon + \varepsilon_i = 6.0\text{ V} + 1.0\text{ V} = 7.0\text{ V}$$

(b) The current is in the sense of the total emf (counter clockwise). The magnitude of the current in the loop

$$i = \frac{\varepsilon_{\text{total}}}{R} = \frac{7.0}{35.0} = 0.2\text{ A}$$

INDUCED CHARGE

When the magnetic field passing through a loop is changed, an induced emf, and hence an induced current is produced in the circuit. If R is the resistance of the circuit, then induced current is given by

$$i = \frac{e}{R} = \frac{1}{R}\left(\frac{-d\phi_B}{dt}\right)$$

Current starts flowing in the circuit, i.e., flow of charge takes place. Charge flown in the circuit in time dt will be given by

$$dq = i\,dt = \frac{1}{R}(-d\phi_B)$$

Thus, for a time interval Δt, we can write

$$\Delta q = i\Delta t = \frac{1}{R}(-\Delta\phi_B)$$

From these equations we can see that e and i are inversely proportional to Δt while Δq is independent of Δt. It depends on the magnitude of change in flux, not the time taken by it.

Important Points:

- Induced charge in any coil (or circuit) does not depends on time in which change in flux occurs i.e., it is independent from rate of change of flux or relative speed of coil–magnet system.
- Induced charge depends on change in flux through the coil and nature of the coil (or circuit) i.e. resistance.

ILLUSTRATION 4.11

A closed coil having 20 turns, area of cross-section 1 cm^2 and resistance 2 ohms are connected to a ballistic galvanometer of resistance 30 ohms. If the normal of the coil is inclined at 60° to the direction of a magnetic field of intensity 1.5 Wb/m^2, the coil is quickly pulled out of the field to a region of zero magnetic field, calculate the charge passed through the galvanometer.

Sol. The total flux linked with the coil having turns N and area A is

$$\phi_{\text{initial}} = N(\vec{B}.\vec{A}) = NBA\cos\theta = NBA\cos 60° = \frac{NBA}{2}$$

when the coil is pulled out, the flux becomes zero, $\phi_{\text{final}} = 0$

The change in flux is $|\Delta\phi| = \dfrac{NBA}{2}$

Hence the charge flow through the circuit is

$$q = \frac{|\Delta\phi|}{R} = \frac{NBA}{2R} = \frac{20\times1.5\times10^{-4}}{2\times30} = 5.0\times10^{-5}\text{ C}$$

LENZ'S LAW

Direction of the Induced Current in a Circuit

Lenz's law states that "when the magnetic flux through a loop changes, a current is induced in the loop such that the magnetic field due to the induced current opposes the change in the magnetic flux through the loop".

The above rule can be systematically applied as follows to determine the direction of the induced currents.

- Identify the loop in which the induced current is to be determined.
- Determine the direction of the magnetic field in this loop (i.e., in or out of the loop).
- The direction of flux is the same as the direction of the magnetic field. Determine if the flux through the loop is increasing or decreasing (because of change in area or change in B).

Choose the appropriate current in the loop that will oppose the change in flux.

- If the flux is into the paper and increasing then the flux due to the induced current should be out of the paper.
- If the flux is into the paper and decreasing, the flux due to the induced current should be into the paper.
- If the flux is out of the paper and increasing, the flux due to the induced current should be into the paper.
- If the flux is out of the paper and decreasing, the flux due to the induced current should be out of the paper.

The above description is the physical interpretation of Lenz's law. We can determine the direction of the induced current mathematically by simply applying Lenz's law, $\xi_{\text{ind}} = -\dfrac{d\Phi_B}{dt}$ with the appropriate conventions.

The right hand sign convention used is as follows.

- For counterclockwise current, emf is positive.
- For clockwise current, emf is negative.
- Magnetic flux out of the paper is positive.
- Magnetic flux into the plane of the paper is negative.
- The rate of change of an increasing positive flux is positive.
- The rate of change of a decreasing positive flux is negative.
- The rate of change of an increasing negative flux is negative.
- The rate of change of a decreasing negative flux is positive.

ILLUSTRATION 4.12

A circular loop of wire is placed on flat horizontal plane. An external magnetic field is directed perpendicular to the plane of the loop. The magnitude of the applied magnetic field is increasing with time. Because of this increasing magnetic field, an induced current is flowing clockwise in the loop, as viewed from above. What is the direction of the external magnetic field?

Sol. As external magnetic field is perpendicular to the plane of the horizontal loop, so it must point either upward or downward. The external magnetic field B is increasing in magnitude, it means the magnetic flux through the loop also increases with time. In order to oppose the increase in magnetic flux, the induced magnetic field B_{ind} must be directed opposite to the external magnetic field B. The drawing shows the loop as viewed from above, with an induced current I_{ind} flowing clockwise. According to Right-Hand Rule, this induced current creates an induced magnetic field B_{ind} that is directed into the page at the center of the loop (and all other points of the loop's interior). Therefore, the external magnetic field B must be directed out of the page. Because we are viewing the loop from above, "out of the page" corresponds to upward toward the viewer.

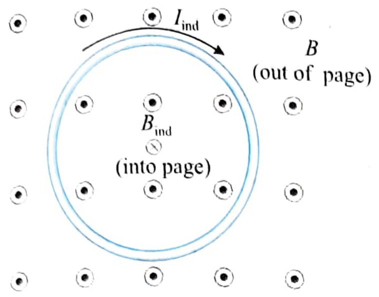
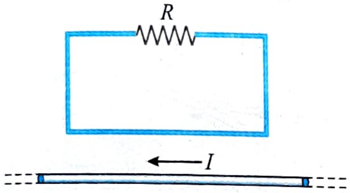


ILLUSTRATION 4.13

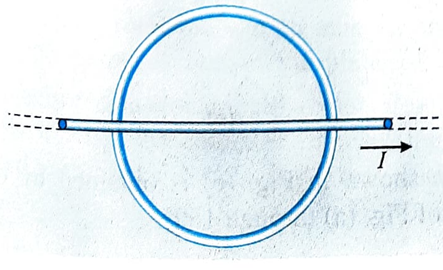
A rectangular loop that contains a resistor R is placed near a long straight wire carrying a current I. The wire and the loop are placed in same plane as shown in figure. If the current I is decreasing in time, what is the direction of the induced current through the resistor R—left-to-right or right-to-left?



Sol. At the location of the loop, the magnetic field produced by the current I is directed into the page .This can be verified by using right hand rule. The current is decreasing, so the magnetic field is decreasing. Therefore, the magnetic flux that penetrates the loop is decreasing. According to Lenz's law, the induced emf has a polarity that leads to an induced current whose direction is such that the induced magnetic field opposes this flux change. The induced magnetic field will oppose this decrease in flux by pointing into the page, in the same direction as the field produced by I. According to RHR-2, the induced current must flow clockwise around the loop in order to produce such an induced field. The current then flows from left-to-right through the resistor.

ILLUSTRATION 4.14

An insulated circular loop of wire rests upon a long, straight wire such that the center of the loop lies on the wire. Both the wire and loop are in same horizontal plane as shown in figure. The current I in the straight wire is decreasing. In what direction is the induced current, if any, in the loop?



Sol. The current I in the straight wire produces a circular pattern of magnetic field lines around the wire. The magnetic field at any point is tangent to one of these circular field lines. Thus, the field points perpendicular to the plane of the table. Furthermore, according to Right-Hand Rule, the field is directed up out of the table surface in region 1 above the wire and is directed down into the table surface in region 2 below the wire.

As the current I decreases, the magnitude of the field that it produces also decreases. Since the fields in these two regions always have opposite directions and equal magnitudes at any given radial distance from the straight wire, the flux through the regions add up to give zero for any value of the current. With the flux remaining constant as time passes, Faraday's law indicates that there is no induced emf in the coil. Since there is no induced current.

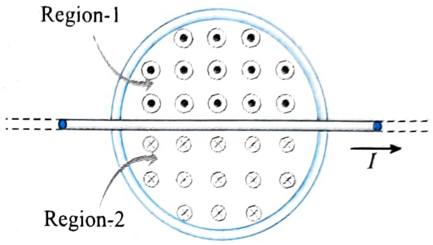
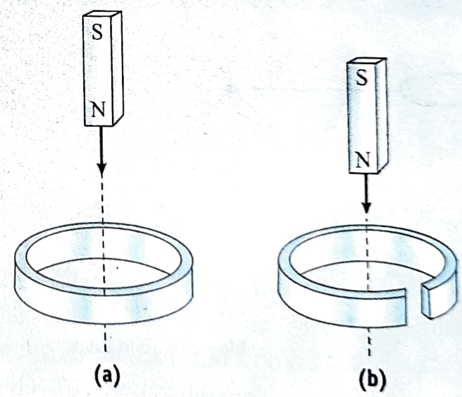


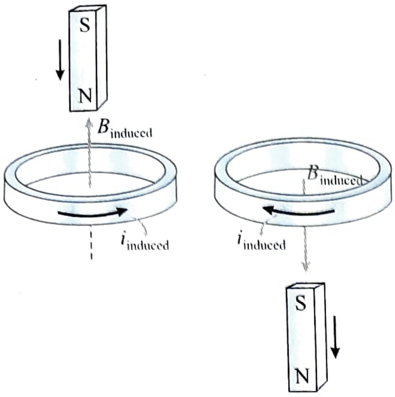
ILLUSTRATION 4.15

A bar magnet is released from rest and it starts falling through a metal ring as shown in the figure. Consider two cases, in case (I) the ring is solid all the way around, but in case (II) it has been cut through. Discuss the motion of the magnet in both the cases when it is above the ring and below the ring as well.



(a) (b)

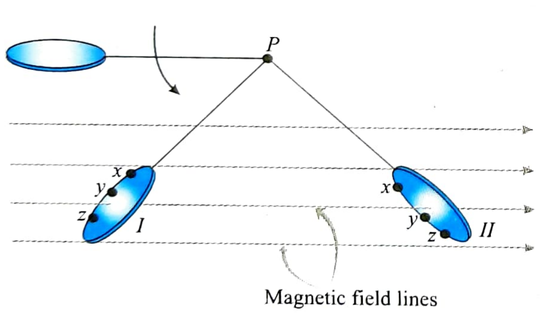
Sol. In case (I), when the magnet is above the ring its magnetic field points down through the ring and is increasing in strength as the magnet falls. According to Lenz's law, an induced magnetic field appears that attempts to reduce the increasing field. Therefore, the induced field must point up. Using Right-Hand Rule, we can see that the induced current in the ring is as shown in the drawing. Because of the induced current, the ring looks like a magnet with its north pole at the top. The north pole of the loop repels the falling magnet and retards its motion. When the magnet is below the ring, its magnetic field still points down through the ring but is decreasing as the magnet falls. According to Lenz's law, the induced magnetic field attempts to bolster the decreasing field and, therefore, must point down. The induced current in the ring is as shown in the drawing, and the ring then looks like a magnet with its north pole at the bottom, attracting the south pole of the falling magnet and retarding its motion.



In case (II), the motion of the magnet is unaffected, since no induced current can flow in the cut ring. No induced current means that no induced magnetic field can be produced to repel or attract the falling magnet.

ILLUSTRATION 4.16

A wire loop is suspended from a string that is attached to point P in the drawing. When released, the loop swings downward, from left to right, through a uniform magnetic field, with the plane of the loop remaining perpendicular to the plane of the paper at all times. Determine the direction of the current induced in the loop as it swings past the locations labeled I and II. Specify the direction of the current in terms of the points x, y, and z on the loop (e.g., $x \rightarrow y \rightarrow z$ or $z \rightarrow y \rightarrow x$). The points x, y, and z lie behind the plane of the paper.

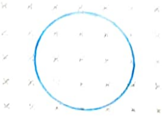


Sol. **Location I:** As the loop swings downward, the normal to the loop makes a smaller angle with the applied field. Hence, the flux through the loop is increasing. The induced magnetic field must point generally to the left to counteract this increase. The induced current flows $\boxed{x \rightarrow y \rightarrow z}$

Location II: The angle between the normal to the loop and the applied field is now increasing, so the flux through the loop is decreasing. The induced field must now be generally to the right, and the current flows $\boxed{z \rightarrow y \rightarrow x}$

ILLUSTRATION 4.17

A circular loop of radius a having n turns is kept in a horizontal plane. A uniform magnetic field B exists in a vertical direction as shown in figure. Find the emf induced in the loop if the loop is rotated with a uniform angular velocity ω about

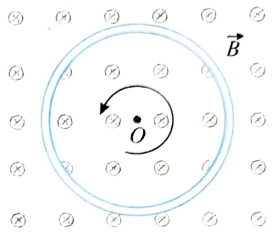


(a) an axis passing through the center and perpendicular to the plane of the loop.

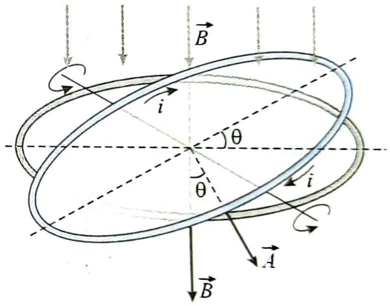
(b) the diameter.

Sol.

(a) The emf induces when there is change of flux. As in this case (figure), there is no change of flux, hence no emf will be induced in the coil.



(b) If the loop is rotated about the diameter there will be change of flux with time. In this case, emf, will be induced in the coil. The area of the loop is $A = \pi a^2$. If the normal of the loop makes an angle $\theta = 0$ with the magnetic field at $t = 0$, this angle will become $\theta = \omega t$ at time t. The flux of the magnetic field at this time is



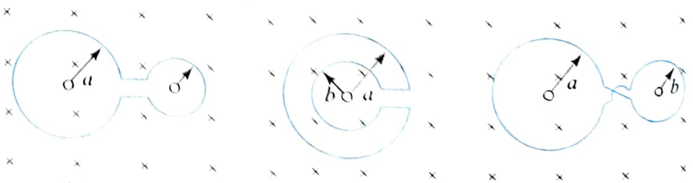
$$\phi = nB\pi a^2 \cos\theta = nB\pi a^2 \cos\omega t$$

The induced emf is $\varepsilon = -\dfrac{d\phi}{dt} = \pi n a^2 B\omega \sin\omega t$.

ε is coming out to be positive, so the direction of induced emf will be as shown in figure. Because here this sense is positive if we look at the direction of area vector.

ILLUSTRATION 4.18

Figure (a) shows two circular rings of radii a and b ($a > b$) joined together with wires of negligible resistance. Figure (b) shows the pattern obtained by folding the small loop in the plane of the large loop.



(a) (b) (c)

The pattern shown in Fig. (c) is obtained by twisting the small loop of Fig. (a) through 180°.

All the three arrangements are placed in a uniform time varying magnetic field $dB/dt = k$, perpendicular to the plane of the loops. If the resistance per unit length of the wire is λ, then determine the induced current in each case.

Sol.

(a) $\phi_B = \pi(a^2 + b^2)B$

$$|\varepsilon| = \frac{d\phi_B}{dt} = \pi(a^2 + b^2) \times \frac{dB}{dt} = \pi(a^2 + b^2)k$$

$$\text{Induced current, } I = \frac{|\varepsilon|}{R} = \frac{\pi(a^2+b^2)k}{\lambda[2\pi(a+b)]} = \frac{k(a^2+b^2)}{2\lambda(a+b)}$$

(b) $\phi_B = \pi a^2 B - \pi b^2 B = \pi B(a^2 - b^2)$

$$|\varepsilon| = \frac{d\phi_B}{dt} = \frac{\pi dB}{dt} \times (a^2 - b^2) = \pi k(a^2 - b^2)$$

$$\text{Induced current, } I = \frac{|\varepsilon|}{R} = \frac{\pi k(a^2-b^2)}{2\pi\lambda(a+b)} = \frac{k(a-b)}{2\lambda}$$

(c) Here induced emf in both loops will oppose each other, hence effective flux is given by

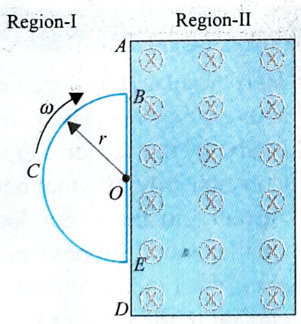
$$\Phi_B = \pi a^2 B - \pi b^2 B = \pi B(a^2 - b^2)$$

$$\Rightarrow \quad |\varepsilon| = \frac{d\Phi_B}{dt} = \pi k\,(a^2 - b^2)$$

$$\text{Induced current, } I = \frac{|\varepsilon|}{R} = \frac{k(a-b)}{2\lambda}$$

ILLUSTRATION 4.19

Space is divided by the line AD into two regions. Region I is field free and region II has a uniform magnetic field B directed into the plane of the paper. BCE is a semicircular conducting loop of radius r with center at O, the plane of the loop being in the plane of the paper. The loop is now made to rotate with a constant angular velocity ω about an axis passing through O and the perpendicular to the plane of the paper. The effective resistance of the loop is R.



(a) Obtain an expression for the magnitude of the induced current in the loop.

(b) Show the direction of the current when the loop is entering into region II.

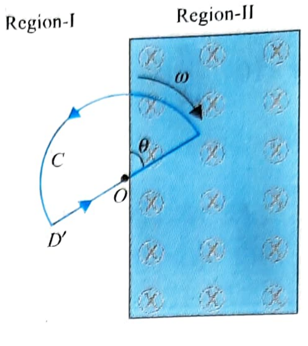
(c) Plot a graph between the induced emf and the time of rotation for the two periods of rotation.

Sol.

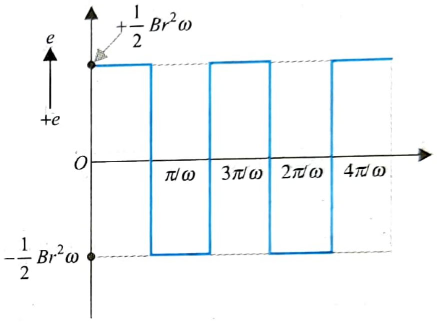
(a) When the loop is rotated about an axis passing through center O and perpendicular to the plane of the paper, the angle between magnetic field vector $\vec{B}$ and area $\vec{A}$ is always 0°. When the loop is in region I, the magnetic flux linked with loop = $BA\cos 0 = 0$ the (since $B = 0$ in region I).

When the loop enters the magnetic field in region II, the magnetic flux linked with it is given by $\phi = BA$ where $A = \frac{1}{2}r^2\theta$. Therefore, emf induced

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}(BA) = -B\frac{dA}{dt} = \frac{-Br^2}{2}\frac{d\theta}{dt} = -\frac{Br^2}{2}\omega$$



(a)



(b)

As resistance of the loop is R, the current induced is given by

$$i = \frac{e}{R} = \frac{1}{2}\frac{Br^2\omega}{R}$$

This is the required expression for current induced in the loop.

(b) According to Lenz's law, the direction of current induced is to oppose the change in magnetic flux. So, when entering into region II the field produced by the current induced must be upward. For this, the current in the loop must be anticlockwise as shown in Fig. (a).

(c) When the loop enters the magnetic field, the magnetic flux linked with it increases and the emf $e = \frac{1}{2}Br^2\omega$ is induced in one direction. When the loop comes out of the field, the flux decreases and emf is induced in opposite sense. The graph for representing the emf induced versus time for two periods ($T = 2\pi/\omega$) is shown in Fig. (b). Here we have taken anticlockwise direction as positive.

ILLUSTRATION 4.20

A current $i = 2.5(1+2t)\times 10^3$ A increases at a steady rate in a long straight wire. A small circular loop of radius 10^{-3} m has its plane parallel to the wire and its center is placed at a distance of 1m from the wire. The resistance of the loop is $5\pi \times 10^{-4}\ \Omega$. Find the magnitude and the direction of the induced current in the loop.

Sol. The field due to the wire at the center of loop is

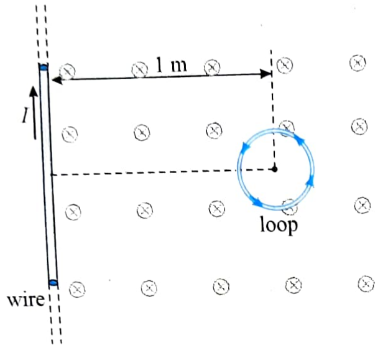
$$B = \frac{\mu_0 I}{2\pi d} = \frac{4\pi \times 10^{-7} \times I}{2\pi \times 1} = 2I \times 10^{-7} \text{ W/m}^2$$

So the flux linked with the loop wire

$$\phi = BA = B \times \pi r^2 = 2I \times 10^{-7} \times \pi \times (10^{-3})^2 = 2\pi I \times 10^{-13} \text{ W}$$

The magnitude of emf induced in the loop due to change of current

$$|e| = \frac{d\phi}{dt} = 2\pi \times 10^{-13} \frac{dI}{dt} \text{ V} \qquad \text{...(i)}$$



$$\because \ I = 2.5(1+2t) \times 10^3 \quad \therefore \ \frac{dI}{dt} = 5.0 \times 10^3 \text{ A/s} \qquad \text{...(ii)}$$

From (i) and (ii), we get $e = 2\pi \times 10^{-13} \times 5.0 \times 10^3 = 10\pi \times 10^{-10}$ V

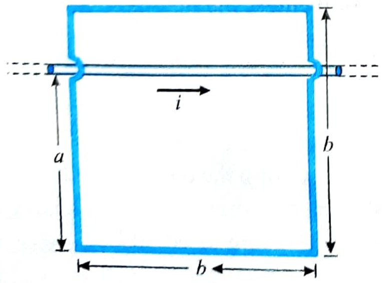
And the induced current in the loop

$$i = \frac{e}{R} = \frac{10\pi \times 10^{-10}}{5\pi \times 10^{-4}} = 2.0 \times 10^{-6} \text{ A}$$

Due to increase in current in the wire the flux linked with the loop will increase, so in accordance with Lenz's law the direction of current induced in the loop will be inverse of that in wire, i.e., anticlockwise.

ILLUSTRATION 4.21

A wire loop with a = 10.0 cm and b = 15.0 cm and a long straight wire is arranged as shown in figure. The current in the wire is changing according with a relation $i = 5.0t^2 - 10.0t$, where i is in amperes and t is in seconds. **(a)** Find the emf in the square loop at t = 4.0 s. **(b)** What is the direction of the induced current in the loop?



Sol.

(a) First, we observe that a large portion of the figure contributes flux that "cancels out." The field (due to the current in the long straight wire) through the part of the rectangle above the wire is out of the page (by the right-hand rule) and below the wire it is into the page. Thus, since the height of the part above the wire is $b - a$, then a strip below the wire (where the strip borders the long wire, and extends a distance $b - a$ away from it) has exactly the equal but opposite flux that cancels the contribution from the part above the wire. Thus, we obtain the non-zero contributions to the flux:

$$\Phi_B = \int B\,dA = \int_{b-a}^{a} \left(\frac{\mu_0 i}{2\pi r}\right)(b\,dr) = \frac{\mu_0 i b}{2\pi} \ln\left(\frac{a}{b-a}\right).$$

According to Faraday's law, the e.m.f induced

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left[\frac{\mu_0 i b}{2\pi} \ln\left(\frac{a}{b-a}\right)\right]$$

$$= -\frac{\mu_0 b}{2\pi} \ln\left(\frac{a}{b-a}\right)\frac{di}{dt}$$

$$= -\frac{\mu_0 b}{2\pi} \ln\left(\frac{a}{b-a}\right)\frac{d}{dt}(5t^2 - 10t)$$

$$= \frac{-\mu_0 b(10t - 10)}{2\pi} \ln\left(\frac{a}{b-a}\right).$$

With a = 0.10 m and b = 0.15 m, then at t = 4.0 s, the magnitude of the emf induced in the rectangular loop is

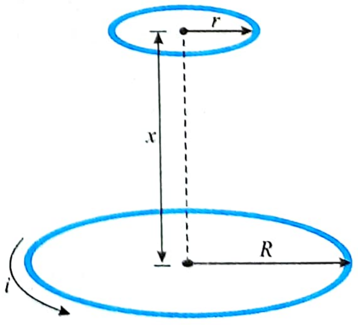
$$|\varepsilon| = \frac{(4\pi \times 10^{-7})(0.15)(10(4) - 10)}{2\pi} \ln\left(\frac{0.10}{0.15 - 0.10}\right)$$

$$= 4.16 \times 10^{-7} \text{ V}.$$

(b) We note that $di/dt > 0$ at $t = 2.0$ s. From Lenz's law, then, the induced emf (hence, the induced current) in the loop is counterclockwise.

ILLUSTRATION 4.22

Two circular loops are placed with common vertical axis one above other as shown in figure. The smaller loop (radius r) is above the larger loop (radius R) by a distance $x >> R$. Consequently, the magnetic field due to the counterclockwise current i in the larger loop is nearly uniform throughout the smaller loop. Suppose that x is increasing at the constant rate $dx/dt = v$. Find the induced emf and the direction of the induced current in the smaller loop,



Sol. Increasing the separation between the two loops changes the flux through the smaller loop and, therefore, induces a current in the smaller loop.In the region of the smaller loop the magnetic field produced by the larger loop may be taken to be uniform and equal to its value at the center of the smaller loop, on the axis.

The magnetic field at the centre of smaller loop, $\vec{B} = \frac{\mu_0 i R^2}{2x^3}\hat{i}$, where the $+x$ direction is upward. The area of the smaller loop is $A = \pi r^2$.

The magnetic flux through the smaller loop is,

$$\Phi_B = BA = \frac{\pi\mu_0 i r^2 R^2}{2x^3}.$$

The emf is given by Faraday's law:

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\left(\frac{\pi\mu_0 i r^2 R^2}{2}\right)\frac{d}{dt}\left(\frac{1}{x^3}\right).$$

$$= -\left(\frac{\pi\mu_0 i r^2 R^2}{2}\right)\left(-\frac{3}{x^4}\frac{dx}{dt}\right) = \frac{3\pi\mu_0 i r^2 R^2 v}{2x^4}.$$

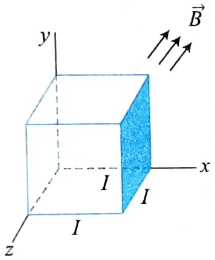
As the smaller loop moves upward, the flux through it decreases. The induced current will be directed so as to produce a magnetic field that is upward through the smaller loop, in the same direction as the field of the larger loop. It will be counterclockwise as viewed from above, in the same direction as the current in the larger loop.

CONCEPT APPLICATION EXERCISE 4.1

1. A cube of edge length $l = 2.50$ cm is positioned as shown in figure. A uniform magnetic field given by

$$\vec{B} = (5.00\hat{i} + 4.00\hat{j} + 3.00\hat{k})\ \text{T}$$

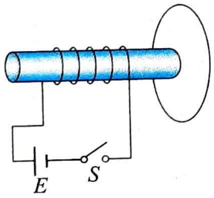
exists throughout the region.



   (a) Calculate the flux through the shaded face.

   (b) What is the total flux through the six faces?

2. A conducting ring is placed near a solenoid as shown in figure. Find the direction of the induced current in the ring.

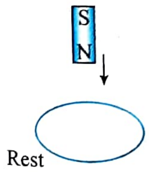


   (a) At the instant the switch in the circuit containing the solenoid is closed.

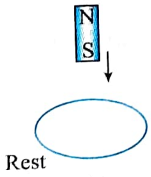
   (b) After the switch has been closed for a long time.

   (c) At the instant the switch is opened.

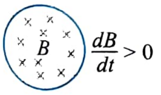
3. Identify the direction of induced current as seen from the above in the following cases.



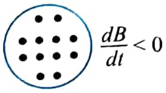
(a)



(b)

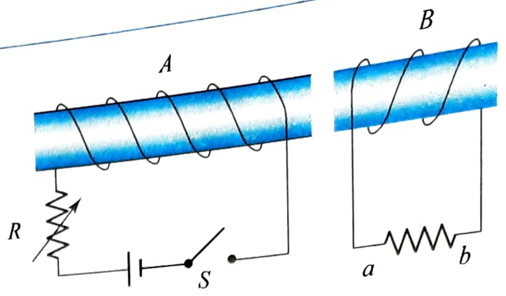


(c)



(d)

4. Using Lenz's law, determine the direction of the current in resistor ab in figure when

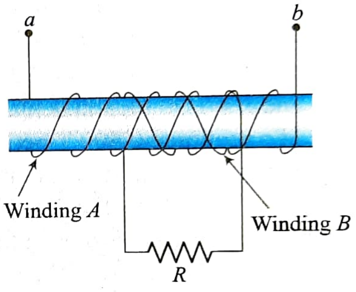


   (a) switch S is opened after having been closed for several minutes.

   (b) coil B is brought closer to coil A with the switch closed.

   (c) the resistance of R is decreased while the switch remains closed.

5. A cardboard tube is wrapped with two windings of insulated wire wound in opposite directions as shown in figure. Terminals a and b of winding A may be connected to a battery through a reversing switch. State whether the induced current in the resistor R is from left to right or from right to left in the following circumstances.

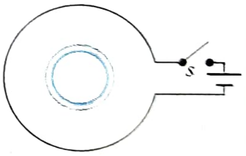


   (a) The current in winding A is from a to b and is increasing.

   (b) The current in winding A is from b to a and is decreasing.

   (c) The current in winding A is from b to a and is increasing.

6. A small, circular ring is inside a larger loop that is connected to a battery and a switch as shown in figure. Use Lenz's law to find the direction of the current induced in the small ring

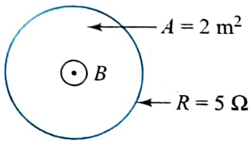


   (a) just after switch S is closed;

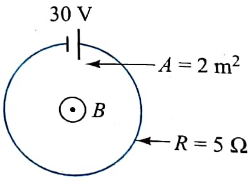
   (b) after S has been closed for a long time;

   (c) just after S has been reopened after being closed for a long time.

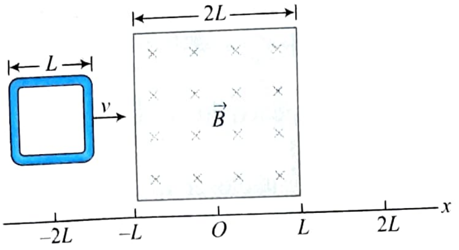
7. Figure shows a coil placed in a decreasing magnetic field applied perpendicular to the plane of the coil. The magnetic field is decreasing at a rate of 10 T s^{-1}. Find out current in magnitude and direction of current.



8. Figure shows a coil placed in a magnetic field decreasing at a rate of 10 T s^{-1}. There is also a source of emf 30 V in the coil. Find the magnitude and direction of the current in the coil.



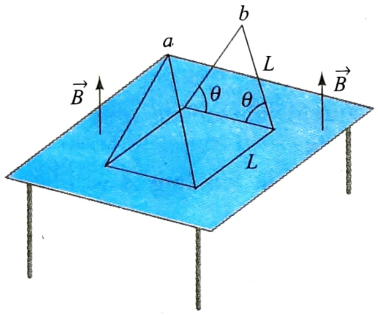
9. A square loop of wire with resistance R is moved at a constant speed v across a uniform magnetic field confined to a square region whose sides are twice the length of those of the square loop (as shown in figure).



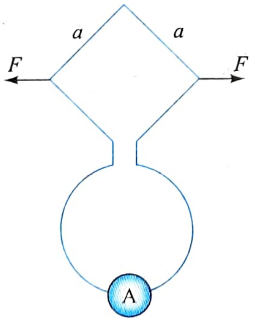
(a) Graph the external force F needed to move the loop at a constant speed as a function of the coordinate x from $x = -2L$ to $x = +2L$. (The coordinate x is measured from the center of the magnetic field region to the center of the loop. It is negative when the center of the loop is to the left of the center of the magnetic field region. Take positive force to be to the right.)

(b) Graph the induced current in the loop as a function of x. Take counterclockwise currents to be positive.

10. The wire shown in figure is bent in the shape of a tent, with $\theta = 60.0°$ and $L = 1.50$ m, and is placed in a uniform magnetic field of magnitude 0.300 T perpendicular to the tabletop. The wire is rigid but hinged at points a and b. If the tent is flattened out on the table in 0.100 s, what is the average induced emf in the wire during this time?



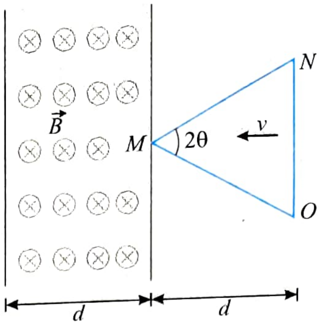
11. The plane of a square loop of wire with edge length $a = 0.200$ m is perpendicular to the Earth's magnetic field at a point where $B = 15.0$ μT, as shown in figure. The total resistance of the loop and the wires connecting it to a sensitive ammeter is 0.50 Ω. If the loop is suddenly collapsed by horizontal forces as shown, what is the total charge passing through the ammeter?



12. A triangular conducting loop having resistance R is placed outside an uniform inward magnetic field B. At $t = 0$,the loop starts moving into the magnetic field with a uniform speed v. Find the

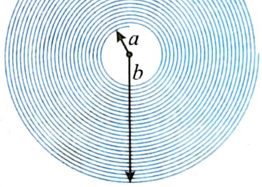
(a) magnitude of induced emf ε at any time t.

(b) the magnitude and direction of induced current at any time t.



13. A circular stretchable conducting ring is kept in a uniform magnetic field (B) that is perpendicular to the plane of the ring. The ring is pulled out uniformly from all sides so as to increase its radius at a constant rate $\frac{dr}{dt} = n$ while maintaining its circular shape. Calculate the rate of work done by the external agent against the magnetic force when the radius of the ring is r_0. Resistance of the ring remains constant at R.

14. A flat spiral coil has a large number of turns N. The turns are wound tightly and the inner and outer radii of the coil are a and b respectively. A uniform external magnetic field (B) is applied perpendicular to the plane of the coil. Find the emf induced in the coil when the field is made to change at a rate $\frac{dB}{dt} = \alpha$.



ANSWERS

1. (a) 3.125×10^{-3} Wb (b) 0

2. (a) anticlockwise (b) no change (c) clockwise

3. (a) anticlockwise (b) clockwise (c) anticlockwise (d) anticlockwise

4. (a) to the right (b) to the left (c) to the left

5. (a) right to left (b) right to left (c) left to right

6. (a) clockwise (b) no change (c) counter-clockwise

7. 4A, anticlockwise **8.** 2A, clockwise **10.** 5.84 V

11. 1.2 μC **12.** (a) $2(Bv^2 \tan\theta)t$

(b) $\frac{2Bv^2 \tan\theta}{R}t$ (counter clockwise sense)

13. $\frac{4\pi^2 B^2 r^2 n^2}{R}$ **14.** $\frac{\pi N\alpha}{3}(a^2 + b^2 + ab)$

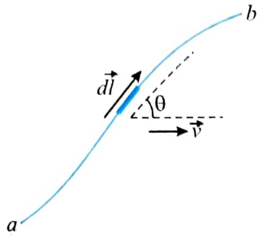
MOTIONAL ELECTROMOTIVE FORCE

When a conductor moves in a magnetic field, then charges inside conductor experience magnetic force given by $\vec{F} = q\vec{v} \times \vec{B}$, where $\vec{v}$ is the velocity of conductor (here we assume that elocity of charges inside conductor is same as the velocity of onductor and random motion of charges inside the conductor is eglected). Due to this, force charges start moving in a particular irection in the conductor or we say that an emf is induced in the onductor. This emf is known as motional emf.

Consider a conductor of arbitrary hape as shown in figure moving in ome magnetic field $\vec{B}$.

The different parts of the conductor nay have different velocity. Consider n element $\vec{dl}$ having velocity $\vec{v}$.

Force due to magnetic field on a harge in conductor.



$$\vec{F} = q\vec{v}\times\vec{B}$$

Work done by this force on charge q in passing through $\vec{dl}$

$$dW = \vec{F}\cdot\vec{dl} = q(\vec{v}\times\vec{B})\cdot\vec{dl}$$

So emf induced within $\vec{dl}$: $\quad d\varepsilon = \dfrac{dW}{q} = (\vec{v}\times\vec{B})\cdot\vec{dl}$

Integrate this to find net emf: $\varepsilon = \int_a^b (\vec{v}\times\vec{B})\cdot\vec{dl}$

This emf is directed from a to b.

Other Approach

Due to the magnetic force, charges start moving in the conductor. An electric field is set up in the conductor. In equilibrium force of electric field balances the force of magnetic field. Let electric electric field induced be $\vec{E}$, then

$$q\vec{v}\times\vec{B} + q\vec{E} = 0 \quad\Rightarrow\quad \vec{E} = -\vec{v}\times\vec{B}$$

Now potential difference developed across the ends of the conductor: $V_b - V_a = -\int_a^b \vec{E}\cdot\vec{dl} = \int_a^b (\vec{v}\times\vec{B})\cdot\vec{dl}$

If circuit is incomplete, then this terminal potential difference will be equal to emf induced. Hence emf induced:

$$\varepsilon = \int_a^b (\vec{v}\times\vec{B})\cdot\vec{dl}$$

Note: We can also write $\varepsilon = \int_a^b \vec{B}\cdot(\vec{dl}\times\vec{v})$

As $\vec{dl}\times\vec{v}$ is the area swept per unit time by length $\vec{dl}$ and hence $\vec{B}\cdot(\vec{dl}\times\vec{v})$ is the flux of induction through the area. Therefore, the motional emf is equal to the flux of induction cut by the conductor per unit time.

We can write the expression for induced emf in various forms:

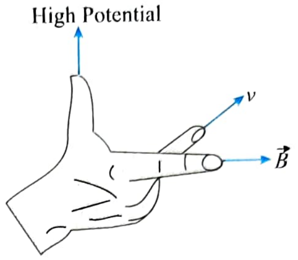
$$\varepsilon = \int_a^b (\vec{v}\times\vec{B})\cdot\vec{dl} = \int_a^b (\vec{B}\times\vec{dl})\cdot\vec{v} = \int_a^b (\vec{dl}\times\vec{v})\cdot\vec{B}$$

if any two out of $\vec{v}$, $\vec{B}$ and $\vec{dl}$ become parallel or antiparallel, ε will become zero.

FINDING DIRECTION OF INDUCED EMF

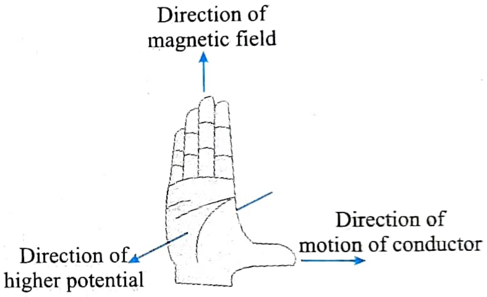
We can find the direction of induced EMF by Flemings Right Hand Rule as given below.

"If we stretch the index finger, middle finger and thumb of right hand in mutually perpendicular directions as shown in figure with index finger pointing toward the direction of magnetic induction and thumb along the velocity of conductor then middle finger will point toward the high potential end of the conductor."



The direction of induced EMF can also be find out by Right Hand Palm Rule.

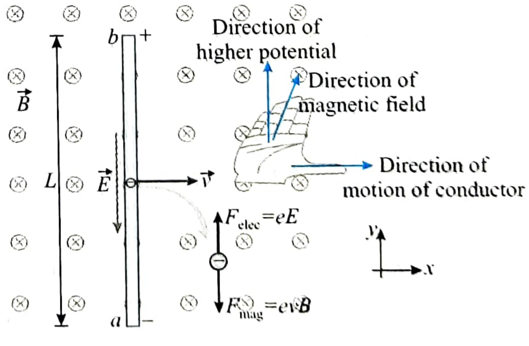
"If we stretch the fingers of right hand towards magnetic field and thumb along the velocity of conductor then the high potential end of the conductor is directed perpendicular to the right-hand palm."


Direction of magnetic field
Direction of higher potential
Direction of motion of conductor

SPECIAL CASES

A Straight Conductor Moving in a Magnetic Field

Consider a straight conductor of length L is moving in a magnetic field B with velocity v. Here all three L, B, and v are mutually perpendicular.


$\vec{B}$
L
b
+
a
−
$\vec{E}$
$\vec{v}$
Direction of higher potential
Direction of magnetic field
Direction of motion of conductor
$F_{elec} = eE$
$F_{mag} = evB$
y
x

We have $\vec{v} = v\hat{i}$, $\vec{B} = -B\hat{k}$, $\vec{dl} = dl\,\hat{j}$

Induced emf in the rod is given by

$$\varepsilon = \int_a^b (\vec{v}\times\vec{B})\cdot\vec{dl} = \int_a^b [v\hat{i}\times(-B\hat{k})]\cdot dl\,\hat{j}$$

$$= \int_a^b (Bv\hat{j})]\cdot dl\,\hat{j} = Bv\int_a^b dl = BVL \quad\Rightarrow\quad \varepsilon = BVL$$

Here end b will be positive w.r.t. end a.

Other Approach

Consider an electron inside the conductor. Magnetic force $F_b = evB$ on the electron will be toward a. So electrons will start accumulating near end a and end b will become positive w.r.t. end a. This will create an electric field E from b to a. Thus E will apply electric force $F_e = eE$ on electrons opposite to F_b.

In equilibrium: $F_b = F_e$

$$\Rightarrow \quad evB = eE \quad \Rightarrow \quad E = vB$$

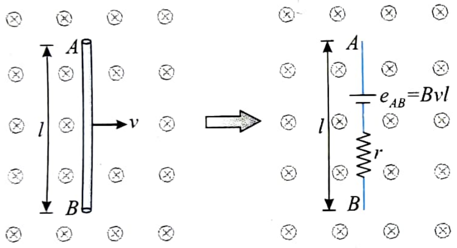
Potential difference between b and a: $V_b - V_a = EL = vBL$

If ends a and b are not connected in external circuit, then this terminal potential diffrence will be equal to emf induced. Hence

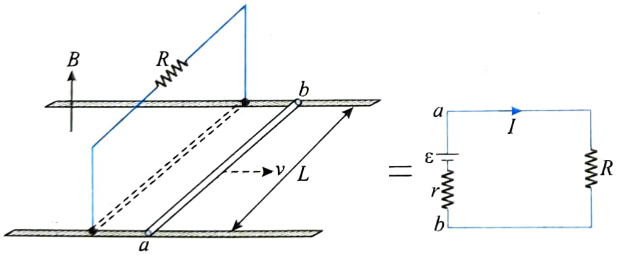
$$\varepsilon = BvL$$

MOTIONAL EMF AS AN EQUIVALENT BATTERY

When a conductor moves in magnetic field, the EMF may develop across its ends. The conductor can be considered like an equivalent battery with internal resistance equal to the resistance of the conductor which can supply current. In the figure a conducting rod of length l, resistance r moving at a velocity v in a uniform magnetic field B. In this case its length (l), velocity (v) and magnetic field (B) are perpendicular to each other and it can be replaced by an equivalent battery of EMF Bvl as shown in figure.



Now what will happen if ends a and b are connected externally through resistance R as shown in Fig. (a). A current will start flowing in the circuit due to induced emf.

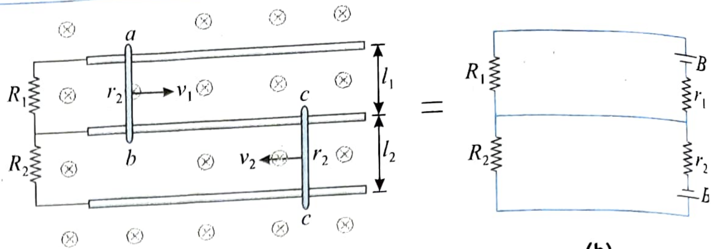


The equivalent circuit is shown in right Fig. (b). The rod will act as a battery of emf $\varepsilon = BvL$ and r is the resistance of the rod which will act as internal resistance of the battery. Current in the circuit is given by: $I = \dfrac{E}{R+r}$

pd between ends a and b:

$$V_b - V_a = \varepsilon - Ir = \varepsilon - \frac{\varepsilon r}{R+r} = \frac{\varepsilon R}{R+r}$$

Let us consider one more situation, shown in Fig. (a) in which two sliding wires ab and cd are sliding on three conducting rails which are connected to some resistances at their ends as shown. In this case we can determine the currents through resistances and different sections of the setup by making an equivalent circuit of this setup as shown in Fig. (b).

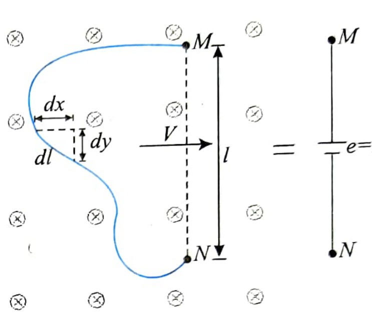


(a)

(b)

MOTIONAL EMF IN A RANDOM SHAPED WIRE MOVING IN MAGNETIC FIELD

Consider a random shaped conducting wire MN in which the distance between its ends M and N is l is moving with a velocity v perpendicular to line MN in a uniform magnetic field B as shown in figure.



Let us consider a small element dl in the wire as shown in figure which will have its two components, one along the line MN and other perpendicular to the line MN. There will not be no induced EMF in component dx, as this component is parallel to the velocity vector $\vec{v}$. The EMF will be induced only in the component of dy, thus EMF induced in this component

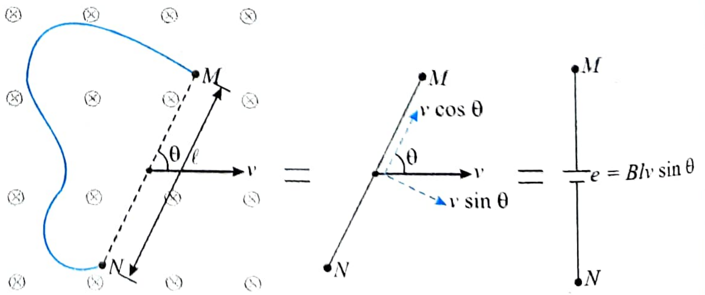
$$de = Bvdy \qquad \text{...(i)}$$

Hence total EMF induced between ends MN of the wire is given as

$$e_{MN} = \int de = Bv\int dy = Bvl$$

Important Point:

We can find the EMF induced in case of different random shaped wires are moving in different directions by using the component of velocity perpendicular to the line MN as shown in given figures.



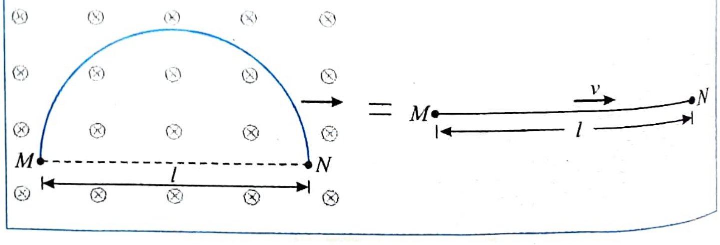
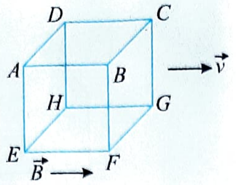


ILLUSTRATION 4.23

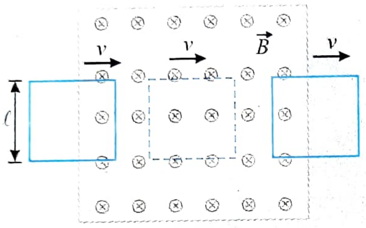
Twelve wires of equal lengths are connected in the form of a skeleton cube which is moving with a velocity $\vec{v}$ in the direction of magnetic field $\vec{B}$. Find the e.m.f. in each arm of cube.



Sol. No e.m.f. is induced in any arm because $\vec{v}$ is parallel to $\vec{B}$. ($\because e_d = \vec{l}.(\vec{B}\times\vec{v})$)

ILLUSTRATION 4.24

A square loop of side l having resistance R moves with constant velocity v, through a constant magnetic field as shown in figure. Find the magnitude and direction of induced current in the situation when



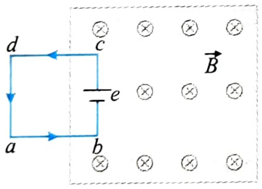
(i) the loop enters into magnetic field

(ii) it is moving in magnetic field.

(iii) it is coming out of magnetic field.

Sol.

(i) When loop starts entering into magnetic field. The side ab is outside the magnetic field, hence no EMF will develop across this side.

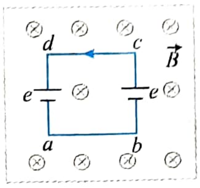


The length of the sides ab and cd are parallel to the velocity hence no EMF will be induced in these sides. The EMF will be induced across the side bc.

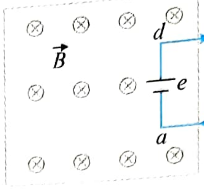
Hence net induced emf $\varepsilon = Blv$

The current in the loop $I = \frac{\varepsilon}{R} = \frac{Blv}{R}$. The current will be in counter clockwise sense.

(ii) When loop is completely submerged in magnetic field. The EMF will be induced insides bc and ad as shown in figure. In this case net EMF in the loop will be zero. Hence no induced current.



(iii) When the loop starts coming out of magnetic field. The EMF will be induced in side ad only. The direction of induced EMF or induced current can be found out by Lenz's law or by right hand rule.

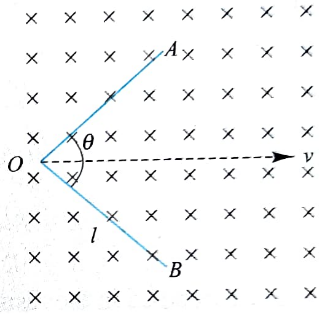


Net induced emf $\varepsilon = Blv$

The current in the loop $I = \frac{\varepsilon}{R} = \frac{Blv}{R}$

ILLUSTRATION 4.25

An angle $\angle AOB$ made of a conducting wire moves along its bisector through a magnetic field B as suggested by figure. Find the emf induced between the two free ends if the magnetic field is perpendicular to the plane at the angle.



Sol. The rod OA is equivalent to a battery of emf $vBl\sin\theta/2$. The positive charges of OA shift toward A due to the force. The positive terminal of the battery appears toward A. Similarly, the rod OB is equivalent to a battery of emf $vBl\sin\theta/2$ with the positive terminal toward O. The equivalent circuit is shown in figure. Clearly, the emf between points A and B is $2Blv\sin\theta/2$.

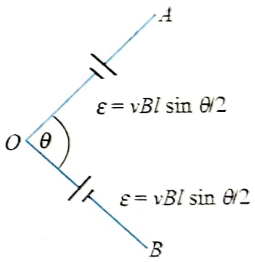
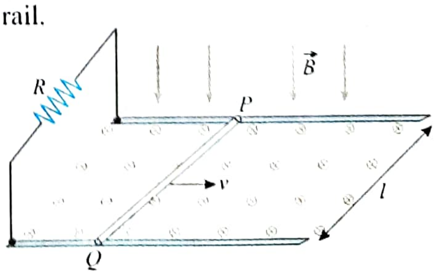

A
$\varepsilon = vBl\sin\theta/2$
O
θ
$\varepsilon = vBl\sin\theta/2$
B

ILLUSTRATION 4.26

A conducting rod of length l slides at constant velocity v on two parallel conducting rails, placed in a uniform and constant magnetic field B perpendicular to the plane of the rails as shown in figure. A resistance R is connected between the two ends of the rail.



(a) Identify the cause which produces change in magnetic flux.

(b) Identify the direction of current in the loop.

(c) Determine the emf induced in the loop.

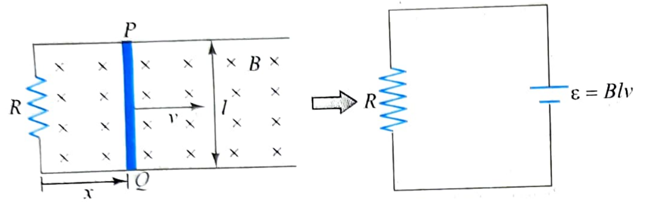
(d) Compute the electric power dissipated in the resistor.

(e) Calculate the mechanical power required to pull the rod at a constant velocity.

Sol.

(a) The change in area produces the change in magnetic flux.

(b) The direction of current in the loop is anticlockwise. As the rod moves toward right, the number of crosses in the loop increases with time and to oppose the increasing number of crosses in the loop, the current in the loop must be anticlockwise.



(c) According to Faraday's law,

$$|\varepsilon| = \frac{d\phi_B}{dt} = \frac{d}{dt}(Blx) = Bl\frac{dx}{dt} = Blv.$$

The emf across rod PQ can also be calculated by using the concept of motional emf.

(d) The magnitude of current is $I = \frac{\varepsilon}{R} = \frac{Blv}{R}$.

The electric power dissipated in the resistor is

$$P_{\text{ele}} = I^2R = \frac{B^2l^2v^2}{R}$$

(e) The mechanical power is $P_{\text{mech}} = F_{\text{ext}}v$.

The external force is equal and opposite to Ampere's force:

$$F_{\text{ext}} = -F_{\text{Ampere}}$$

Ampere's force is given by

$$F_{\text{Ampere}} = \int I d\vec{l} \times \vec{B} \text{ or } F_{\text{Ampere}} = BIl$$

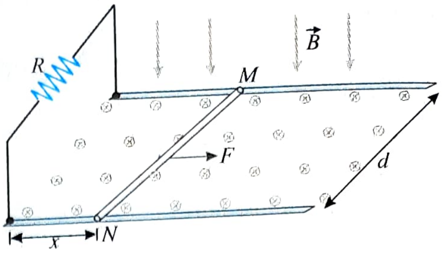
Now, $P_{\text{mech}} = (BIl)v$.

Substituting the value of I, we get $P_{\text{mech}} = \frac{B^2l^2v^2}{R}$.

We see that mechanical power is same as electric power. It means mechanical power supplied to the rod is dissipated in the form of heat through resistor.

ILLUSTRATION 4.27

Two long parallel horizontal rails, a distance d apart and each having a resistance λ per unit length, are joined at one end by a resistance R. A perfectly conducting rod MN of mass m is free to slide along the rails without friction (see figure). There is a uniform magnetic field of induction B normal to the plane of the paper and directed into the paper. A variable force F is applied to rod MN such that as the rod moves, a constant current i flows through R.

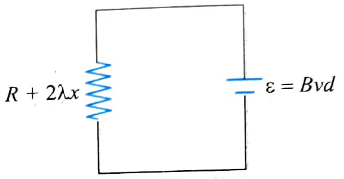


(a) Find the velocity of the rod and the applied force F as a function of the distance x of the rod from R.

(b) What fraction of the work done per second by F is converted into heat?

Sol.

(a) If the rod has instantaneous velocity v at a distance x from R, the induced emf is Bvd.



Instantaneous resistance of current $= R + 2\lambda x$

$\therefore$ Induced current $i = \frac{Bvd}{R+2\lambda x}$ = constant

$\therefore$ Velocity $v = \frac{(R+2\lambda x)i}{Bd}$

Magnetic force on the rod $= Bid$.

This force will be opposite to F. Hence, net force acting on rod,

$$F - Bid = m\frac{dv}{dt}$$

$$\text{or} \quad F - Bid = m\frac{dv}{dx}\frac{dx}{dt} = mv\frac{d}{dx}\left\{\frac{(R+2\lambda x)i}{Bd}\right\} = mv\frac{2\lambda i}{Bd}$$

$$\therefore \quad F = Bid + 2m\lambda\frac{(R+2\lambda x)}{B^2d^2}i^2$$

(b) Work done per second $= Fv$

Heat produced per second $= i^2(R + 2\lambda x)$

$$\text{Required ratio} = \frac{i^2(R+2\lambda x)}{Fv} = \frac{i^2(R+2\lambda x)Bd}{F(R+2\lambda x)i} = \frac{iBd}{F}$$

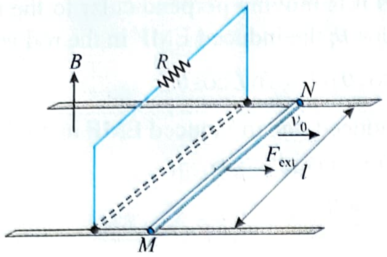
$$= \frac{Bid}{Bid + \frac{2m\lambda(R+2\lambda x)i^2}{B^2d^2}}$$

$$= \frac{1}{1 + \frac{2m\lambda(R+2\lambda x)i}{B^3d^3}}$$

ILLUSTRATION 4.28

Figure shows a setup with two long conducting rails with separation l in a plane perpendicular to a uniform magnetic field of magnetic induction B. A sliding rod MN of mass m is placed on rails as shown. The rails are connected with a resistance R and the sliding rod is pulled with constant velocity v_0 as shown in the figure.

Then find



(a) ampere force acting on the rod

(b) power developed by the external force

(c) electrical power delivered by the external agent

(d) thermal power generated in the resistor

Sol.

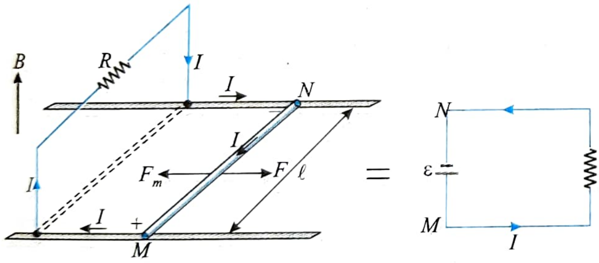
(a) In this case the rod moves with a velocity v_0, normal tothe magnetic field of induction B.

The emf induced in rod $\varepsilon_i = Blv_0$.

The induced current in rod, $i = \dfrac{Blv_0}{R}$

The magnetic field experiences a magnetic force

$$F_m = ilB = \frac{B^2l^2v_0}{R}$$



(b) Power developed by the external agent

$$P_{ext} = \vec{F}_{ext}\cdot\vec{v} = F_{ext}.v_0\cos 0°$$

As the bar moves with constant velocity v, it experiences no net force, $F_{ext} = F_m = \dfrac{B^2l^2v_0}{R}$.

Thus, $P_{ext} = F_{ext}v_0 = \dfrac{B^2l^2v_0^2}{R}$

(c) Thermal power generated in the resistor, $P_{ther.} = i^2R$

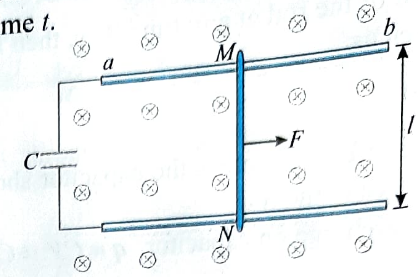
Substitution of i in this equation yields $P_{ther.} = \dfrac{B^2l^2v_0^2}{R}$

(d) Electrical power generated

$$P_{el} = \varepsilon_i i = (Blv_0)\left(\frac{Blv_0}{R}\right) = \frac{B^2l^2v_0^2}{R}$$

ILLUSTRATION 4.29

Figure shows two long smooth conducting rails ab and cd separated by a distance l. Ends a and c are connected with capacitor of capacitance C. A rod MN of mass m is horizontally kept in touch with both rails as shown and it can slide along the rails without friction. The entire system is placed in a downward magnetic field of induction B. At $t = 0$, a force F is applied on the rod as shown in figure. Find the velocity of the rod at any time t.



Sol. When the rod moves then at any instant when its speed is v, the motional emf across the rod is given as

$$e = Blv$$

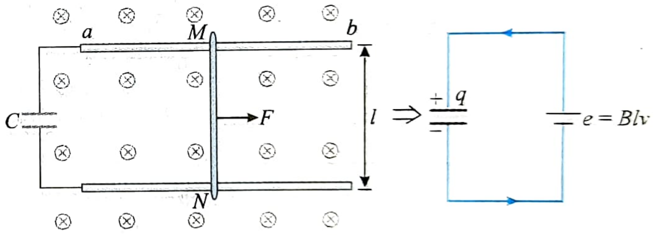
As we can neglect the resistance of all connecting wires and rails the charge on capacitor is given as

$$q = Ce = CBlv \qquad ...(i)$$

Current through the capacitor is given as

$$i = \frac{dq}{dt} = CBla$$

The rod experiences an leftward magnetic force rightward force as shown in figure.



The acceleration a, of the rod is given as

$$a = \frac{F - Bil}{m} = \frac{F - B^2l^2Ca}{m}$$

$$\Rightarrow \quad a = \frac{F}{m + B^2l^2C}$$

As acceleration is constant, after time t its velocity is given as

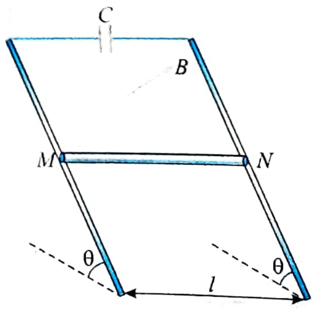
$$v = at = \frac{Ft}{m + B^2l^2C}$$

Thus charge on capacitor after time t is given by equation as

$$q = CBl\left(\frac{Ft}{m + B^2l^2C}\right) = \frac{CBlFt}{m + B^2l^2C}$$

ILLUSTRATION 4.30

Two smooth conducting fixed parallel rails are inclined at an angle θ to the horizontal. The top end of the rails are connected with a capacitor of capacitance C. A straight horizontal conducting rod MN of length l, and mass m slides down on these rails as shown in figure.



The system is placed in a uniform magnetic field, in the direction perpendicular to the inclined plane formed by the rails. If the resistance of the bars and the sliding conductor are negligible, calculate the acceleration of sliding conductor as a function of time if it is released from rest at $t = 0$.

Sol. Let speed of the rod at any time be v, then EMF induced in the rod is given as

$$e_{MN} = Blv$$

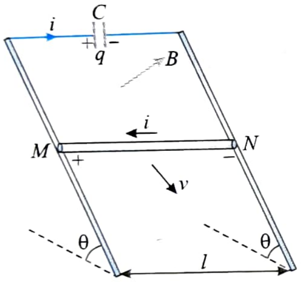
Hence potential difference across the capacitor should be equal to the EMF induced in the rod.

The instantaneous charge on capacitor, $q = CV = CBlv$

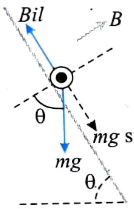
Current through the rod, $i = \frac{dq}{dt} = CBi\frac{dv}{dt} = CBla$

In magnetic field the current carrying conductor experiences a magnetic force in upward direction by right hand palm rule as shown in Fig. (b) which shows the side view of sliding conductor and its free body diagram. This gives

$$F_{\text{mag}} = Bil = B^2l^2Ca$$



(a)



(b)

If a is the acceleration of conductor, then its equation of motion is given as

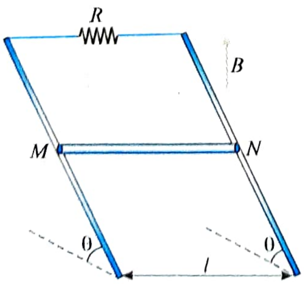
$$ma = mg\sin\theta - B^2l^2Ca$$

$$\Rightarrow \quad a[m + B^2l^2C] = mg\sin\theta$$

$$\Rightarrow \quad a = \frac{mg\sin\theta}{m + B^2l^2C}$$, Here we can see that a is constant in time.

ILLUSTRATION 4.31

Two smooth conducting fixed parallel rails are inclined at an angle θ to the horizontal. The top end of the rails are connected with a resistance R. A straight horizontal conducting rod MN of length l, and mass m slides down on these rails as shown in figure. The system is located in a uniform magnetic field of induction B in vertically upward direction as shown in figure. The resistances of the bars, the rod and the sliding contacts are considered to be negligible. If the rod is released from rest, find the steady state velocity of the rod.

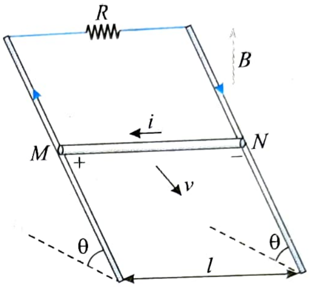


Sol. The rod starts moving from rest, it cuts the magnetic flux and an EMF is induced in it with polarity given by right hand palm rule as shown in figure. Let the steady state velocity of the rod be v and as it is moving perpendicular to the magnetic field component B cos θ, the induced EMF in the rod is given as

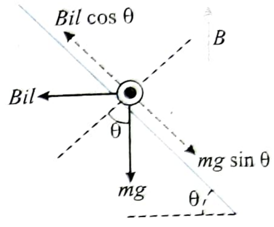
$$e = (B\cos\theta)vL = BvL\cos\theta$$

Thus current induced due to induced EMF in the loop containing resistance as shown is given as

$$i = \frac{BvL\cos\theta}{R}$$



(a)



(b)

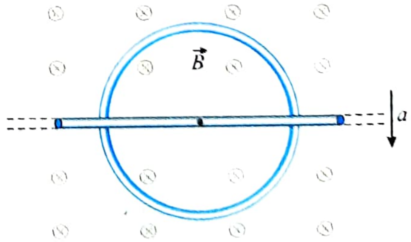
In Fig. (b), we can observe the side view of the sliding rod. By right hand rule, we can observe that magnetic force will act along leftward direction and if rod slides with constant velocity. At this time

$$mg\sin\theta = BiL\cos\theta \Rightarrow mg\sin\theta = \frac{B^2L^2v\cos^2\theta}{R}$$

Hence steady state velocity, $v = \frac{mgR\sin\theta}{B^2L^2\cos^2\theta}$

ILLUSTRATION 4.32

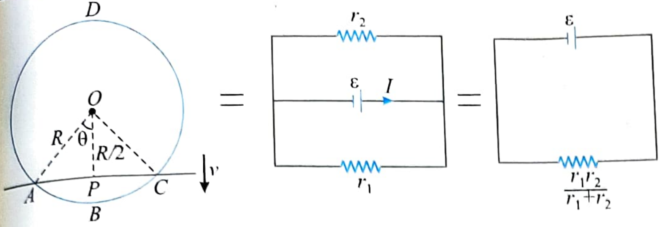
A circular conducting wire ring of radius R is fixed in a horizontal plane. The resistance per unit length of wire of the ring has a resistance of $\lambda\Omega\text{m}^{-1}$. There is a uniform vertical downward magnetic field B in entire space. A perfectly conducting rod (l) is kept along the diameter of the ring. Therod is made to move with a constant acceleration a in a direction perpendicular to its own length. Find the current through the rod at the instant it has travelled through a distance $x = \frac{R}{2}$.



Sol. The velocity of the rod when it has travelled a distance

$$x = \frac{R}{2} \text{ is } v = \sqrt{2ax} = \sqrt{2a\frac{R}{2}} = \sqrt{aR}$$

Length of rod inside the ring, $L = AC = 2\sqrt{R^2 - \frac{R^2}{4}} = \sqrt{3}R$



Emf induced in the rod at this instant is

$$\varepsilon = BvL = B\cdot\sqrt{aR}\sqrt{3}R = \sqrt{3a}\,BR^{3/2}$$

$$\Delta AOP,\ \cos\theta = \frac{R/2}{R} = \frac{1}{2}$$

It means $\theta = 60^\circ = \dfrac{\pi}{3}$ radian

$\therefore$ Length of lower arc ABC, $l_1 = 2\pi R\left(\dfrac{\pi/3}{2\pi}\right) = \dfrac{\pi R}{3}$

Length of lower arc ADC, $l_2 = 2\pi - \dfrac{\pi R}{3} = \dfrac{5\pi R}{3}$

Resistance of arc ABC; $r_1 = l_1\lambda = \dfrac{\pi R}{3}\lambda$

Resistance of arc ADC; $r_2 = l_2\lambda = \dfrac{5\pi R}{3}\lambda$

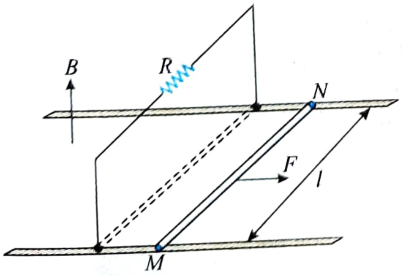
Equivalent resistance connected across equivalent battery,

$$r = \frac{r_1r_2}{r_1+r_2} = \frac{5\pi}{18}\lambda R$$

$\therefore$ Current through the rod $I = \dfrac{\varepsilon}{r_{eq}} = \dfrac{18B}{5\pi}\dfrac{\sqrt{3aR}}{\lambda}$

ILLUSTRATION 4.33

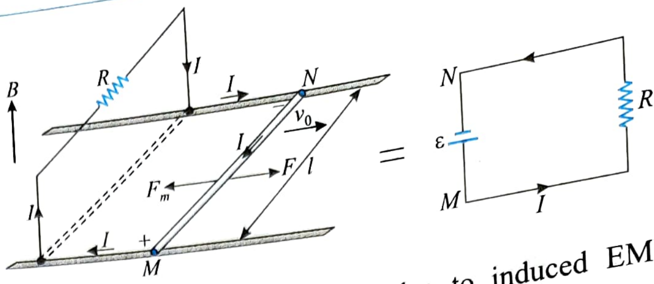
Figure shows a setup with two long conducting rails with separation l in a plane perpendicular to a uniform magnetic field of magnetic induction B. A sliding rod MN of mass m is placed on rails as shown. The rails are connected with a resistance R and the sliding rod is pulled with a constant external force F. If initially the sliding rod is at rest at $t = 0$,



Find the velocity of the rod at any time t and terminal velocity of the rod.

Sol. At a general instant of time t, let the rod is moving with velocity v then at this instant motional EMF induced in this wire is given as

$$e = Bvl \text{ (with end } M \text{ of the rod to be positive)}$$



The current through the resistance due to induced EMF is given as ...(i)

$$i = \frac{e}{R} = \frac{Bvl}{R}$$

Due to the direction of induced EMF current flows from N to M in the sliding wire because of which magnetic field exerts a magnetic force on the sliding wire which is given as

$$F_m = Bil = \frac{B^2l^2v}{R} \qquad \text{...(ii)}$$

Using the right-hand palm rule we can find the direction of this magnetic force on sliding wire MN is toward left which is in opposition to the external force. This is also validating Lenz's law that the effects produced by induction oppose the causes of induction.

If at a general instant of time t acceleration of wire MN is a then we have

$$F - F_m = ma \qquad \text{...(iii)}$$

$$\Rightarrow \quad a = \frac{F - F_m}{m}$$

$$\Rightarrow \quad a = \frac{F - \frac{B^2l^2}{R}v}{m} \qquad \text{...(iv)}$$

$$\Rightarrow \quad \frac{dv}{dt} = a = \frac{FR - B^2l^2v}{mR}$$

$$\Rightarrow \quad \frac{dv}{dt} = a = \frac{FR - B^2l^2v}{mR}$$

To find the velocity of wire MN as a function of time we integrate the above expression from initial instant $t = 0$ to a general time instant $t = t$ given as

$$\int_0^v \frac{dv}{FR - B^2l^2v} = \int_0^t \frac{dt}{mR}$$

$$\Rightarrow \quad -\frac{1}{B^2l^2}[\ln(FR - B^2l^2v)]_0^v = \frac{1}{mR}[t]_0^t$$

$$\Rightarrow \quad -\frac{1}{B^2l^2}[\ln(FR - B^2l^2v) - \ln(FR)] = \frac{1}{mR}[t-0]$$

$$\Rightarrow \quad \ln\left(\frac{FR - B^2l^2v}{FR}\right) = -\frac{B^2l^2t}{mR}$$

$$\Rightarrow \quad \frac{FR - B^2l^2v}{FR} = e^{-\frac{B^2l^2t}{mR}}$$

$$\Rightarrow \quad FR - B^2l^2v = FRe^{-\frac{B^2l^2t}{mR}}$$

$$\Rightarrow \quad B^2l^2v = FR\left(1 - e^{-\frac{B^2l^2t}{mR}}\right)$$

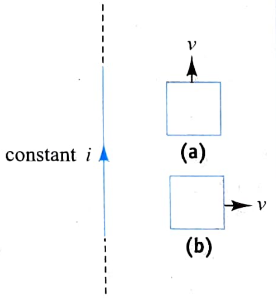
$$\Rightarrow \quad v = \frac{FR}{B^2l^2}\left(1 - e^{-\frac{B^2l^2t}{mR}}\right) \qquad \text{...(v)}$$

With the above expression given in Eq. (v) it can be seen that with time velocity of sliding wire approaches to a steady value and at $t \to \infty$ velocity approaches to the steady velocity called terminal velocity and it is given by Eq. (v) as

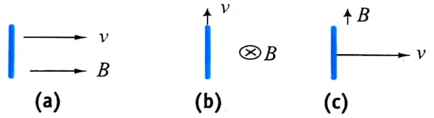
$$v_{\text{terminal}} = \frac{RF}{B^2l^2} \qquad \text{...(vi)}$$

CONCEPT APPLICATION EXERCISE 4.2

1. Figure shows a long current-carrying wire and two rectangular loops moving with velocity v. Find the direction of current in each loop.

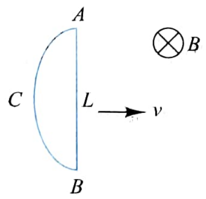


2. A rod of length l is moving with velocity v in magnetic field B as seen in figure. Find the emf induced in all three cases.

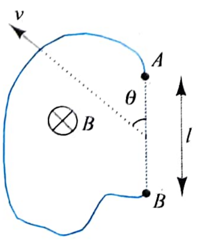


(a) (b) (c)

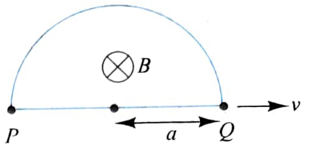
3. Figure shows a closed coil $ABCL$ moving in a uniform magnetic field B with a velocity v.
Find
(a) emf induced in the coil.
(b) emf induced in curved part ACB and straight part AB



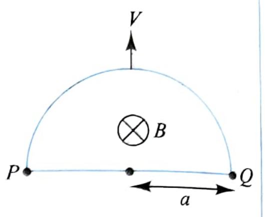
4. Figure shows an irregular shaped wire AB moving with velocity v. Find the emf induced in the wire.



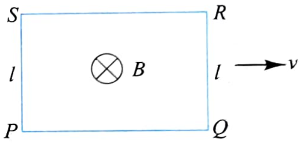
5. Find the emf across points P and Q which are diametrically opposite points of a semicircular closed loop moving in a magnetic field as shown in figure.



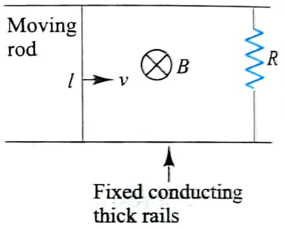
6. Find the emf across points P and Q which are diametrically opposite points of a semicircular closed loop moving in a magnetic field as shown in figure. Also draw the electrical equivalence of each branch.



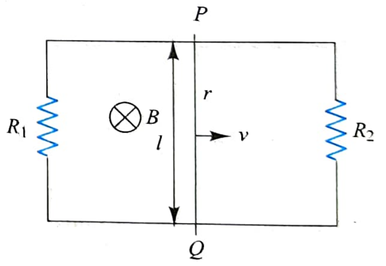
7. Figure shows a rectangular loop moving in a uniform magnetic field. Show the electrical equivalence of each branch. What is the net induced emf?



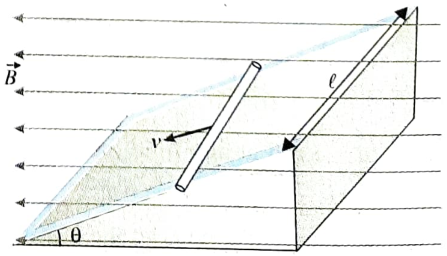
8. Figure shows a rod of length l and resistance r moving on two rails shorted by a resistance R. A uniform magnetic field B is present normal to the plane of rod and rails. Show the electrical equivalence of each branch.



9. Figure shows rod PQ of mass m and resistance r moving on two fixed, resistanceless, smooth conducting rails (closed on both sides by resistances R_1 and R_2). Find the current in the rod (at the instant its velocity is v).

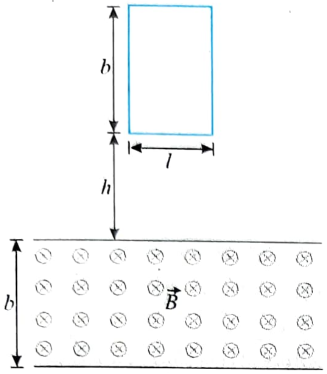


10. A rod of mass m, length l and resistance R is sliding down on a smooth inclined parallel rails with a constant velocity v. If a uniform horizontal magnetic field B exists, then find the value of B.

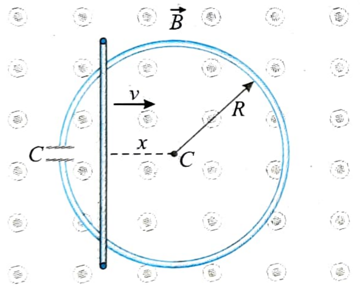


11. A rectangular loop of dimensions $b \times l$ having resistance R, is released from a certain height h into a region of

horizontal magnetic field of induction B. What must be the height of fall 'h' so that the loop will move with a constant velocity in the magnetic field?



12. The conducting rod slides on a conducting ring of radius R which is placed in horizontal plane . A capacitor having capacity C is connected with ring as shown in figure. The rod moves towards right with a constant velocity v in an outward magnetic field B. **(a)** When the rod is at a distance $x = \frac{R}{2}$ from the centre of the circular wire, find the current through the capacitor. **(b)** If we substitute the capacitor by a resistor R_0, find the current as the function of time.



ANSWERS

1. loop (a) —zero, loop (b) —clockwise.

2. (a) 0 as $\vec{B} \| \vec{v}$ (b) 0 as $\vec{l} \| \vec{v}$ (c) 0 as $\vec{l} \| \vec{B}$

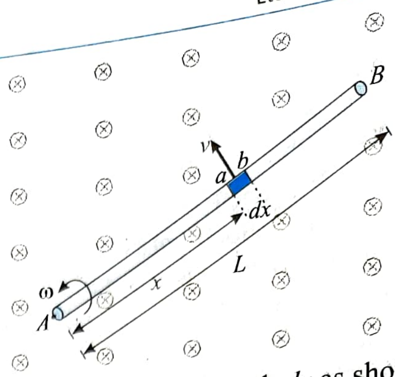
3. (a) 0 (b) vBL **4.** $Bvl \sin\theta$ **5.** 0 **6.** $2Bav$

7. 0 **9.** $\dfrac{Blv}{r + \dfrac{R_1 R_2}{R_1 + R_2}}$ **10.** $\sqrt{\dfrac{mgR}{l^2 v \sin\theta}}$

11. $\dfrac{m^2 g R^2}{2B^4 l^4}$ **12.** (a) $\dfrac{2BCv^2}{\sqrt{3}R}$ (b) $\dfrac{2Bv}{R_0}\sqrt{R^2 - (R - vt)^2}$

EMF ACROSS ROTATING STRAIGHT CONDUCTOR

Consider a straight conductor AB of length L rotating about end A with angular velocity ω. Magnetic field B is perpendicular to the plane of rotation as shown in figure. We want to find emf induced in the conductor.



Take a very small element ab of length dx as shown.

Velocity of this element: $v = \omega x$

Small emf induced in this element: $d\varepsilon = (\vec{v} \times \vec{B}) \cdot \overrightarrow{dx}$

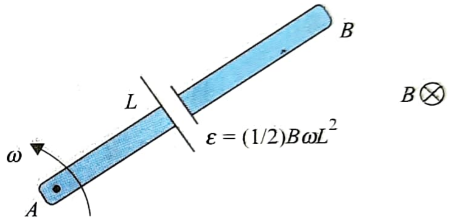
$\Rightarrow \quad d\varepsilon = -vBdx$

(because $\vec{v} \times \vec{B}$ will be opposite to $\overrightarrow{dx}$)

Negative sign indicates that a will be positive w.r.t. b.

Integrating: $\varepsilon = -\int_0^L Bv\,dx = -B\omega \int_0^L x dx = -\frac{1}{2} B\omega L^2$

Negative sign indicates that end A will be at higher potential than B. Hence, magnitude of induced emf in the rod is $\varepsilon = \frac{1}{2} B\omega L^2$ with the polarities induced as shown in figure.



Electric Field at Any Point in the Rod

PD between b and a: $dV_{ba} = -vBdx$

Electric field in this element ab:

$$E = \frac{dV_{ba}}{dx} \quad \rightarrow \quad \text{directed from } b \text{ to } a$$

$$= \frac{-vB\,dx}{dx} \quad \rightarrow \quad \text{directed from } b \text{ to } a$$

$$= -B\omega x \quad \rightarrow \quad \text{directed from } b \text{ to } a$$

$$= B\omega x \quad \rightarrow \quad \text{directed from } a \text{ to } b$$

At end A: $x = 0 \quad \Rightarrow \quad E = 0$

At end B: $x = L \quad \Rightarrow \quad E = B\omega L$ (max)

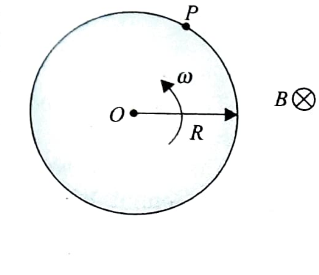
Electric field in the rod is directed from A to B.

A Rotating Conducting Disc

Consider a conducting disc of radius R rotating about its axis in a uniform magnetic field B. Magnetic field is perpendicular to the plane of disc.

Due to the rotation of the disc, electrons inside it experience magnetic force due to which emf is induced in the disc.

In the present situation electrons experience force away from the center. So center acquires positive potential w.r.t. circumference. Induced emf between center O and circumference P is



$$\varepsilon = \frac{1}{2} B\omega R^2$$

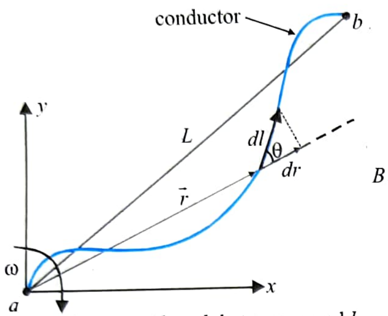
This expression is similar to the emf induced in a rotating rod with replacing L with R.

A Rotating Conductor of Arbitrary Shape

Consider an arbitrary shaped conductor ab which is rotating about end a with angular velocity ω as shown in figure. We want to find induced emf between ends a and b.

Let us consider an element 'dl' of the conductor. The emf induced across the element $d\varepsilon = (\vec{v} \times \vec{B}) \cdot \vec{dl}$

$\vec{v}$ is the velocity of the element.



L is shortest length between a and b

Induced emf developed between ends a and b is given by:

$$\varepsilon = \int d\varepsilon = \int (\vec{v} \times \vec{B}) \cdot \vec{dl}$$

$$\Rightarrow \quad \varepsilon = \int [(\vec{\omega} \times \vec{r}) \times \vec{B}] \cdot \vec{dl} \qquad \text{Here } \vec{r} = x\hat{i} + y\hat{j}$$

$$\varepsilon = \int [\{-\omega\hat{k} \times (x\hat{i} + y\hat{j})\} \times (-B\hat{k})] \cdot \vec{dl}$$

$$= \int [(-\omega x\hat{j} + \omega y\hat{i}) \times (-B\hat{k})] \cdot \vec{dl}$$

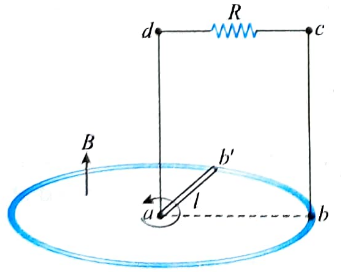
$$= \int (\omega x B\hat{i} + \omega B y\hat{j}) \cdot \vec{dl} = B\omega \int (x\hat{i} + y\hat{j}) \cdot \vec{dl}$$

$$= B\omega \int \vec{r} \cdot \vec{dl} = B\omega \int r\, dl \cos\theta = B\omega \int_0^L r\,dr$$

$$= \frac{1}{2} B\omega L^2$$

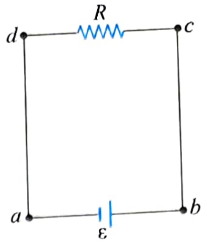
ILLUSTRATION 4.34

A metal rod ab of length l, rotates with constant angular velocity ω about one of its end in a vertically upward magnetic field B. Other end of the rod slides on a circular conducting rail as shown in figure. A resistance R is connected between the fixed points a and b. Find the induced current in the resistor.



Sol. The EMF induced across the rod $\varepsilon = \frac{1}{2} B\omega l^2$

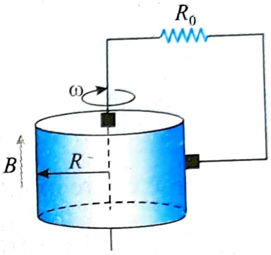
From right hand law we can verify that end b of the rod will be at higher potential. We can draw an equivalent circuit diagram of this situation as shown in figure.



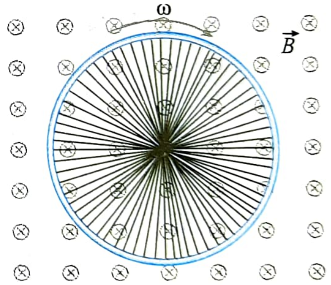
The induced current $i = \frac{\varepsilon}{R} = \frac{Bl^2\omega}{2R}$.

ILLUSTRATION 4.35

A cylinder of radius R is rotating about its axis with constant angular velocity ω in a vertically upward magnetic field B. A resistance R_0 is connected between the axis and the periphery of the cylinder as shown in figure. Find the induced current in the resistor.



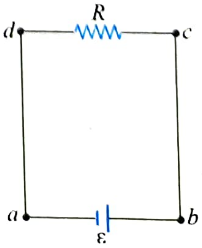
Sol. This case of rotating metallic cylinder (or disc) can considered as a ring connected with a number of spokes connected between center and circumference of ring.



The EMF induced across any of spoke, $\varepsilon = \frac{1}{2} B\omega l^2$

The potential difference across each spoke is same as they are connected in parallel. Hence the potential difference across the axis and the periphery of the cylinder should be $\frac{1}{2} B\omega l^2$.

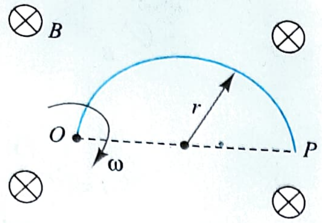
From right hand rule we can verify that the periphery of the cylinder will be at higher potential. We can draw an equivalent circuit diagram of this situation as shown in figure.



The induced current $i = \frac{\varepsilon}{R} = \frac{Bl^2\omega}{2R}$.

ILLUSTRATION 4.36

A wire is in the form of a semicircle of radius r. One end is attached to an axis about which it rotates with an angular speed ω. The axis is normal to the plane of the semicircle. The wire is immersed in a uniform magnetic field B parallel to the axis. Find the induced emf between points O and P of the semicircle.



Sol. It is given that the magnetic field is uniform.

Join the end points O and P and replace the semicircle by a straight rod of length $2r$.

We now have a straight rod rotating in a uniform magnetic field in a plane perpendicular to the magnetic field.

Therefore, the induced emf between O and P will be

$$\xi_{\text{ind}} = \frac{1}{2} B\omega (2r)^2 = 2B\omega r^2$$

From the right hand rule, we see that electrons will accumulate at end O. Therefore, end P is at a higher potential than O.

ILLUSTRATION 4.37

A metal disc of radius $R = 25$ cm rotates with a constant angular velocity $\omega = 130$ rad s^{-1} about its axis. Find the potential difference between the center and rim of the disc if

(a) the external magnetic field is absent,

(b) the external uniform magnetic field $B = 5.0$ mT is directed perpendicular to the disc.

Sol.

(a) Centripetal force required for circular motion of electron is generated by a radial electric field caused by the redistribution of the electrons in the disc.

$$F = eE = mr\omega^2 \Rightarrow E = \frac{mr\omega^2}{e}$$

From $dV = -E\,dr$, we have

$$dV = -\frac{m\omega^2}{e} r\,dr$$

$$\Rightarrow \int_{V_1}^{V_2} dV = -\frac{m\omega^2}{e} \int_0^R r\,dr$$

$$V_1 - V_2 = \frac{m\omega^2 R^2}{2e} = \frac{(9.1\times10^{-31})(130)^2(0.25)^2}{(2)(1.6\times10^{-19})}$$

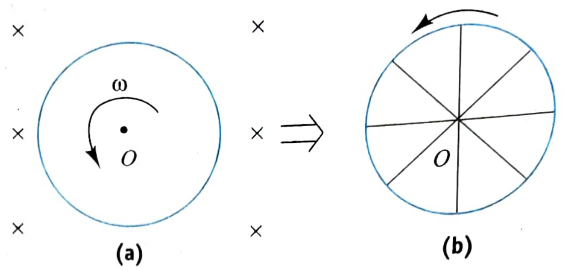
$$= 3.0 \times 10^{-9}\text{ V} = 3.0\text{ nV}$$

$V_1 > V_2$, i.e., potential at center is more than the potential at edge.

(b) A disc may be assumed to be made up of a large number of radial, conducting, differential elements rotating with angular velocity ω about the center of disc O. Thus,

$$V_{\text{center}} - V_{\text{edge}} = \frac{1}{2} BR^2\omega$$

Now, $V_{\text{center}} > V_{\text{edge}}$ for anticlockwise rotation and $V_{\text{edge}} > V_{\text{center}}$ for clockwise rotation.



(a)
(b)

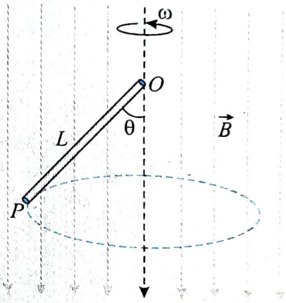
Substituting the values, we have

$$V_{\text{center}} - V_{\text{edge}} = \frac{1}{2} \times 5.0 \times 10^{-3} \times 0.25 \times 0.25 \times 130$$

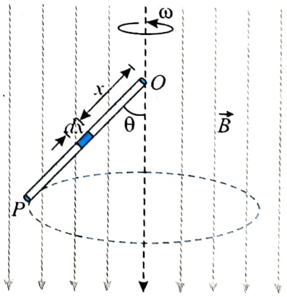
$$= 0.02\text{ V} = 20\text{ mV}$$

ILLUSTRATION 4.38

A metallic rod of length L rotates in the form of a conical pendulum with an angular velocity ω about a fixed vertical axis passing through its end O as shown in figure. A uniform magnetic field B present in vertical downward direction. The rod makes an angle θ with the direction of the magnetic field. Find the emf induced across the ends of the rod.



Sol. Let us consider an elemental length dx at a distance x from O. The radius of rotation of the element is $x \sin\theta$. The speed of the element is perpendicular to the plane of the figure and its magnitude is $v = \omega(x\sin\theta)$



Emf induced in the element is

$$d\varepsilon = Bv(dx)\sin\theta \; [\theta \text{ is angle between } dx \text{ and } B]$$

$$d\varepsilon = B\omega \sin^2\theta\, x dx \qquad \text{...(i)}$$

For total emf can be calculated by integrating Eq. (i)

$$\therefore \quad \varepsilon = \int d\varepsilon = B\omega \sin^2\theta \int_0^L x dx = \frac{1}{2} B\omega \sin^2\theta L^2$$

From right hand rule we can verify that the free electrons experience force towards P. There will be deficiency of electron at O, hence O is positive.

MOTIONAL EMF WHEN THE MAGNETIC FIELD IS NON-UNIFORM

In some of the cases, motion of a conductor may be in a non-uniform magnetic field. Take the following steps while calculating motional emf.

Step 1: Determine the magnetic field at all points on the rod.

Step 2: Consider a small element at some distance from one end of the rod.

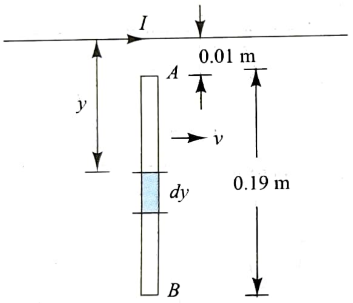
Step 3: Assuming B to be uniform over this element, calculate the potential difference across this element using the procedures outlined earlier.

Step 4: Integrate over the entrie rod to calculate the total induced emf.

Let us learn to calculate the induced emf through the illustrations given below.

ILLUSTRATION 4.39

A copper rod of length 0.19 m is moving with uniform velocity 10 m s^{-1} parallel to a long straight wire carrying a current of 5.0 A. The rod is perpendicular to the wire with its ends at distances 0.01 and 0.2 m from it. Calculate the emf induced in the rod.



Sol. As shown in figure, consider an element of length dy at a distance y from the wire, then at this position of the element, the magnetic field due to the current-carrying wire PQ will be $B = \dfrac{\mu_0}{4\pi}\dfrac{2I}{y}$ into the plane of the paper.

So, the emf induced in the element $d\varepsilon = Bv\,dy = \dfrac{\mu_0}{4\pi}\dfrac{2I}{y}v\,dy$

and hence the emf induced across the ends of the rod due to its motion in the field of the wire,

$$\varepsilon = \int_a^b d\varepsilon = \frac{\mu_0}{4\pi}2Iv\int_a^b \frac{dy}{y}, \text{ i.e., } \varepsilon = \frac{\mu_0}{4\pi}2Iv\log_e\left(\frac{b}{a}\right)$$

Substituting the given data with $b = (a + l)$,

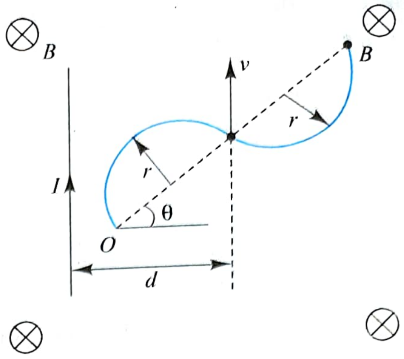
$$\varepsilon = 10^{-7}\times 2\times 5\times 10\log_e\frac{0.20}{0.01}$$

$$= 10^{-5}\times \log_e 20$$

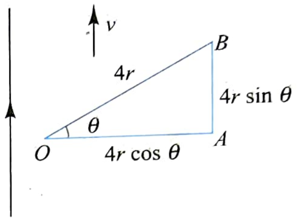
$$\simeq 30\ \mu\text{V}$$

ILLUSTRATION 4.40

An infinite wire carries a current I. An S-shaped conducting rod of two semicircles each of radius r is placed at an angle θ to the wire. The center of the conductor is at a distance d from the wire. If the rod translates parallel to the wire with a velocity v as shown in figure, calculate the emf induced across the ends OB of the rod.



Sol. If we join the end points O and B and replace the two semicircles by a straight rod of length $4r$ (figure).



Induced emf in straight rod OB will be same as in the actual conductor. Now this straight rod can be replaced by a combination of two rods OA and AB. Induced emf in OB will be same as the sum of induced emf in OA and induced emf in AB. But induced emf in AB will be zero because its velocity will be parallel to its length. Hence, induced emf in OA will be the net induced emf in actual conductor.

We now have a rod of effective length $4r\cos\theta$ translating in a non-uniform magnetic field. Consider a small element of the wire of length dx located at a distance x from the wire. The magnetic field at this element is $B = \dfrac{\mu_0 I}{2\pi x}$

The potential difference across this element is $dV = \dfrac{\mu_0 I}{2\pi x}v\,dx$.

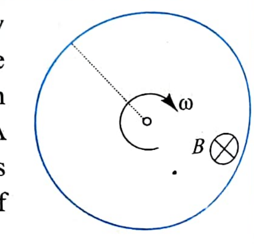
The potential difference across the ends of the rod can be calculated by integrating over the whole end. Therefore,

$$V = \int dV = \int_{d-2r\cos\theta}^{d+2r\cos\theta}\frac{\mu_0 I}{2\pi x}v\,dx = \frac{\mu_0 Iv}{2\pi}\ln\left[\frac{d+2r\cos\theta}{d-2r\cos\theta}\right]$$

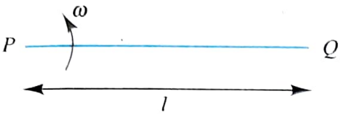
From the right hand rule, we see that electrons will accumulate at end B. Therefore, end O is at a higher potential than B.

CONCEPT APPLICATION EXERCISE 4.3

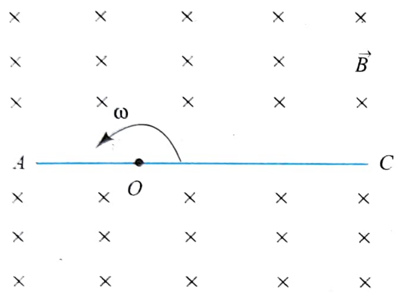
1. A ring rotates with angular velocity ω about an axis perpendicular to the plane of the ring passing through the center of the ring (figure). A constant magnetic field B exists parallel to the axis. Find the emf induced in the ring.



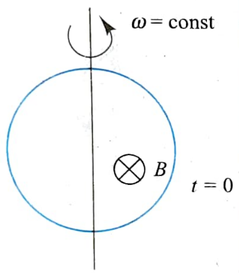
2. A rod PQ of length l is rotating about end P, with an angular velocity ω (figure). Due to centrifugal forces the free electrons in the rod move toward the end Q and an emf is created. Find the induced emf.



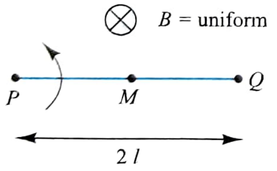
3. A conducting rod AC of length $4l$ is rotated about point O in a uniform magnetic field $\vec{B}$ directed into the plane of the paper. $AO = l$ and $OC = 3l$. Find $V_A - V_C$.



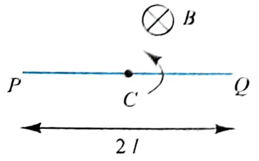
4. A ring rotates with angular velocity ω about an axis in the plane of the ring which passes through the center of the ring. A constant magnetic field B exists perpendicular to the plane of the ring (figure). Find the emf induced in the ring as a function of time.



5. Rod PQ of length $2l$ is rotating about one end P in a uniform magnetic field B which is perpendicular to the plane of rotation of the rod (figure). Point M is the mid-point of the rod. Find the induced emf between M and Q if the potential between P and Q is 100 V.

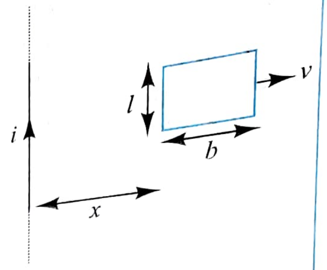


6. Rod PQ of length $2l$ is rotating about its midpoint C in a uniform magnetic field B which is perpendicular to the plane of rotation of the rod (figure). Find the induced emf between PQ and PC. Draw the circuit diagram of parts PC and CQ.

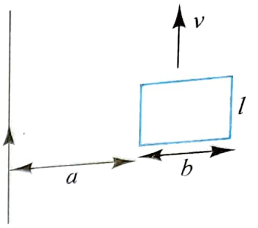


7. A rod of length l is kept parallel to a long wire carrying constant current i. It is moving away from the wire with a velocity v. Find the emf induced in the wire when its distance from the long wire is x.

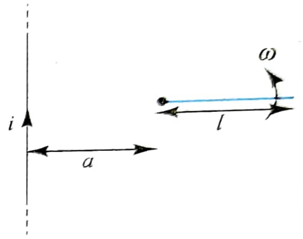
8. A rectangular loop, as shown in figure, moves away from an infinitely long wire carrying a current i. Find the emf induced in the rectangular loop.



9. A rectangular loop is moving parallel to a long wire carrying current i with a velocity v. Find the emf induced in the loop (figure) if its nearest end is at a distance a from the wire. Draw equivalent electrical diagram.



10. A rod of length l is rotating with an angular speed ω about one of its ends which is at a distance a from an infinitely long wire carrying current i. Find the emf induced in the rod at the instant shown in figure.



ANSWERS

1. 0 **2.** $\dfrac{m_e\omega^2 l^2}{2e}$ **3.** $4B\omega l^2$ **4.** $BA\omega \sin \omega t$

5. 75 V **6.** $\varepsilon_{PQ} = 0, \varepsilon_{PC} = \dfrac{B\omega l^2}{2}$ **7.** $\dfrac{\mu_0 ilv}{2\pi x}$

8. $\dfrac{\mu_0 iblv}{2\pi x\,(l+b)}$ **9.** 0 **10.** $\dfrac{\mu_0 i\omega}{2\pi}\left[l - a\ln\left(\dfrac{l+a}{a}\right)\right]$

INDUCED ELECTRIC FIELD

INDUCED EMF IN A STATIONARY CONDUCTOR

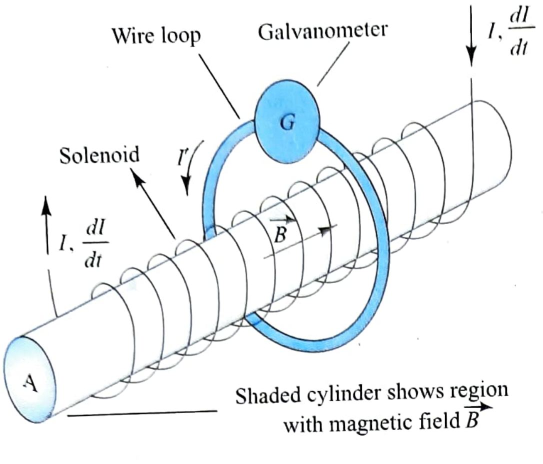
According to Faraday's law, it is the relative motion between the loop and the magnet which produces the induced emf; it does not matter whether the loop moves toward the magnet or the magnet moves toward the loop. When the loop moves toward the magnet, it is the magnetic force which drives the charge to flow. But what causes the induced current in a stationary loop when magnet moves toward it? A magnetic field cannot exert a force on a stationary conductor. Whenever a magnetic field is varying with time, an induced electric field E is produced in any closed path whether in matter or in empty space.

$$\oint \vec{E}\cdot d\vec{l} = -A\frac{dB}{dt}$$

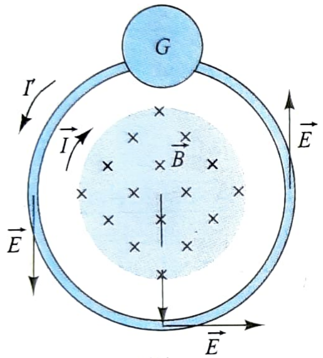
The induced electric field is different from the electrostatic field of the Coulomb field.

- Unlike electrostatic field, the lines of induced field form closed loops. It is also called a circuital field or vortex field.
- It is not a conservative field, i.e., $\oint \vec{E}\cdot d\vec{l} \neq 0$.

- The line integral of the electrostatic field between any two points is the potential difference while the line integral of the induced electric field between any two points is the electromotive force.



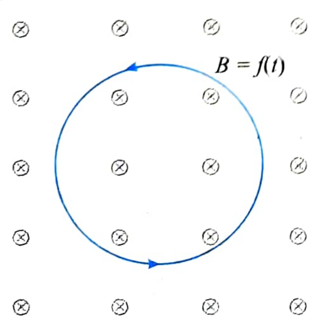
(a)



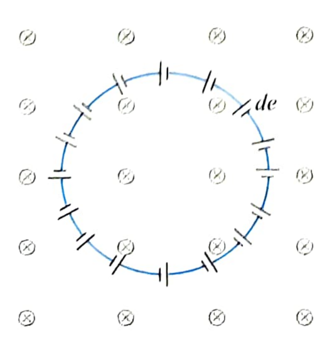
(b)

Figure (a) shows the windings of a long solenoid carrying current I that is increasing at a rate of dI/dt. The magnetic flux in the solenoid is increasing at a rate of $d\Phi_B/dt$, and this changing flux passes through a wire loop. An emf $\varepsilon = -d\Phi_B/dt$ is induced in loop, inducing a current I' that is measured by the galvanometer G. Figure (b) shows the cross-sectional view.

Figure (a) shows a single turn circular coil placed in time varying magnetic field in inward direction of which magnitude is increasing with the function $B = f(t)$. According to Lenz's law due to increase in magnetic flux through the coil an anticlockwise current is induced in it to oppose the increasing flux. Actually, this induced current is caused by the EMF induced in every element of the coil as shown in Fig. (b). Every small element of the coil behaves like an elemental EMF de and all such elemental EMFs are in series. Sum of all these EMFs in the coil is called loop EMF .



(a)



(b)

Note:

There are two basic mechanisms of induced emf generation.

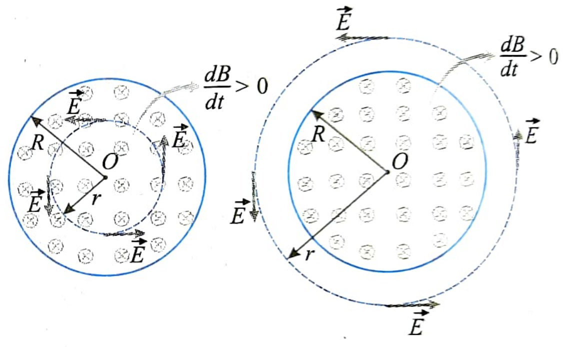
1. The first one involves the motion of a conductor relative to magnetic field lines, called the motional emf.
2. The second one involves the generation of an electric field associated with a time-varying magnetic field. In the modified form, Faraday's law may be stated as:

$$\varepsilon = \oint (\vec{v} \times \vec{B}) \cdot d\vec{l} - \vec{A} \cdot \frac{d\vec{B}}{dt}$$

TIME-VARYING MAGNETIC FIELD

Consider a conducting loop of area A in a uniform but time-varying magnetic field. Rate of change of magnitude of magnetic field $= dB/dt$ for the loop, flux linked with it $= BA = \phi$ (say) (take area vector directed along $\vec{B}$).

Hence, rate of change of flux ϕ is $A\frac{dB}{dt}$ and hence induced emf $= -A\frac{dB}{dt}$.



$r < R$
(a)

$r > R$
(b)

For non-zero values of dB/dt, there could be a definite current in the loop, whose direction can be obtained using Lenz's rule. For example, if $\frac{dB}{dt} > 0$, i.e., B is increasing with time, magnetic field produced by the induced current would oppose the existing magnetic field. Hence, the induced current would be anticlockwise.

The current in the loop can be easily known if the resistance of the loop is known as $I = \varepsilon/R$.

The field associated with the induced emf in case of time-varying magnetic field is non-conservative as, then only we would have non-zero value for $\oint \vec{E}_n \cdot d\vec{l}$. Here $\vec{E}_n$ denotes the induced field caused by the time-varying magnetic field.

For the path described by the loop, $\varepsilon_{\text{induced}} = -\frac{d\phi}{dt} = \oint \vec{E}_n \cdot d\vec{l}$.

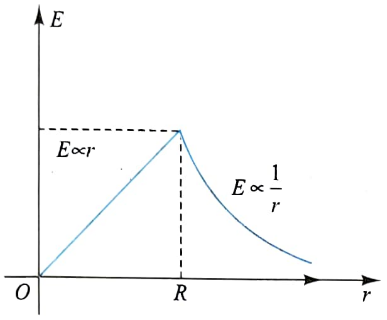
Consider a magnetic field where B (magnitude of the magnetic field) is a function of $r (r < R)$. The distance of the point from O is shown in Fig. (a). For the circular path.

For $r < R$, $E_n(2\pi r) = \pi r^2 \left|\frac{dB}{dt}\right| \Rightarrow E_n = \frac{r}{2}\left|\frac{dB}{dt}\right|$

Now follow Fig. (b).

$$\text{For } r > R,\ E_n 2\pi r = \pi R^2 \left|\frac{dB}{dt}\right| \quad \Rightarrow \quad E_n = \frac{R^2}{2r}\left|\frac{dB}{dt}\right|$$

Direction of $\vec{E}_n$ can be easily obtained as it would be responsible for the induced current when a conducting loop is placed on the given path. For example, in the present case, for $\frac{dB}{dt} > 0$, path is in anticlockwise direction. Variation of E with r is shown in figure.



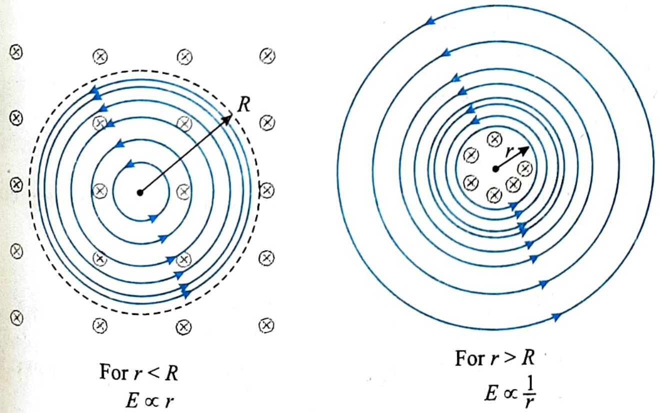
ELECTRIC FIELD LINES OF FORCES

The induced electric field in time varying magnetic field have closed loop electric lines of forces as there are no static charges or source of electric field is present in this situation for which the field is non-conservative in nature as for a charge going round the loop electric force acts in same direction and hence work done is non zero. Thus for the induced electric field in time varying magnetic field we have for a general closed path

$$\oint \vec{E}\cdot\vec{dl} \neq 0$$

Thus in regions of time varying magnetic fields at any point in space we cannot define potential as electric field in this region is non-conservative in nature.

Inside the magnetic field region the electric field increases with distance from axis of region thus the density of electric lines of forces increases as we move away from the central axis. The configuration of electric lines is shown in Fig. (a). But outside the magnetic field region the electric field decreases with distance from axis of region thus the density of electric lines of forces decreases as we move away from the central axis. The configuration of electric lines is shown in Fig. (b).



For $r < R$
$E \propto r$
(a)

For $r > R$
$E \propto \frac{1}{r}$
(b)

ILLUSTRATION 4.41

A thin non-conducting ring of mass m carrying a charge q can freely rotate about its axis. At the initial moment, the ring was at rest and no magnetic field was present. Then a uniform magnetic field was switched on, which was perpendicular to the plane of the ring and increased with time according to a certain law: $\frac{dB}{dt} = k$.

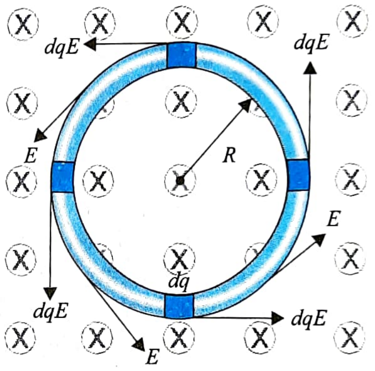
Find the angular velocity ω of the ring as a function of k.

Sol. $E = \frac{1}{2}R\frac{dB}{dt}$

Electric force on charge dq is given by

$$dF = Edq = \frac{1}{2}R\left(\frac{dB}{dt}\right)dq$$

$$\Rightarrow \quad d\tau = RdF$$



$$\Rightarrow \quad d\tau = \frac{1}{2}R^2\left(\frac{dB}{dt}\right)dq$$

$$\Rightarrow \quad \tau = \frac{1}{2}R^2kq$$

$$\text{Now,} \quad \tau = \frac{1}{2}R^2kq$$

$$\Rightarrow \quad I\alpha = \frac{1}{2}R^2kq \quad \Rightarrow \quad mR^2\alpha = \frac{1}{2}R^2kq$$

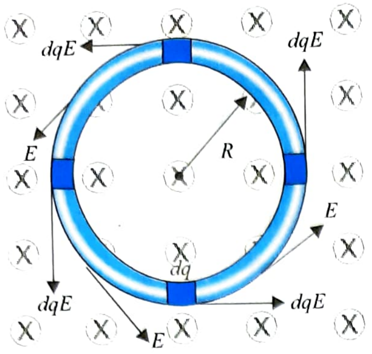
$$\Rightarrow \quad \alpha = \frac{kq}{2m} \quad \Rightarrow \quad \omega = \frac{kq}{2m}t$$

ILLUSTRATION 4.42

A thin non-conducting ring of mass m carrying a charge q can rotate freely about its axis. At $t = 0$, the ring was at rest and no magnetic field was present. Then suddenly a magnetic field B was set perpendicular to the plane. Find the angular velocity acquired by the ring.

Sol. Due to the sudden charge in flux, an electric field is set up and the ring experiences an impulsive torque and suddenly acquires an angular velocity.

$$E = \frac{r}{2}\frac{dB}{dt}$$



Force experienced by an element of ring $dF = dqE$

Torque experienced by this element $d\tau = dFr = dq\,Er$

Total torque experienced by ring: $\tau = qEr$

$$\Rightarrow \qquad \tau = q\frac{r}{2}\frac{dB}{dt}r = \frac{q}{r}r^2\frac{dB}{dt}$$

Angular impulse experienced

$$L = \int \tau\,dt = \frac{qr^2}{2}\int\frac{dB}{dt}dt = \frac{qr^2}{2}B$$

$$L = I\omega \Rightarrow mr^2\omega = \frac{qr^2}{2}B \Rightarrow \omega = \frac{qB}{2m}$$

ILLUSTRATION 4.43

A non-conducting ring of mass m and radius R has charge Q uniformaly distributed over its circumference. The ring is placed on a rough horizontal surface such that the plane of the ring is parallel to the surface. A vertical magnetic field $B = B_0t^2$ tesla is switched on. After 2 s from switching on the magnetic field, the ring is just about to rotate about vertical axis through its center.

(a) Find friction coefficient μ between the ring and the surface.

(b) If magnetic field is switched off after 4 s, then find the angular velocity of the ring just after switching off the magnetic field.

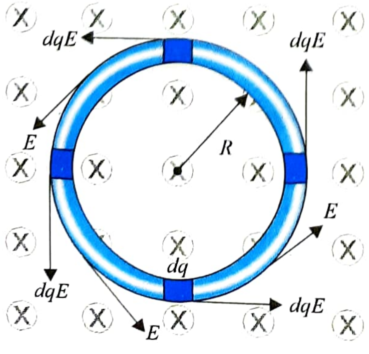
(c) Find the angle rotated by the ring before coming to rest after switching off the magnetic field.

Sol.

(a) $E = \frac{R}{2}\frac{dB}{dt} \Rightarrow E = B_0Rt$

Force on the ring $F = QE = B_0QRt$. This force is tangential to the ring. The ring starts rotating when torque of this force is greater than the torque due to maximum friction ($f_{\max} = \mu mg$).

$$\tau_F \geq \tau_{\text{fric, max}},\ FR \geq \mu mgR \Rightarrow F > \mu mg$$



$$B_0QRt = \mu mg$$

Hence, $\mu = \dfrac{B_0QRt}{mg}$

Given, $t = 2$ s $\Rightarrow \mu = \dfrac{2B_0QR}{mg}$

(b) After 2 s

Net torque $\tau = \tau_F - \tau_{f\max} = B_0QR^2t - \mu mgR$

$$= B_0QR^2t - \left(\frac{2B_0QR}{mg}\right)mgR$$

$$\Rightarrow \tau = B_0QR^2(t-2) \Rightarrow I\alpha = B_0QR^2(t-2)$$

$$mR^2\left(\frac{d\omega}{dt}\right) = B_0QR^2(t-2)$$

$$\Rightarrow d\omega = \frac{B_0QR^2}{mR^2}(t-2)\,dt$$

$$\Rightarrow \int_0^\omega d\omega = \frac{B_0Q}{m}\int_2^4 (t-2)\,dt$$

$$\Rightarrow \omega = \frac{2B_0Q}{m}$$

(c) If magnetic field is switched off after 4 s, only force present is frictional force which will retard the motion.

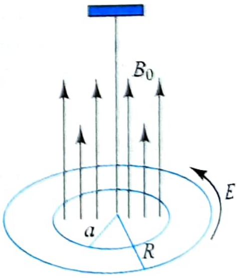
Retarding torque $\tau = \tau_{\text{friction}}$

Angular retardation $\alpha = \dfrac{\tau_{\text{friction}}}{I} \Rightarrow \alpha = \dfrac{\mu mgR}{mR^2} = \dfrac{\mu g}{R}$

Using $\omega^2 = \omega_0^2 + 2\alpha\theta \Rightarrow 0 = \left(2\dfrac{B_0Q}{m}\right)^2 - 2\left(\dfrac{\mu g}{R}\right)\theta$

$$\Rightarrow \theta = 2\left(\frac{B_0Q}{m}\right)^2\frac{R}{\mu g}$$

ILLUSTRATION 4.44



A line charge with linear charge density λ is wound around an insulating disc of mass M and radius R, which is then suspended horizontally as shown in figure, so that it is free to rotate. In the central region, of radius a, there is a uniform magnetic field B_0, pointing up. Now the magnetic field is switched off, which causes the disc to rotate.

Find the angular speed with which the disc starts rotating.

Sol. The induced electric field E due to the changing magnetic field is given by (from Faraday's law)

$$\Rightarrow \qquad E \cdot 2\pi R = -\pi a^2\frac{dB}{dt} \Rightarrow E = \frac{-a^2}{2R}\frac{dB}{dt}$$

Hence, induced electric field is tangential to the disc as shown in figure and its magnitude is $E = \dfrac{a^2}{2R}\dfrac{dB}{dt}$.

This electric field causes the disc to rotate. Now torque on the disc is $\tau = (\lambda 2\pi R)\, ER = \pi\lambda a^2 R\,\dfrac{dB}{dt}$.

Angular impulse:

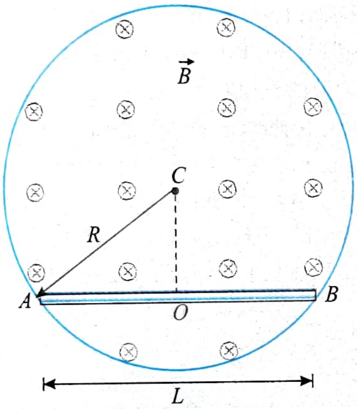
$$L = \int \tau dt = \int \pi\lambda a^2 R \frac{dB}{dt} dt = \pi\lambda a^2 R \int dB = \pi\alpha a^2 RB_0$$

Now $L = I\omega$

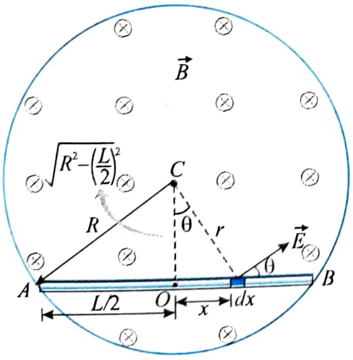
$$\Rightarrow \quad \pi\lambda a^2 RB_0 = \frac{MR^2}{2}\omega \;\Rightarrow\; \omega = \frac{2\pi\lambda a^2 B_0}{MR}$$

ILLUSTRATION 4.45

A cylindrical region of radius R is filled with a uniform magnetic field B as shown in the figure. A metal rod AB of length L is placed inside the field such that its ends are symmetrically located with respect to the center of the circular cross section of the region. If the magnetic field is changed at a rate $\dfrac{dB}{dt} = \alpha$, find the emf induced across the metal rod.



Sol. When the rodis placed in time varying magnetic field then due to induced electric field free electrons of the metal are displaced which causes an opposing static electric field to be developed inside the metal rod due to static charges and at every point inside the wire its magnitude is equal to that of non-conservative induced electric field of the region. To understandthis we will discuss on an illustration shown in figure.



Let us consider an element of length dx on rod at a distance x from point O as shown in figure above. The induced electric field at the location of element dx can be given by

$$E_i = \frac{1}{2} r \frac{dB}{dt} = \frac{1}{2} r\alpha \qquad \text{...(i)}$$

Due to the component of this electric field along the length of the rod its free electrons will drift toward left and left end A of rod becomes negative and due to deficiency of electrons end B becomes positive. These charges establishes another electric field inside the rod from B to A. In steady condition this electric field balances the component of E_i along the rod so that no further drift of free electrons take place. Thus the electric field inside the rod due to static charges which is conservative in nature is given as

$$E = E_i \cos\theta \qquad \text{...(ii)}$$

$$\Rightarrow \quad E = \frac{1}{2} r\alpha \times \frac{\sqrt{R^2 - \left(\frac{L}{2}\right)^2}}{r}$$

$$\Rightarrow \quad E = \frac{1}{2}\alpha\sqrt{R^2 - \left(\frac{L}{2}\right)^2} \qquad \text{...(iii)}$$

The expression in above Eq. (iii) shows that the static electric field inside the rod is uniform and depends only upon the distance of axis of rod from the center of the cylindrical region in which magnetic field is confined.

If this electric field is uniform, the potential difference across the rod is given as

$$V_B - V_A = EL$$

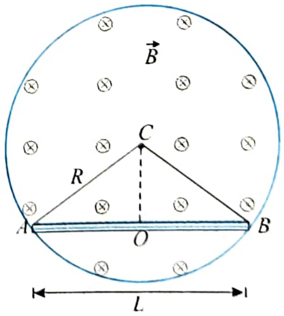
$$\Rightarrow \quad V_B - V_A = \left(\frac{1}{2}\alpha\sqrt{R^2 - \left(\frac{L}{2}\right)^2}\right) L$$

$$\Rightarrow \quad V_B - V_A = \frac{1}{2}\alpha L\sqrt{R^2 - \left(\frac{L}{2}\right)^2}$$

2nd Approach

Let us join end A and B of the rod with center of the circle and consider a triangular closed loop ABC

Area of the loop $A = \dfrac{1}{2} \times L \times \sqrt{R^2 - \left(\dfrac{L}{2}\right)^2}$

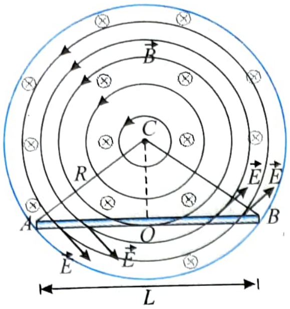


Flux through loop ABC is $\phi = B.\left(\dfrac{1}{2} \times L \times \sqrt{R^2 - \left(\dfrac{L}{2}\right)^2}\right)$

The magnitude of the emf induced in the loop is

$$\varepsilon = \frac{d\phi}{dt} = A\frac{dB}{dt} = \left(\frac{1}{2} \times L \times \sqrt{R^2 - \left(\frac{L}{2}\right)^2}\right).\alpha$$

Because of symmetry the induced field lines will be circular.



It means field lines of E are perpendicular to CA and CB. The emf in the loop ABC can also be written as

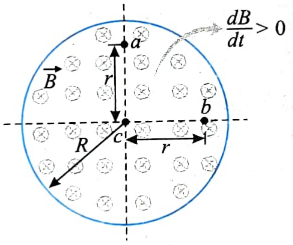
$$\int_C^A \vec{E}\cdot\vec{dl} + \int_A^B \vec{E}\cdot\vec{dl} + \int_B^C \vec{E}\cdot\vec{dl} = \frac{1}{2}\alpha L\sqrt{R^2 - \left(\frac{L}{2}\right)^2}$$

$$0 + \int_A^B \vec{E}\cdot\vec{dl} + 0 = \frac{1}{2}\alpha L\sqrt{R^2 - \left(\frac{L}{2}\right)^2}$$

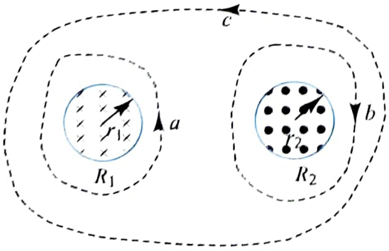
$$\int_A^B \vec{E}\cdot\vec{dl} = V_B - V_A = \frac{1}{2}\alpha L\sqrt{R^2 - \left(\frac{L}{2}\right)^2}$$

CONCEPT APPLICATION EXERCISE 4.4

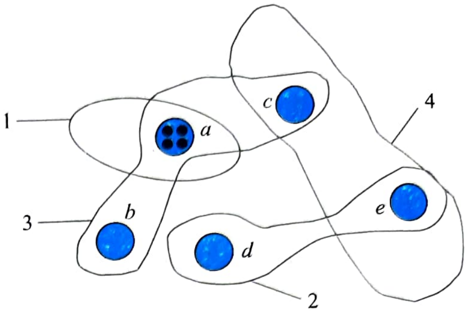
1. The magnetic field at all points within a circular region of radius R is uniform in space and directed into the plane of the page in figure (the region could be a cross section inside the windings of a long, straight solenoid). If the magnetic field is increasing at a rate dB/dt, what are the magnitude and direction of the force on a stationary positive point charge q located at points a, b, and c? (Point a is a distance r above the center of the region, point b is at a distance r to the right of the center, and point c is at the center of the region.)



2. Figure shows two circular regions R_1 and R_2 with radii $r_1 = 20.0$ cm and $r_2 = 25.0$ cm, respectively. In R_1 there is a uniform magnetic field $B_1 = 48.6$ mT into the page and in R_2 there is a uniform magnetic field $B_2 = 77.2$ mT out of the page (ignore any fringing of these fields). Both fields are decreasing at the rate 8.0 mT s^{-1}. Calculate the integral $\oint \vec{E}\cdot\vec{dl}$ for each of the three identical paths.

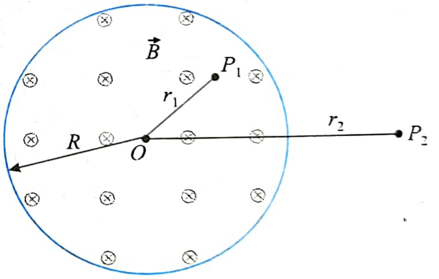


3. Figure shows five lettered regions in which a uniform magnetic field extends directly either out of the page (as in region a) or into the page. The field is increasing in magnitude at the same steady rate in all the five regions; the regions are identical in area. Also shown are four numbered paths along which $\oint \vec{E}\cdot\vec{dl}$ has the magnitudes given below in terms of a quantity *mag*. Determine whether the magnetic fields in regions b through e are directed into or out of the page.

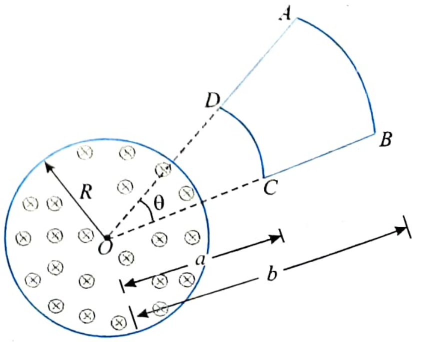


Path:	1	2	3	4
$\oint \vec{E}\cdot\vec{dl}$	*mag*	2(*mag*)	3(*mag*)	0

4. In a cylindrical region of radius $R = 2.5$ cm the magnetic field changes with time according to,
$B = (2.00t^3 - 4.00t^2 + 0.8)$ T and
   - **(a)** Find the force on an electron located at point P_2 at a distance $r_2 = 5.0$ cm, from the centre of magnetic field region at time $t = 2$ s.
   - **(b)** What is the magnitude and direction of the electric field at P_1 when $t = 3$ s and $r_1 = m$.



5. A uniform magnetic field present in cylindrical region of radius R centered at O as shown in figure. The magnetic field is changing at a rate $\frac{dB}{dt} = \beta$. There present a conducting loop $ABCD$ in the plane of the figure with its arms BC and DA along two radial direction from O having an angle θ between them. AB and CD are two circular arcs of radius b and a respectively centered at O. Find

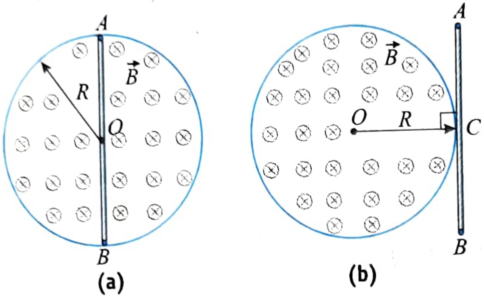


(a) the emf induced in the loop $ABCD$.

(b) the emf induced in arc AB

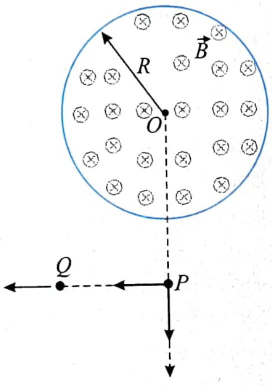
(c) the emf induced in arc BC

6. A uniform magnetic field B is present in a cylindrical region of radius R. The field is increasing at a constant rate of $\frac{dB}{dt} = \alpha$. A straight conducting rod AB of length $2R$ is placed in two situations as shown in Figs. (a) and (b). Find the emf induced in the rod when it is placed
   (a) along the diameter shown in Fig. (a)
   (b) along the tangent shown in Fig. (b) point C is midpoint of the rod.

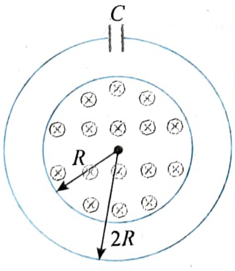


(a) (b)

7. A particle having charge q is placed at a point P outside the cylindrical magnetic field region, where uniform magnetic field is perpendicular to the plane of the figure in a cylindrical region of radius R. The magnetic field is increasing at a constant rate of βTs^{-1}. If the particle is slowly moved to infinity, calculate work done by the external agent on the particle in case
   (a) the particle is moved in radial direction OP.
   (b) the particle is moved in a direction perpendicular to OP along PQ.



8. A long solenoid of radius R and n be the number of turns per unit length carries a linearly increasing current $i = \frac{i_0}{T}t$. A co-axial circular conducting loop of radius $2R$ is fitted with a parallel capacitor whose plate area is A and separation is d. Find the:
   (a) charge deposited in the capacitor
   (b) static electric field in the capacitor
   (c) induced electric field in the capacitor
   (d) total electric field in the capacitor



ANSWERS

1. Force on a: $\frac{qr}{2}\left(\frac{dB}{dt}\right)(-\hat{i})$; on b: $\frac{qr}{2}\left(\frac{dB}{dt}\right)(-\hat{j})$; on c: zero

2. For loop a: $-3.2\pi \times 10^{-4}$ V, For loop b: $= -5\pi \times 10^{-4}$ V, For loop c: $-1.8\pi \times 10^{-4}$ V

3. $b, c \rightarrow$ out of the page; $d, e \rightarrow$ into the page

4. (a) 8.0×10^{-21} N (b) 0.3 V/m

5. (a) Zero (b) $\frac{R^2\theta\beta}{2}$ (c) Zero

6. (a) Zero (b) $\frac{\pi R^2}{4} \cdot \alpha$ **7.** (a) Zero (b) $\frac{\pi q R^2 \beta}{4}$

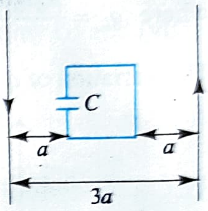
8. (a) $\frac{\pi R^2 \mu_0 n i_0 \varepsilon_0 A}{dT}$ (b) $\frac{\mu_0 \pi R^2 n i_0}{dT}$ (c) $\frac{R\mu_0 n i_0}{4T}$

(d) $\frac{\mu_0 n i R}{T}\left[\frac{\pi R}{d} - 1\right]$

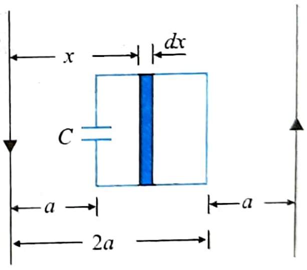
Solved Examples

EXAMPLE 4.1

Two infinitely long parallel wires carrying currents $I = I_0 \sin \omega t$ in opposite direction are placed at a distance $3a$ apart. A square loop of side a of negligible resistance with a capacitor of capacitance C is placed in the plane of wires as shown in figure. Find the maximum current in the square loop. Also sketch the graph showing the variation of charge on the upper plate of the capacitor as a function of time for one complete cycle taking anticlockwise direction for the current in the loop as positive.



Sol. Let us first find the flux through the loop due to left wire.



$$\phi_L = \int_a^{2a} \frac{\mu_0 I}{2\pi x} a dx = \frac{\mu_0 I a}{2\pi} (\log_e 2)$$

This flux will be in outward direction because magnetic field due to wire is in outward direction and area vector is also in outward direction because anticlockwise direction is positive. Now flux due to right wire will also be same because loop is placed symmtrically w.r.t. both wires. So net flux through the loop is

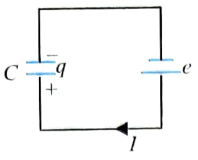
$$\phi = 2\phi_L = \frac{\mu_0 I a}{\pi} \log_e 2$$

Induced emf in the loop:

$$e = -\frac{d\phi}{dt} = -\frac{\mu_0 a(\log_e 2)}{\pi}\left(\frac{dI}{dt}\right)$$

$$= -\frac{\mu_0 I_0 \omega a}{\pi}(\log_e 2)\cos \omega t$$

Negative sign indicates that induced emf will be in clockwise sense because anticlockwise sense is positive. Finally, induced emf in the loop is $e = \frac{\mu_0 \omega a I_0 (\log_e 2)}{\pi} \cos \omega t$ as shown in figure.



Let q be the charge on the capacitor at any time, then

$$q = Ce = \frac{\mu_0 I_0 \omega a C \log_e 2 \cos \omega t}{\pi}$$

Current I in the circuit: $I = \frac{dq}{dt} = -\frac{\mu_0 I_0 \omega^2 a C \log_e 2}{\pi} \sin \omega t$

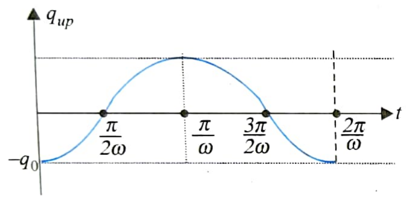
Hence, maximum current in the loop is

$$I_{\max} = \frac{\mu_0 I_0 \omega^2 a C \log_e 2}{\pi}$$

Charge on the upper plate: $q_{\text{up}} = -q = -q_0 \cos \omega t$

where $q_0 = \frac{\mu_0 I_0 \omega a C (\log_e 2)}{\pi}$

The variation of q_{up} with time is as shown in figure.



EXAMPLE 4.2

A thermocol vessel contains 0.5 kg of distilled water at 30°C. A metal coil of area 5×10^{-3} m^2, number of turns 100, mass 0.06 kg and resistance 1.6 Ω is lying horizontally at the bottom of the vessel. A uniform time-varying magnetic field is set up to pass vertically through the coil at time $t = 0$. The field is first increased from zero to 0.8 T at a constant rate between 0 and 0.2 s and then decreased to zero at the same rate between 0.2 and 0.4 s. The cycle is repeated 12000 times. Make sketches of the current through the coil and the power dissipated in the coil as function of time for the first two cycles. Clearly indicate the magnitude of the quantities on the axes. Assume that no heat is lost to the vessel or the surroundings. Determine the final temperature of water under thermal equilibrium. Specific heat of metal = 500 J kg^{-1} K^{-1} and the specific heat of water = 4200 J kg^{-1} K^{-1}. Neglect the inductance of coil.

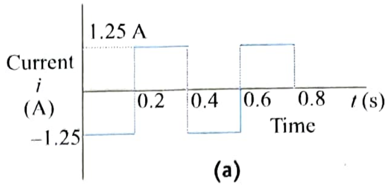
Sol. Induced emf due to changing magnetic field

$$\varepsilon = -\frac{d\phi}{dt} = -\frac{d}{dt}(NBA \cos 0^\circ) = -NA\frac{dB}{dt}$$

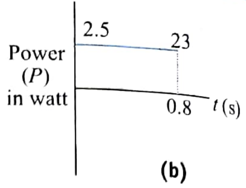
$$= -100 \times 5 \times 10^{-3} \times \frac{0.8}{0.2} = -2 \text{ V}$$

It is negative between 0 and 0.2 s when field is increased and positive between 0.2 and 0.4 s when field is decreased.

Current induced $i = \frac{\varepsilon}{R} = \frac{2}{1.6} = 1.25$ A



(a)



(b)

Power dissipated $= i^2R = (1.25)^2 \times 1.6 = 2.5$ W

Total energy supplied $= Pt = 12{,}000(2.5 \times 0.4) = 12 \times 10^3$ J

Now from the conservation of energy

$$12 \times 10^3 = m_1c_1\Delta\theta + m_2c_2\Delta\theta$$

$$\Rightarrow \quad = (m_1c_1 + m_2c_2)\Delta\theta$$

$$\Rightarrow \quad \Delta\theta = \frac{12 \times 10^3}{m_1c_1 + m_2c_2}$$

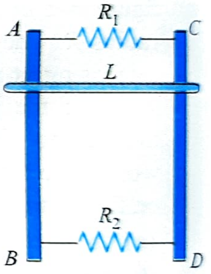
$$= \frac{12 \times 10^3}{0.5 \times 4200 + 0.06 \times 500} = \frac{12 \times 10^3}{2130} = 5.6^\circ$$

$$\Delta\theta = \theta_2 - \theta_1$$

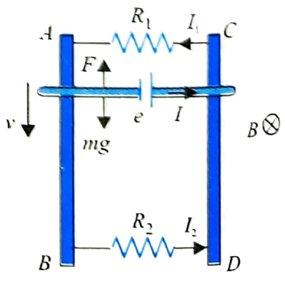
$\Rightarrow$ Final temperature: $\theta_2 = \theta_1 + \Delta\theta = 30 + 5.6 = 35.6$°C

EXAMPLE 4.3

Two parallel vertical metallic rails AB and CD are separated by 1 m. They are connected at two ends by resistances R_1 and R_2 as shown in figure. A horizontal metallic bar L of mass 0.2 kg slides without friction vertically down the rails under the action of gravity. There is a uniform horizontal magnetic field of 0.6 T perpendicular to the plane of the rails. It is observed that when the terminal velocity is attained, the power dissipated in R_1 and R_2 are 0.76 and 1.2 W, respectively. Find the terminal velocity of the bar L and the values of R_1 and R_2.



Sol. Let magnetic field $B = 0.6$ T be inward as shown figure. When the rod moves down due to gravity, emf is induced in the rod as shown. Due to this, current I flows in the rod which results in force F on the rod in upward direction. Let at any instant of time, velocity of rod be v.



Then $e = Bvl$ and $I = \frac{e}{R_{\text{eq}}}$,

where $\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2}$

$$F = IlB = \frac{e}{R_{\text{eq}}}lB = \frac{B^2l^2v}{R_{\text{eq}}}$$

Initially, F is less than mg. But as the velocity increases, F also increases. At a certain time, F will be equal to mg. At that time net force on the rod will become zero and velocity of rod will become constant. This constant velocity is known as terminal velocity. Let it be v_t. So putting $F = mg$

$$\Rightarrow \quad \frac{B^2l^2v_t}{R_{\text{eq}}} = mg \quad \text{...(i)}$$

But at this time, power disspated in resistances is

$$P_1 + P_2 = 0.76 + 1.2 = 1.96\ \text{W}$$

So we have $\dfrac{e^2}{R_{eq}} = P_1 + P_2 \Rightarrow \dfrac{B^2 l^2 v_t^2}{R_{eq}} = 1.96$ ...(ii)

From Eqs. (i) and (ii) $v_t = \dfrac{1.96}{mg} = \dfrac{1.96}{0.2 \times 9.8} = 1\ \text{m s}^{-1}$

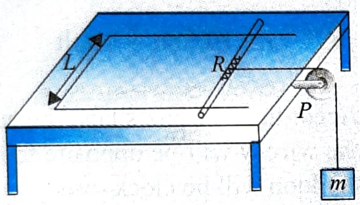
From Eqs. (i) $R_{eq} = \dfrac{B^2 l^2 v_t}{mg} = \dfrac{(0.6)^2 \times 1^2 \times 1}{0.2 \times 9.8} = \dfrac{9}{49}\ \Omega$

$$\frac{P_1}{P_2} = \frac{e^2/R_1}{e^2/R_2} \Rightarrow \frac{0.76}{1.20} = \frac{R_2}{R_1} \Rightarrow \frac{19}{30} = \frac{R_2}{R_1}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow \frac{49}{9} = \frac{1}{R_1} + \frac{30}{19R_1}$$

$$\Rightarrow \quad R_1 = \frac{9}{19}\ \Omega \quad \text{and} \quad R_2 = \frac{19}{30} R_1 = \frac{19}{30} \times \frac{9}{19} = 0.3\ \Omega$$

EXAMPLE 4.4



A pair of parallel horizontal conducting rails of negligible resistance shorted at one end is fixed on a table. The distance between the rails is *L*. A conducting massless rod of resistance *R* can slide on the rails frictionlessly. The rod is tied to a massless string which passes over a pulley fixed to the edge of the table. A mass *m* tied to the other end of the string hangs vertically. A constant magnetic field *B* exists perpendicular to the table if the system is released from rest. Calculate

(a) the terminal velocity achieved by the rod, and

(b) the acceleration of the mass at the instant when the velocity of the rod is half the terminal velocity.

Sol.

(a) If *v* is the instantaneous velocity of the rod at any time *t*, the induced emf will be $\varepsilon = BvL$.

$\therefore$ Induced current in rod

$$i = \frac{\varepsilon}{R} = \frac{RvL}{R} \qquad \text{...(i)}$$

Due to this current, the rod in magnetic field *B* will experience a force

$F = BiL$ opposite to motion. ...(ii)

If v_T is terminal velocity of rod, and *T* the tension in the string, then free-body diagrams of rod and mass *m* are shown in figure.

So equation of motion of mass *m* is $mg - T = ma$ ...(iii)

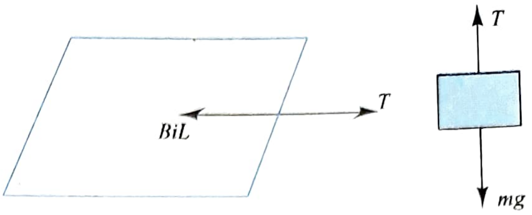
For terminal velocity, $a = 0$

$mg - T = 0 \Rightarrow T = mg$ ...(iv)

Equation of motion of rod is

$T - BiL = 0 \times a = 0$ (as rod is massless)

$T = BiL.$ ...(v)



Using Eq. (iv), we get, $mg = BiL$

$$mg = B\left(\frac{Bv_T L}{R}\right)L \Rightarrow v_T = \frac{mgR}{B^2L^2} \qquad \text{...(vi)}$$

(b) When velocity of the rod is half the terminal velocity

$$v = \frac{v_T}{2} = \frac{1}{2}\frac{mgR}{B^2L^2} \qquad \text{...(vii)}$$

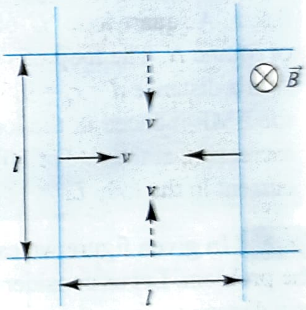
From Eq. (iii), acceleration

$$A = g - \frac{T}{m} = g - \frac{BiL}{m} \qquad \text{[using (v)]}$$

$$= g - \frac{BL}{m}\frac{BL}{R}\frac{\frac{1}{2}mgR}{B^2L^2} \qquad \text{[using (i) and (vii)]}$$

$$= g - \frac{g}{2} = \frac{g}{2}$$

EXAMPLE 4.5



In figure, the four rods have λ resistance per unit length. The arrangement is kept in a magnetic field of constant magnitude *B* and directed perpendicular to the plane of the figure and directing inward. Initially, the sides as shown form a square. Now each wire starts moving with constant velocity *v* toward the opposite wire.

Find as a function of time:

(a) induced emf in the circuit.

(b) induced current in the circuit with direction.

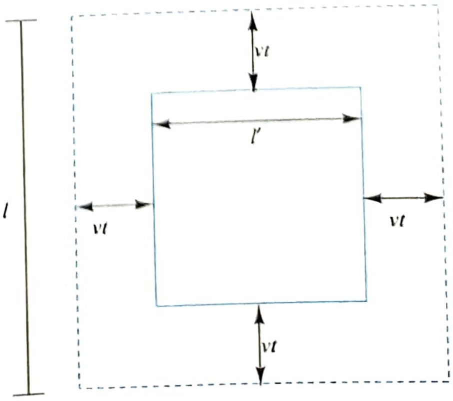
(c) force required on each wire to keep its velocity constant.

(d) total power required to maintain constant velocity.

(e) thermal power developed in the circuit.

Sol.

(a) $E_{net} = 4Bl'v$



Equivalent circuit:

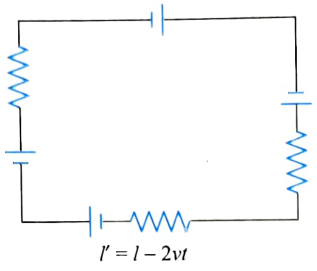
$\therefore \quad E_{\text{net}} = 4B(l-2vt)v$

$E_{\text{net}} = 4Bv(l-2vt)$

(b) $I = \dfrac{E_{\text{net}}}{4r} = \dfrac{4Bl'v}{4\lambda l'} \Rightarrow I = \dfrac{Bv}{\lambda}$

(c) Force required on each wire $= Il'B$

Force $= \dfrac{B^2v}{\lambda}(l-2vt)$



(d) Total power required to maintain constant velocity

$= 4Fv = \dfrac{4B^2v^2}{\lambda}(l-2vt)$

(e) Thermal power developed in the circuit

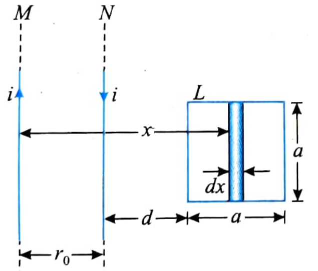
$= 4I^2r = 4I^2\lambda\,(l-2vt) = 4\dfrac{B^2v^2}{\lambda^2}\lambda(l-2vt)$

Thermal power $= \dfrac{4B^2v^2}{\lambda}(l-2vt)$

EXAMPLE 4.6

Two infinite long straight parallel wires '*M*' and '*N*' each carry equal currents but in opposite directions, are separated by $r_0 =$ 0.5 m. A square loop '*L*' of side $a = r_0 = 0.5$ m lies in the plane of *M* and *N*. The loop of wire *L* is kept parallel to both *M* and *N* at a distance $d = r_0 = 0.5$ m from the nearest wire. Calculate the EMF induced in the loop *L* if the currents in *M* and *N* are increasing at the rate of 10^3 A/s. Also indicate the direction of current in the loop *L*.

Sol. In given figure we can observe the situation described in the problem. Let us consider an element of width *dx* in the loop *L* at a distance *x* from the wire *M* as shown.



Magnetic field at the location of the element due to current in *M*

$$B = \frac{\mu_0 i}{2\pi x}$$

Area of the element $dA = a \times dx$

Magnetic flux linked with the elemental area is given as

$$d\phi = \frac{\mu_0 i}{2\pi x} \times (adx)$$

The magnetic flux linked with the entire loop *L* is given as

$$\phi_1 = \int d\phi = \int_{2r_0}^{3r_0}\left(\frac{\mu_0 i}{2\pi x}\right)adx = \frac{\mu_0 ia}{2\pi}\int_{2r_0}^{3r_0}\frac{dx}{x}$$

$$\Rightarrow \quad \phi_1 = \frac{\mu_0 ia}{2\pi}[\ln x]_{2r_0}^{3r_0} = \frac{\mu_0 ia}{2\pi}\ln\left(\frac{3}{2}\right)$$

This flux is directed into the plane of paper. Similarly, the magnetic flux linked with the loop *L* due to the current *i* in wire *N* is given as

$$\phi_2 = \int_{r_0}^{2r_0}\left(\frac{\mu_0 i}{2\pi x}\right)adx = \frac{\mu_0 ia}{2\pi}\ln(2)$$

This flux is directed outward perpendicular to the plane of paper. Net magnetic flux linked with the loop *L* is given as

$$\phi_C = \phi_2 - \phi_1$$

$$\phi_C = \frac{\mu_0 ia}{2\pi}\ln\left(\frac{4}{3}\right)$$

The induced EMF in the loop is given as

$$e = \left|\frac{d\phi}{dt}\right| = \frac{\mu_0 a}{2\pi}\log_e\left(\frac{4}{3}\right)\frac{di}{dt}$$

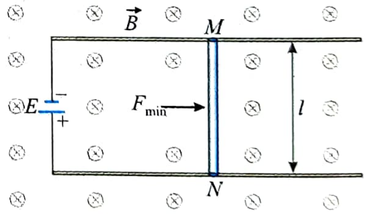
$$\Rightarrow \quad e = 2\times10^{-7}\times0.5\times\ln\left(\frac{4}{3}\right)\times10^3$$

$$\Rightarrow \quad e = 10^{-4}\ln\left(\frac{4}{3}\right)\text{ volt}$$

According to Lenz's law, the current in the arm of loop *L* nearest the wire *N* will be opposite to the current in *N*, so the current in the loop will be clockwise.

EXAMPLE 4.7

Two long parallel conducting rails are fixed at a distance *l* apart in a horizontal plane. A rod of mass *m* and resistance *R* can slide along the rails. A uniform magnetic field of induction *B* exists along vertically downward direction and coefficient of friction between rails and rod is *m*. If a battery of emf *E* and negligible internal resistance is connected between the rails at left end, as shown in figure.



(i) Calculate minimum value, F_{min} of force required to be applied horizontally rightwards on the rod to move it to the right.

If $F = 2F_{\text{min}}$ calculate:

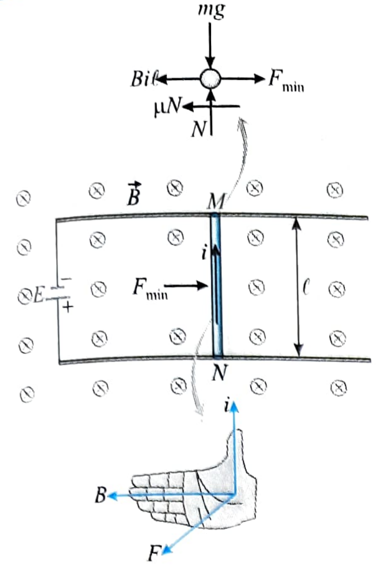
(ii) Steady state velocity of the rod,

(iii) Power required to keep the rod moving with that steady velocity.

Sol.

(i) Current supplied by battery, $i = \dfrac{E}{R}$

Considering free body diagram of the rod

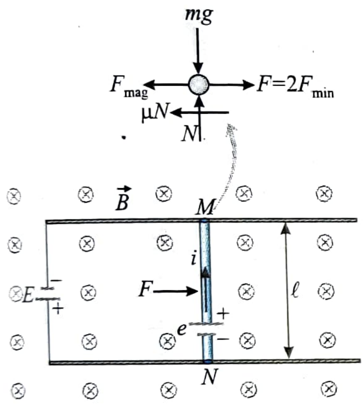


For vertical forces, $N = mg$

For horizontal forces, $F_{\min} = Bil + \mu N$

$$F_{\min} = \left(\frac{BEl}{R} + \mu mg\right)$$

(ii) Let the steady state velocity of the rod be v_0.



Induced emf in rod, $e = Blv_0$

Total emf of the circuit, $E + e = (E + Blv_0)$

Current through the circuit, $i = \dfrac{(E + Blv_0)}{R}$

Due to current, leftward force on the rod,

$$F'_{mag} = Bil = \frac{Bl(E + Blv_0)}{R}$$

For steady state velocity, $F'_{mag} + \mu mg = 2F_{\min}$

$$\frac{Bl}{R}(E + Blv_0) + \mu mg = 2\left(\frac{BEl}{R} + \mu mg\right)$$

$$Bl(E + Blv_0) + \mu mgR = 2(BEl + \mu mgR)$$

$$B^2l^2v_0 = BEl + \mu mgR$$

$$v_0 = \left(\frac{BEl + \mu mgR}{B^2l^2}\right)$$

(iii) Power required to keep the rod moving with this steady velocity v_0 is, $P = Fv_0$

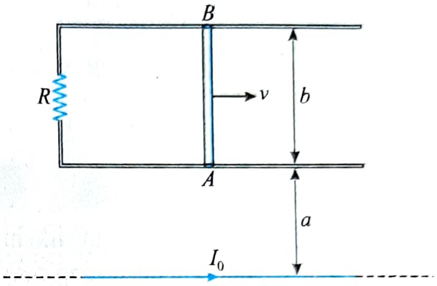
$$P = 2\left(\frac{BEl}{R} + \mu mg\right) \times \left(\frac{BEl + \mu mgR}{B^2l^2}\right)$$

$$= 2\left(\frac{BEl + \mu mgR}{R}\right) \times \left(\frac{BEl + \mu mgR}{B^2l^2}\right)$$

$$P = \frac{2(BEl + \mu mgR)^2}{B^2l^2R}$$

EXAMPLE 4.8

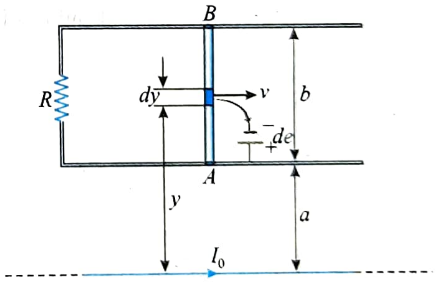
A long straight conductor carries a current i_0. At distances a and $(a + b)$ from it, there are two identical rails, each having resistance λ per unit length, which are inter-connected by a resistance R as shown in figure. A conducting rod AB of length b can slide along the wires without friction. At $t = 0$, the rod is in extreme left position and starts to move to the right with constant velocity v.



Calculate force F required to maintain velocity of the rod constant as function of time t.

Sol. Magnetic field due to long wire $B = \dfrac{\mu_0 i_0}{2\pi y}$

EMF induced in the elemental length, $de = Bvdy = \dfrac{\mu_0 i_0}{2\pi y}.v\,dy$



Total emf induced in the rod,

$$e = \int_{y=a}^{y=(a+b)} \frac{\mu_0 i_0 v}{2\pi y} dy = \frac{\mu_0 i_0 v}{2\pi} \int_a^{(a+b)} \frac{dy}{y} = \frac{\mu_0 i_0 v}{2\pi}[\log y]_a^{(a+b)}$$

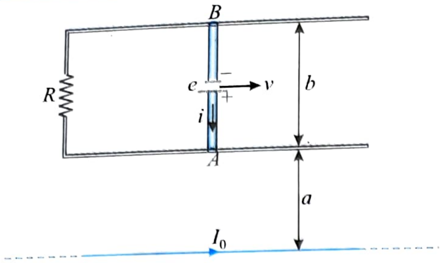
$$\Rightarrow \quad e = \frac{\mu_0 I_0 v}{2\pi} \log\left(\frac{a+b}{a}\right)$$

At time t, distance of rod from left end $= vt$.

Total resistance of the circuit at this instant $= (R + 2vt\lambda)$.

$$\text{Induced current, } i = \frac{\text{induced emf}}{\text{resistance}} = \frac{\dfrac{\mu_0 i_0 v}{2\pi}\log\left(\dfrac{a+b}{a}\right)}{(R + 2vt\lambda)}$$

The mechanical power given to system by pulling the rod is appearing as electrical power in the circuit.



Hence, Mechanical power supplied to rod is equal to Electrical power generated in the circuit.

Mechanical power = Electrical power $\Rightarrow Fv = ei$

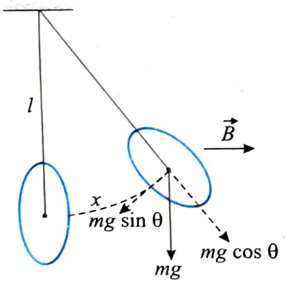
$$\Rightarrow \quad Fv = \frac{\mu_0 i_0 v}{2\pi}\log\left(\frac{a+b}{a}\right)\cdot\frac{\frac{\mu_0 i_0 v}{2\pi}\log\left(\frac{a+b}{a}\right)}{(R+2vt\lambda)}$$

$$\Rightarrow \quad F = \frac{\mu_0^2 i_0^2 v}{4\pi^2(R+2vt\lambda)}\left[\log\left(\frac{a+b}{a}\right)\right]^2$$

EXAMPLE 4.9

A circular wire frame of area 4.0 × 10^{-4} m^2 having resistance 20 Ω is suspended freely from a 0.40 m long thread. There is a uniform magnetic field of 0.8 T and the plane of wire-frame is perpendicular to the magnetic field. The frame is made to oscillate under gravity by displacing it through 2 × 10^{-2} m from its initial position along the direction of magnetic field. The plane of the frame is always along the direction of thread and does not rotate about it. What is the induced EMF in wire-frame as a function of time? Also find the maximum current in the frame.

Sol. The situation given in problem is shown in figure.



The instantaneous flux through the frame when displaced by an angle θ is given as

$$\phi = BA\cos\theta$$

Instantaneous induced EMF

$$e = -\frac{d\phi}{dt}BA\sin\theta\frac{d\theta}{dt}$$

$$e = BA\theta\frac{d\theta}{dt} \quad [As\ \sin\theta \approx \theta] \qquad ..(i)$$

As the loop is very small in size compared to the length of string so we can consider it like a simple pendulum of which the motion equation is given as

$$\theta = \theta_0 \sin\omega t$$

Substituting the value of θ in Eq. (i), we have

$$e = BA(\theta_0\sin\omega t)\frac{d}{dt}(\theta_0\sin\omega t)$$

$$\Rightarrow \quad e = BA\theta_0\sin\omega t\,\theta_0\cos\omega t$$

$$\Rightarrow \quad e = \frac{1}{2}BA\omega\theta_0^2\sin 2\omega t$$

Here we have $\omega = \sqrt{\left(\frac{g}{l}\right)} = \sqrt{\left(\frac{10}{0.40}\right)} = 5$ rad/s

and $\quad \theta_0 = \frac{x_0}{l} = \frac{2\times10^{-2}}{0.40}$ rad

Substituting the values, we get

$$e = \frac{1}{2}\times0.80\times4.0\times10^{-4}\times5\times\left(\frac{2\times10^{-2}}{0.40}\right)^2\sin 10t$$

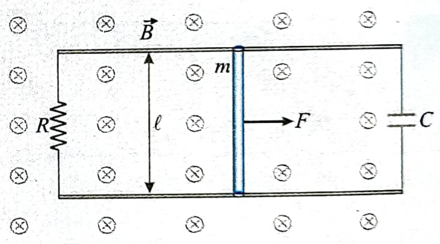
$$e = 2\times10^{-6}\sin 10t$$

$$e_{\max} = 2\times10^{-6}\text{ V}$$

and $\quad i_{\max} = \frac{e_{\max}}{R} = \frac{2\times10^{-6}}{20} = 10^{-7}$ A

EXAMPLE 4.10

Two long rails are horizontal and parallel to each other. On one end, the rails are connected by a resistance R and on the other end a capacitor of capacitance C is connected as shown in figure.



A connector of mass m and length l can slide on the rails with friction. Vertical component of earth's magnetic induction is B (downward). A constant horizontal force F starts acting on the connector.

(i) Calculate steady state velocity of the connector and steady charge on capacitor.

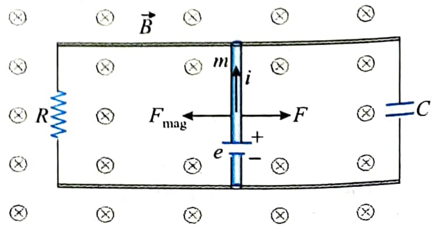
(ii) If at $t = 0$, the connector was at rest, calculate its velocity as a function of time t.

Sol.

(i) Let the steady state velocity of the rod be v_0.

Induced emf in rod, $e = Blv_0$

Induced current $i = \frac{e}{R} = \frac{Blv_0}{R}$



Magnetic force due to induced current

$$F_{mag} = B\left(\frac{Blv_0}{R}\right).l = \frac{B^2l^2v_0}{R}$$

But $F_{mag} = F$

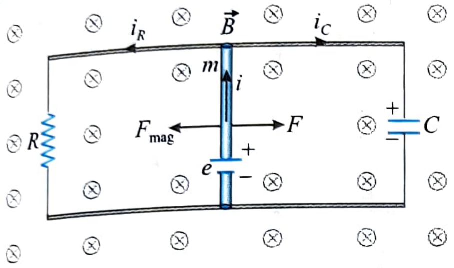
$$\frac{B^2l^2v_0}{R} = F \text{ or } v_0 = \frac{FR}{B^2l^2}$$

In steady state, charge on capacitor is $q_0 = Ce$.

$$q_0 = C(Blv_0) = C\left(Bl\times\frac{FR}{B^2l^2}\right) = \frac{CFR}{Bl}$$

(ii) Let at some instant t, velocity of connector be v.

Charge in capacitor, $q = C(Blv)$



At this instant, current through resistance is $i_R = \frac{e}{R} = \frac{Blv}{R}$

The current through capacitor branch,

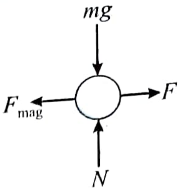
$$i_C = \frac{dq}{dt} = CBl\frac{dv}{dt} = CBla$$

a = acceleration of the connector

Total induced current through the connector is, $i = i_R + i_C$

$$i = \frac{Blv}{R} + CBla$$

From F.B.D of connector



$$F - F_{mag} = ma \Rightarrow F - Bil = ma$$

$$F - B\left(\frac{Blv}{R} + CBla\right)l = ma$$

$$F - \left(\frac{B^2l^2v}{R} + CB^2l^2a\right) = ma$$

$$F - \frac{B^2l^2v}{R} = (m + CB^2l^2)\frac{dv}{dt}$$

$$FR - B^2l^2v = R(m + CB^2l^2)\frac{dv}{dt}$$

$$\therefore \quad \frac{dv}{FR - B^2l^2v} = \frac{dt}{R(m + CB^2l^2)}$$

Integrating both sides

$$\int_0^v \frac{dv}{FR - B^2l^2v} = \int_0^t \frac{dt}{R(m + CB^2l^2)}$$

$$\frac{1}{-B^2l^2}\Big[\log\big(FR - B^2l^2v\big)\Big]_0^v = \frac{t}{R(m + CB^2l^2)}$$

$$\left[\log\frac{\left(FR - B^2l^2v\right)}{FR}\right] = -\frac{B^2l^2t}{R(m + CB^2l^2)}$$

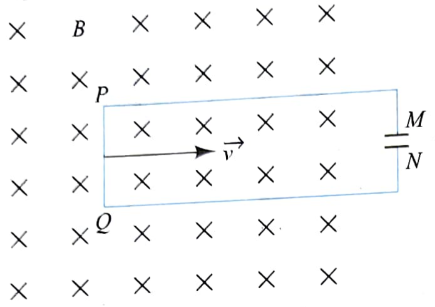
$$\frac{\left(FR - B^2l^2v\right)}{FR} = e^{-\frac{B^2l^2t}{R(m+CB^2l^2)}} \Rightarrow 1 - \frac{B^2l^2}{FR}v = e^{-\frac{B^2l^2t}{R(m+CB^2l^2)}}$$

$$v = \frac{FR}{B^2l^2}(1 - e^{-\frac{B^2l^2t}{R(m+CB^2l^2)}})$$

Exercises

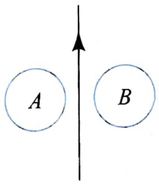
Single Correct Answer Type

1. A rod PQ is connected to the capacitor plates. The rod is placed in a magnetic field (B) directed downward perpendicular to the plane of the paper. If the rod is pulled out of magnetic field with velocity $\vec{v}$ as shown in figure,



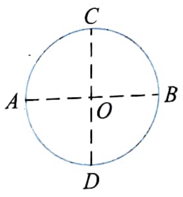
(1) Plate M will be positively charged.
(2) Plate N will be positively charged.
(3) Both plates will be similarly charged.
(4) No charge will be collected on plates.

2. A and B are two metallic rings placed at opposite sides of an infinitely long straight conducting wire as shown in figure. If current in the wire is slowly decreased, the direction of the induced current will be



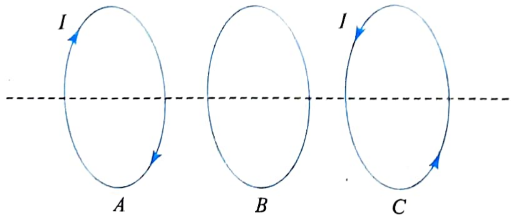
(1) clockwise in A and anticlockwise in B
(2) anticlockwise in A and clockwise in B
(3) clockwise in both A and B
(4) anticlockwise in both A and B

3. A vertical conducting ring of radius R falls vertically with a speed V in a horizontal uniform magnetic field B which is perpendicular to the plane of the ring. Which of the following statements is correct?



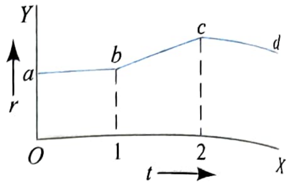
(1) A and B are at the same potential
(2) C and D are at the same potential
(3) current flows in clockwise direction
(4) current flows in anticlockwise direction

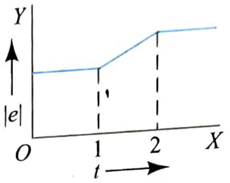
4. Three identical coils A, B, and C carrying currents are placed coaxially with their planes parallel to one another. A and C carry currents as shown in figure. B is kept fixed, while A and C both are moved toward B with the same speed. Initially, B is equally separated from A and C. The direction of the induced current in the coil B is

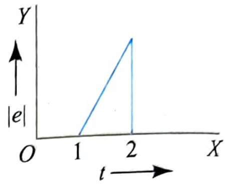


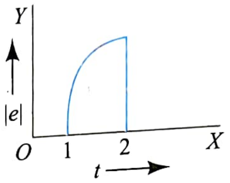
(1) same as that in coil A (1) same as that in coil B
(3) zero (4) none of these

5. A flexible wire bent in the form of a circle is placed in a uniform magnetic field perpendicularly to the plane of the coil. The radius of the coil changes as shown in figure. The graph of magnitude of induced emf in the coil is represented by



(1)


(2)


(3)


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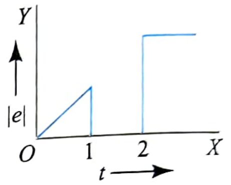
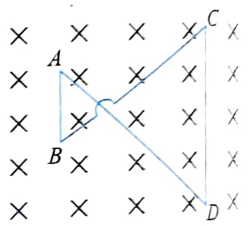
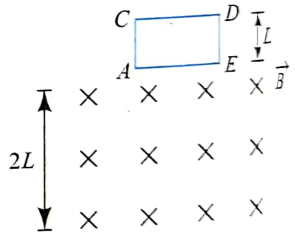


6. A conducting wire frame is placed in a magnetic field which is directed into the plane of the paper (figure). The magnetic field is increasing at a constant rate. The directions of induced currents in wires AB and CD are



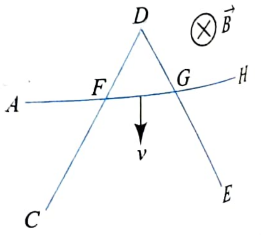
(1) B to A and D to C (2) A to B and C to D
(3) A to B and D to C (4) B to A and C to D

7. A square coil $ACDE$ with its plane vertical is released from rest in a horizontal uniform magnetic field $\vec{B}$ of length $2L$ (figure). The acceleration of the coil is



(1) less than g for all the time till the loop crosses the magnetic field completely
(2) less than g when it enters the field and greater than g when it comes out of the field
(3) g all the time
(4) less than g when it enters and comes out of the field but equal to g when it is within the field

8. A long conducting wire AH is moved over a conducting triangular wire CDE with a constant velocity v in a uniform magnetic field $\vec{B}$ directed into the plane of the paper. Resistance per unit length of each wire is ρ. Then

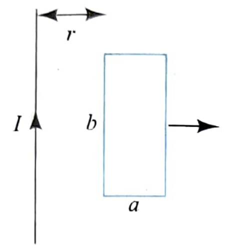


(1) a constant clockwise induced current will flow in the closed loop
(2) an increasing anticlockwise induced current will flow in the closed loop

(3) a decreasing anticlockwise induced current will flow in the closed loop

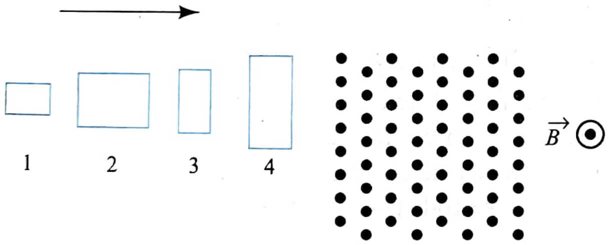
(4) a constant anticlockwise induced current will flow in the closed loop

9. A rectangular loop of wire with dimensions shown in figure is coplanar with a long wire carrying current I. The distance between the wire and the left side of the loop is r. The loop is pulled to the right as indicated. What are the directions of the induced current in the loop and the magnetic forces on the left and the right sides of the loop when the loop is pulled?



Induced current	Force on left side	Force on right side
(1) Counterclockwise	To the left	To the left
(2) Counterclockwise	To the right	To the left
(3) Clockwise	To the right	To the left
(4) Clockwise	To the left	To the right

10. The four wire loops shown in figure have vertical edge lengths of either L, $2L$, or $3L$. They will move with the same speed into a region of uniform magnetic field $\vec{B}$ directed out of the page. Rank them according to the maximum magnitude of the induced emf greatest to least.



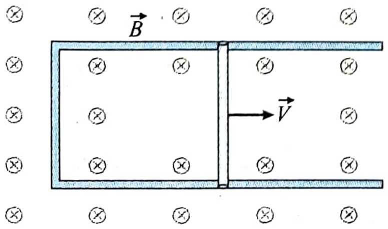
(1) 1 and 2 tie, then 3 and 4 tie

(2) 3 and 4 tie, then 1 and 2 tie

(3) 4, 2, 3, 1

(4) 4 then 2 and 3 tie, and then 1

11. A rod lies across frictionless rails in a uniform magnetic field $\vec{B}$ as shown in figure. The rod moves to the right with speed V. In order to make the induced emf in the circuit to be zero, the magnitude of the magnetic field should



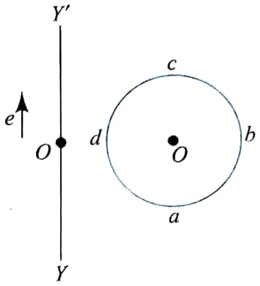
(1) not change

(2) increase linearly with time

(3) decrease linearly with time

(4) decrease nonlinearly with time

12. An electron moves on a straight line path YY' as shown in figure. A coil is kept on the right such that YY' is in the plane of the coil. At the instant when the electron gets closest to the coil (neglect self-induction of the coil),



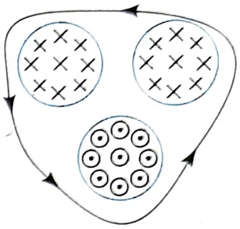
(1) the current in the coil flows clockwise

(2) the current in the coil flows anticlockwise

(3) the current in the coil is zero

(4) the current in the coil does not change the direction as the electron crosses point O

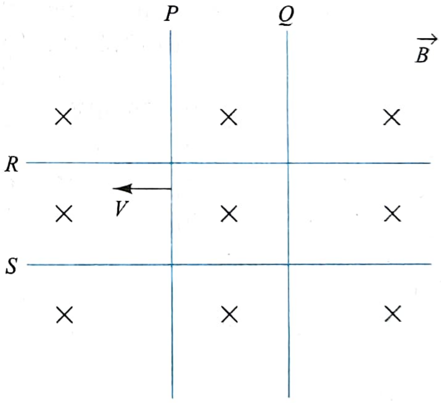
13. Figure shows three regions of magnetic field each of area A, and in each region, magnitude of magnetic field decreases at a constant rate α. If $\vec{E}$ is the induced electric field, then the value of the line integral $\oint \vec{E}\cdot d\vec{r}$ along the given loop is equal to



(1) αA (2) $-\alpha A$

(3) $3\alpha A$ (4) $-3\alpha A$

14. Two identical conductors P and Q are placed on two frictionless rails R and S in a uniform magnetic field directed into the plane. If P is moved in the direction shown in figure with a constant speed, then rod Q



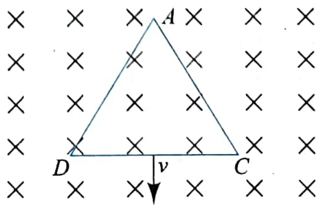
(1) will be attracted toward P

(2) will be repelled away from P

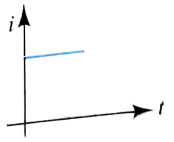
(3) will remain stationary

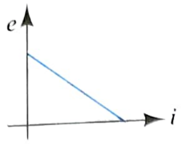
(4) may be repelled away or attracted toward P

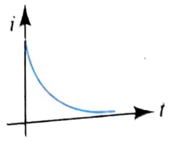
15. An equilateral triangular loop ADC having some resistance is pulled with a constant velocity v out of a uniform magnetic field directed into the paper (figure). At time $t = 0$, side DC of the loop is at edge of the magnetic field.



The induced current (i) versus time (t) graph will be as

(1)


(2)


(3)


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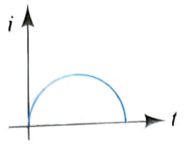
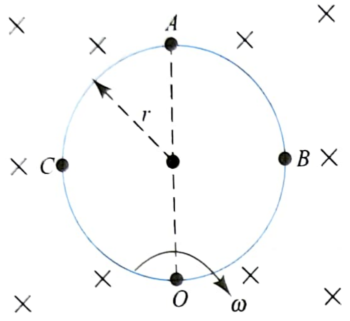


16. In figure, there is a conducting ring having resistance R placed in the plane of paper in a uniform magnetic field B_0. If the ring is rotating in the plane of paper about an axis passing through point O and perpendicular to the plane of paper with constant angular speed ω in clockwise direction, then

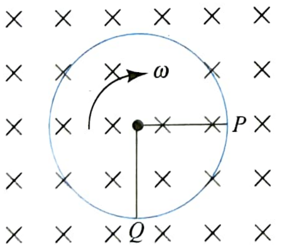


(1) point O will be at higher potential than A
(2) the potential of point B and C will be different
(3) the current in the ring will be zero
(4) the current in the ring will be $\dfrac{2B_0\omega r^2}{R}$

17. A 0.1 m long conductor carrying a current of 50 A is perpendicular to a magnetic field of 1.21 mT. The mechanical power to move the conductor with a speed of 1 m s^{-1} is

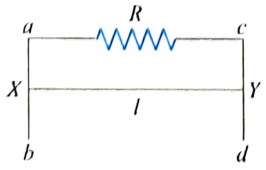
(1) 0.25 mW	(2) 6.25 mW
(3) 0.625 W	(4) 1 W

18. A conducting ring of radius r is rolling without slipping with a constant angular velocity ω (figure). If the magnetic field strength is B and is directed into the page then the emf induced across PQ is



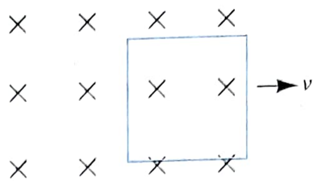
(1) $B\omega r^2$	(2) $\dfrac{B\omega r^2}{2}$
(3) $4B\omega r^2$	(4) $\dfrac{\pi^2 r^2 B\omega}{8}$

19. A conducting wire xy of length l and mass m is sliding without friction on vertical conduction rails ab and cd as shown in figure. A uniform magnetic field B exists perpendicular to the plane of the rails, x moves with a constant velocity of



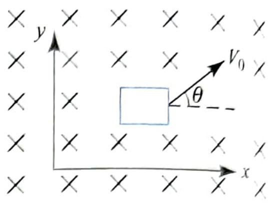
(1) $\dfrac{mgR}{Bl}$	(2) $\dfrac{mgR}{Bl^2}$
(3) $\dfrac{mgR}{B^2l^2}$	(4) $\dfrac{mgR}{B^2l}$

20. Figure shows a square loop of side 0.5 m and resistance 10 Ω. The magnetic field has a magnitude $B = 1.0$ T. The work done in pulling the loop out of the field slowly and uniformly in 2.0 s is



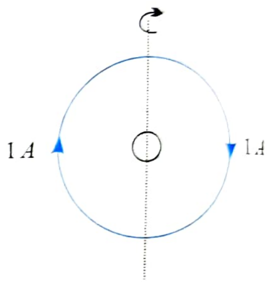
(1) 3.125×10^{-3} J	(2) 6.25×10^{-4} J
(3) 1.25×10^{-2} J	(4) 5.0×10^{-4} J

21. In the space shown a non-uniform magnetic field $\vec{B} = B_0(1+x)(-\hat{k})$ tesla is present. A closed loop of small resistance, placed in the x–y plane is given velocity V_0. The force due to magnetic field on the loop is



(1) zero	(2) along $+x$ direction
(3) along $-x$ direction	(4) along $+y$ direction

22. The inner loop has an area of 5×10^{-4} m^2 and a resistance of 2 Ω (figure). The larger circular loop is fixed and has a radius of 0.1 m. Both the loops are concentric and coplanar. The smaller loop is rotated with an angular velocity ω rad s^{-1} about its diameter. The magnetic flux with the smaller loop is

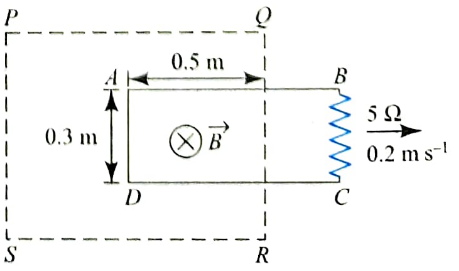


(1) $2\pi \times 10^{-6}$ Wb	(2) $\pi \times 10^{-9}$ Wb
(3) $\pi \times 10^{-9} \cos \omega t$ Wb	(4) zero

23. A gold rod of length l is accelerated in the horizontal direction with an acceleration a_0. The rod is held between two perfectly insulating clamps. Calculate the electric field set up in the rod. Take the mass of electron as m.

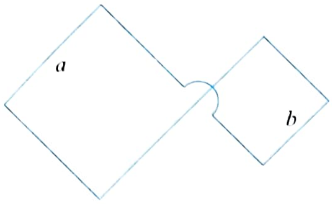
(1) $E = \dfrac{ma_0}{e}$	(2) $E = ma_0 l$
(3) zero	(4) none of these

24. A circuit $ABCD$ is held perpendicular to the uniform magnetic field of $B = 5 \times 10^{-2}$ T extending over the region $PQRS$ and directed into the plane of the paper. The circuit is moving out of the field at a uniform speed of 0.2 m s^{-1} for 1.5 s. During this time, the current in the 5 Ω resistor is



(1) 0.6 mA from B to C	(2) 0.9 mA from B to C
(3) 0.9 mA from C to B	(4) 0.6 mA from C to B

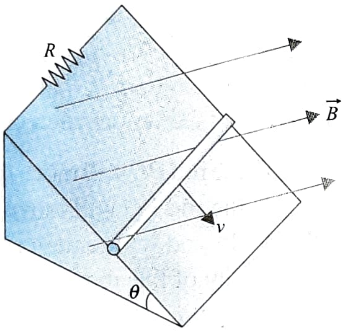
25. A plane loop, shaped as two squares of sides $a = 1$ m and $b = 0.4$ m is introduced into a uniform magnetic field ⊥ to the plane of loop (figure). The magnetic field varies as $B = 10^{-3}\sin(100t)$ T. The amplitude of the current induced in the loop if its resistance per unit length is $r = 5\ \text{m}\Omega\ \text{m}^{-1}$ is



(1) 2 A
(2) 3 A
(3) 4 A
(4) 5 A

26. A conductor AB of length l moves in x–y plane with velocity $\vec{v} = v_0(\hat{i} - \hat{j})$. A magnetic field $\vec{B} = B_0(\hat{i} + \hat{j})$ exists in the region. The induced emf is
(1) zero
(2) $2\,B_0lv_0$
(3) B_0lv_0
(4) $\sqrt{2}\,B_0lv_0$

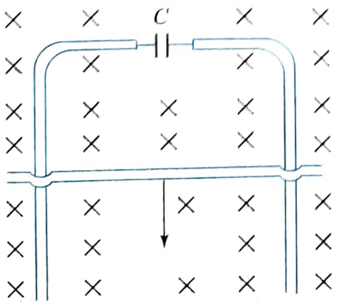
27. A conducting wire of mass m slides down two smooth conducting bars, set at an angle θ to the horizontal as shown in figure. The separation between the bars is l. The system is located in the magnetic field B, perpendicular to the plane of the sliding wire and bars. The constant velocity of the wire is



(1) $\dfrac{mg\,R\sin\theta}{B^2l^2}$
(2) $\dfrac{mg\,R\sin\theta}{Bl^3}$
(3) $\dfrac{mg\,R\theta}{B^2l^5}$
(4) $\dfrac{mg\,R\sin\theta}{Bl^4}$

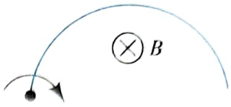
28. A uniform magnetic field exists in a region given by $\vec{B} = 3\hat{i} + 4\hat{j} + 5\hat{k}$. A rod of length 5 m along y-axis moves with a constant speed of 1 ms^{-1} along x axis. Then the induced emf in the rod will be
(1) 0
(2) 25 V
(3) 20 V
(4) 15 V

29. A conductor of length l and mass m can slide without any friction along the two vertical conductors connected at the top through a capacitor (figure). A uniform magnetic field B is set up ⊥ to the plane of paper. The acceleration of the conductor



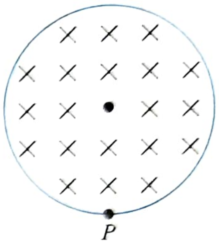
(1) is constant
(2) increases
(3) decreases
(4) cannot say

30. A semicircular wire of radius R is rotated with constant angular velocity about an axis passing through one end and perpendicular to the plane of the wire. There is a uniform magnetic field of strength B. The induced emf between the ends is



(1) $B\omega R^2/2$
(2) $2B\omega R^2$
(3) is variable
(4) none of these

31. A uniform magnetic field of induction B is confined to a cylindrical region of radius R. The magnetic field is increasing at a constant rate of dB/dt T s^{-1}. An electron placed at the point P on the periphery of the field, experiences an acceleration



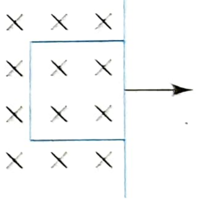
(1) $\dfrac{1}{2}\dfrac{eR}{m}\dfrac{dB}{dt}$ toward left
(2) $\dfrac{1}{2}\dfrac{eR}{m}\dfrac{dB}{dt}$ toward right
(3) $\dfrac{eR}{m}\dfrac{dB}{dt}$ toward left
(4) zero

32. Magnetic flux linked with a stationary loop of resistance R varies with respect to time during the time period T as follows:

$$\phi = at\,(T - t)$$

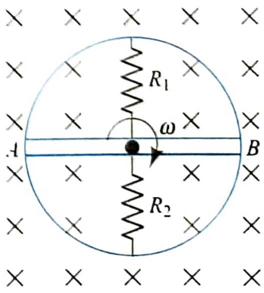
The amount of heat generated in the loop during that time (inductance of the coil is negligible) is
(1) $\dfrac{aT}{3R}$
(2) $\dfrac{a^2T^2}{3R}$
(3) $\dfrac{a^2T^2}{R}$
(4) $\dfrac{a^2T^3}{3R}$

33. A square loop of area 2.5× 10^{-3} m^2 and having 100 turns with a total resistance of 100 Ω is moved out of a uniform magnetic field of 0.40 T in 1 s with a constant speed. Then the work done in pulling the loop is



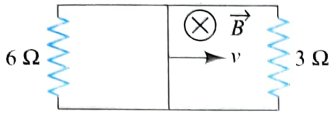
(1) 0
(2) 1 mJ
(3) 1 μJ
(4) 0.1 mJ

34. AB is a resistanceless conducting rod which forms a diameter of a conducting ring of radius r rotating in a uniform magnetic field B as shown in figure. The resistors R_1 and R_2 do not rotate. Then the current through the resistor R_1 is



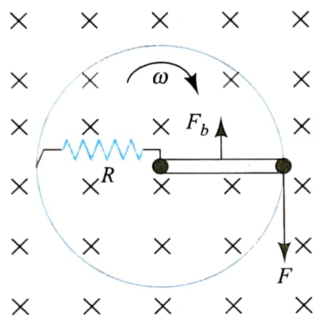
(1) $\dfrac{B\omega r^2}{2R_1}$
(2) $\dfrac{B\omega r^2}{2R_2}$
(3) $\dfrac{B\omega r^2}{2R_1R_2}(R_1 + R_2)$
(4) $\dfrac{B\omega r^2}{2(R_2 + R_2)}$

35. A rectangular loop with a sliding connector of length $l = 1.0$ m is situated in a uniform magnetic field $B = 2$ T perpendicular to the plane of loop. Resistance of connector is $r = 2\ \Omega$. Two resistances of $6\ \Omega$ and $3\ \Omega$ are connected as shown in figure. The external force required to keep the connector moving with a constant velocity $v = 2\text{ m s}^{-1}$ is



(1) 6 N (2) 4 N
(3) 2 N (4) 1 N

36. A metallic ring of radius r with a uniform metallic spoke of negligible mass and length r is rotated about its axis with angular velocity ω in a perpendicular uniform magnetic field B as shown in figure. The central end of the spoke is connected to the rim of the wheel through a resistor R as shown. The resistor does not rotate, its one end is always at the center of the ring and the other end is always in contact with the ring. A force F as shown is needed to maintain constant angular velocity of the wheel. F is equal to (the ring and the spoke has zero resistance)



(1) $\dfrac{B^2\omega r^2}{8R}$ (2) $\dfrac{B^2\omega r^2}{2R}$

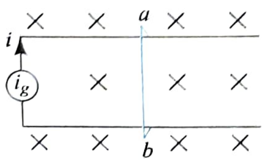
(3) $\dfrac{B^2\omega r^3}{2R}$ (4) $\dfrac{B^2\omega r^3}{4R}$

37. A metal rod of resistance $20\ \Omega$ is fixed along a diameter of a conducting ring of radius 0.1 m and lies on x–y plane. There is a magnetic field $\vec{B} = (50\text{ T})\bar{k}$. The ring rotates with an angular velocity $\omega = 20\text{ rad s}^{-1}$ about its axis. An external resistance of $10\ \Omega$ is connected across the center of the ring and rim. The current through external resistance is

(1) $\dfrac{1}{4}$ (2) $\dfrac{1}{2}$

(3) $\dfrac{1}{3}$ (4) 0

38. The current generator i_g, shown in figure, sends a constant current i through the circuit. The wire ab has a length l and mass m slide on the smooth, horizontal rails connected to i_g. The entire system lies in a vertical magnetic field B. The velocity of the wire as a function of time is



(1) $\dfrac{ilBt}{m}$ (2) $\dfrac{ilBt}{2m}$

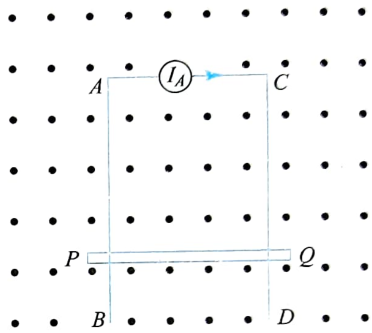
(3) $\dfrac{2ilBt}{m}$ (4) $\dfrac{ilBt}{3m}$

39. A metal disc of radius a rotates with a constant angular velocity ω about its axis. The potential difference between the center and the rim of the disc is (m = mass of electron, e = charge on electron)

(1) $\dfrac{m\omega^2 a^2}{e}$ (2) $\dfrac{1}{2}\dfrac{m\omega^2 a^2}{e}$

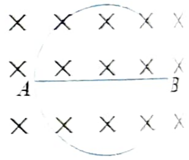
(3) $\dfrac{e\omega^2 a^2}{2m}$ (4) $\dfrac{e\omega^2 a^2}{m}$

40. AB and CD are fixed conducting smooth rails placed in a vertical plane and joined by a constant current source at its upper end. PQ is a conducting rod which is free to slide on the rails. A horizontal uniform magnetic field exists in space as shown. If the rod PQ is released from rest then,



(1) the rod PQ will move downward with constant acceleration
(2) the rod PQ will move upward with constant acceleration
(3) the rod will remain at rest
(4) any of the above

41. The radius of the circular conducting loop shown in figure is R. Magnetic field is decreasing at a constant rate α. Resistance per unit length of the loop is ρ.



Then, the current in wire AB is (AB is one of the diameters)

(1) $\dfrac{R\alpha}{2\rho}$ from A to B (2) $\dfrac{R\alpha}{2\rho}$ from B to A

(3) $\dfrac{R\alpha}{\rho}$ from A to B (4) 0

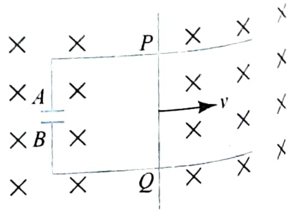
42. The magnetic field in a region is given by $\vec{B} = B_0\left(1+\dfrac{x}{a}\right)\hat{k}$.

A square loop of edge length d is placed with its edge along the x- and y-axes. The loop is moved with a constant velocity $\vec{v} = v_0\hat{i}$. The emf induced in the loop is

(1) $\dfrac{v_0 B_0 d^2}{a}$ (2) $\dfrac{v_0 B_0 d^3}{a^2}$

(3) $v_0 B_0 d$ (4) zero

43. A conducting rod PQ of length $L = 1.0$ m is moving with a uniform speed $v = 2.0\text{ m s}^{-1}$ in a uniform magnetic field $B = 4.0$ T directed into the plane of the paper.



A capacitor of capacity $C = 10\ \mu\text{F}$ is connected as shown in figure, then

(1) $q_A = +80\ \mu\text{C}$ and $q_B = -80\ \mu\text{C}$

(2) $q_A = -80\ \mu\text{C}$ and $q_B = +80\ \mu\text{C}$

(3) $q_A = 0 = q_B$

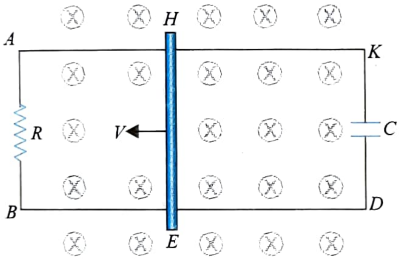
(4) charge stored in the capacitor increases exponentially with time

44. A metallic wire is folded to form a square loop of side a. It carries current i and is kept perpendicular to the region of uniform magnetic field B. If the shape of the loop is changed from square to an equilateral triangle without changing the length of the wire and current, the amount of work done in doing so is

(1) $Bia^2\left(1-\dfrac{4\sqrt{3}}{9}\right)$ (2) $Bia^2\left(1-\dfrac{\sqrt{3}}{9}\right)$

(3) $\dfrac{2}{3}Bia^2$ (4) zero

45. In the circuit shown in figure, a conducting wire HE is moved with a constant speed v toward left. The complete circuit is placed in a uniform magnetic field $\vec{B}$ perpendicular to the plane of the circuit inward. The current in $HKDE$ is



(1) clockwise (2) anticlockwise
(3) alternating (4) zero

46. A flexible wire loop in the shape of a circle has a radius that grows linearly with time. There is a magnetic field perpendicular to the plane of the loop that has a magnitude inversely proportional to the distance from the center of the loop, $B(r) \propto \dfrac{1}{r}$. How does the emf E vary with time?

(1) $E \propto t^2$ (2) $E \propto t$

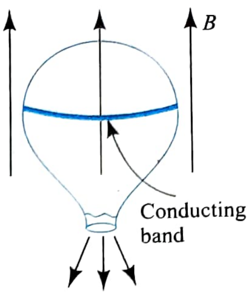
(3) $E \propto \sqrt{t}$ (4) E is constant

47. A vertical ring of radius r and resistance R falls vertically. It is in contact with two vertical rails which are joined at the top. The rails are without friction and resistance. There is a horizontal uniform magnetic field of magnitude B perpendicular to the plane of the ring and the rails. When the speed of the ring is v, the current in the top horizontal of the rail section is

(1) 0 (2) $\dfrac{2Brv}{R}$

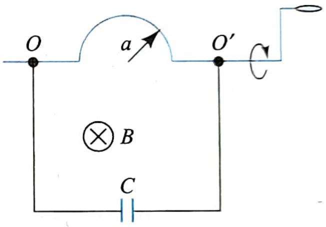
(3) $\dfrac{4Brv}{R}$ (4) $\dfrac{8Brv}{R}$

48. An elasticized conducting band is around a spherical balloon (figure). Its plane passes through the center of the balloon. A uniform magnetic field of magnitude 0.04 T is directed perpendicular to the plane of the band. Air is let out of the balloon at 100 cm^3s^{-1} at an instant when the radius of the balloon is 10 cm. The induced emf in the band is



(1) 15 μV (2) 25 μV
(3) 10 μV (4) 20 μV

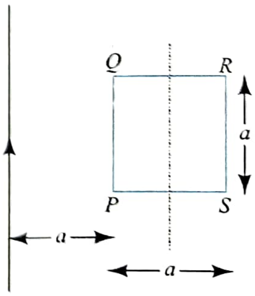
49. A copper rod is bent into a semi-circle of radius a and at ends straight parts are bent along diameter of the semi-circle and are passed through fixed, smooth, and conducting ring O and O' as shown in figure. A capacitor having capacitance C is connected to the rings. The system is located in a uniform magnetic field of induction B such that axis of rotation OO' is perpendicular to the field direction. At initial moment of time ($t = 0$), plane of semi-circle was normal to the field direction and the semicircle is set in rotation with constant angular velocity ω. Neglect the resistance and inductance of the circuit. The current flowing through the circuit as function of time is



(1) $\dfrac{1}{4}\pi\omega^2a^2CB\cos\omega t$ (2) $\dfrac{1}{2}\pi\omega^2a^2CB\cos\omega t$

(3) $\dfrac{1}{4}\pi\omega^2a^2CB\sin\omega t$ (4) $\dfrac{1}{2}\pi\omega^2a^2CB\sin\omega t$

50. In figure, a square loop $PQRS$ of side a and resistance r is placed near an infinitely long wire carrying a constant current I. The sides PQ and RS are parallel to the wire. The wire and the loop are in the same plane. The loop is rotated by 180° about an axis parallel to the long wire and passing through the mid-points of the sides QR and PS. The total amount of charge which passes through any point of the loop during rotation is



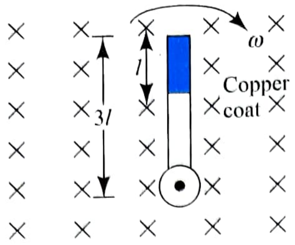
(1) $\dfrac{\mu_0 Ia}{2\pi r}\ln 2$

(2) $\dfrac{\mu_0 Ia}{\pi r}\ln 2$

(3) $\dfrac{\mu_0 Ia^2}{2\pi r}$

(4) Cannot be found because time of rotation is not given.

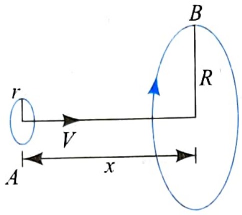
51. A wooden stick of length $3l$ is rotated about an end with constant angular velocity ω in a uniform magnetic field B perpendicular to the plane of motion. If the upper one-third of its length is coated with copper, the potential difference across the whole length of the stick is



(1) $\dfrac{9B\omega l^2}{2}$ (2) $\dfrac{4B\omega l^2}{2}$

(3) $\dfrac{5B\omega l^2}{2}$ (4) $\dfrac{B\omega l^2}{2}$

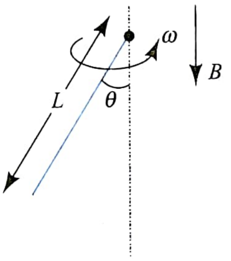
52. Loop A of radius $r << R$ moves toward loop B with a constant velocity V in such a way that their planes are always parallel. What is the distance between the two loops (x) when the induced emf in loop A is maximum?



(1) R (2) $\dfrac{R}{\sqrt{2}}$

(3) $\dfrac{R}{2}$ (4) $R\left(1-\dfrac{1}{\sqrt{2}}\right)$

53. A rod of length L rotates in the form of a conical pendulum with an angular velocity ω about its axis as shown in figure. The rod makes an angle θ with the axis. The magnitude of the motional emf developed across the two ends of the rod is



(1) $\dfrac{1}{2}B\omega L^2$ (2) $\dfrac{1}{2}B\omega L^2\tan^2\theta$

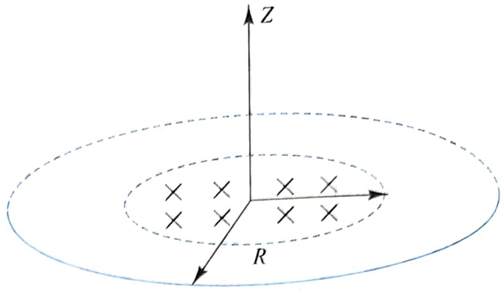
(3) $\dfrac{1}{2}B\omega L^2\cos^2\theta$ (4) $\dfrac{1}{2}B\omega L^2\sin^2\theta$

54. The magnetic flux density B is changing in magnitude at a constant rate dB/dt. A given mass m of copper, drawn into a wire of radius a and formed into a circular loop of radius r is placed perpendicular to the field B. The induced current in the loop is i. The resistivity of copper is ρ and density is d. The value of the induced current i is

(1) $\dfrac{m}{2\pi\rho d}\dfrac{dB}{dt}$ (2) $\dfrac{m}{4\pi a^2 r}\dfrac{dB}{dt}$

(3) $\dfrac{m}{4\pi a d}\dfrac{dB}{dt}$ (4) $\dfrac{m}{4\pi\rho d}\dfrac{dB}{dt}$

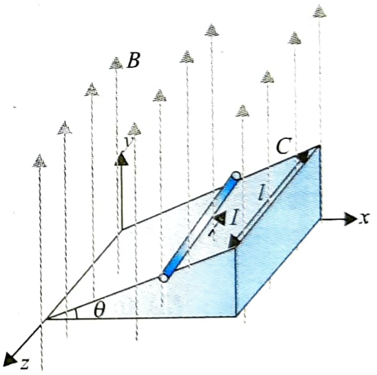
55. A line charge λ per unit length is pasted uniformly on to the rim of a wheel of mass m and radius R. The wheel has light non-conducting spokes and is free to rotate about a vertical axis as shown in figure. A uniform magnetic field extends over a radial region of radius r given by $B=-B_0\,\hat{k}\,(r\le a;\, a<R)=0$ (otherwise). What is the angular velocity of the wheel when this field is suddenly switched off?



(1) $\dfrac{-2B_0\pi a^2 r}{mR}\hat{k}$ (2) $\dfrac{-B_0\pi a^2 r}{3mR}\hat{k}$

(3) $\dfrac{B_0\pi a^2\lambda}{mR}\hat{k}$ (4) $\dfrac{-B_0\pi a^2\lambda}{mR}\hat{k}$

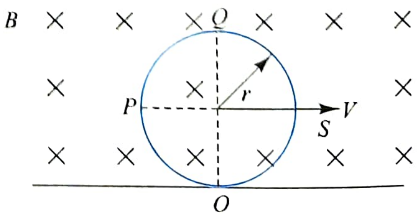
56. A conducting wire of length l and mass m is placed on two inclined rails as shown in figure. A current I is flowing in the wire in the direction shown. When no magnetic field is present in the region, the wire is just on the verge of sliding. When a vertically upward magnetic field is switched on, the wire starts moving up the incline. The distance travelled by the wire as a function of time t will be



(1) $\dfrac{1}{2}\left[\dfrac{IBl}{m}-2g\right]t^2$ (2) $\dfrac{1}{2}\left[\dfrac{IBl}{m}\times\dfrac{1}{\cos\theta}-2g\sin\theta\right]t^2$

(3) $\dfrac{1}{2}\left[\dfrac{IBl}{m}-2g\sin\theta\right]t^2$ (4) $\dfrac{1}{2}\left[\dfrac{IBl}{m}\dfrac{\cos 2\theta}{\cos\theta}-2g\sin\theta\right]t^2$

57. A conducting ring of radius r and resistance R rolls on a horizontal surface with constant velocity v. The magnetic field B is uniform and is normal to the plane of the loop. Choose the correct option.



(1) The induced emf between O and Q is Brv.

(2) An induced current $I=\dfrac{2Bvr}{R}$ flows in the clockwise direction.

(3) An induced current $I = \frac{2Bvr}{R}$ flows in the anticlockwise direction.

(4) No current flows.

58. In a region at a distance r from z-axis, magnetic field $\vec{B} = B_0 rt\hat{k}$ is present where B_0 is constant and t is time. Then the magnitude of induced electric field at a distance r from z-axis is given by

(1) $\frac{r}{2}B_0$ (2) $\frac{r^2}{2}B_0$

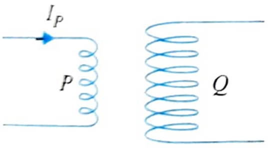
(3) $\frac{r^2}{3}B_0$ (4) none

59. A small square loop of edge length a and resistance R is moved with velocity v_0 away from an infinitely long current carrying conductor carrying current i so that the conductor and side of square are always in same plane. Find induced current in loop at a separation of r.

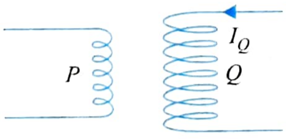
(1) $\frac{\mu_0 ia^2 v}{\pi r^2 R}$ (2) $\frac{\mu_0 ia^2 v}{4\pi r^2 R}$

(3) $\frac{\mu_0 ia^2 v}{2\pi r^2 R}$ (4) $\frac{\mu_0 iav}{2\pi rR}$

60. In Figs. (a) and (b), two air-cored solenoids P and Q have been shown. They are placed near each other. In Fig. (a), when I_p the current in P, changes at the rate of 5 A/s, an emf of 2 mV is induced in Q. The current in P is then switched off, and a current changing at 2 A/s is fed through Q as shown in diagram. What emf will be induced in P?



(a)

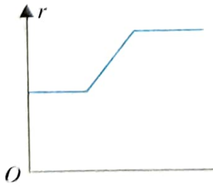


(b)

(1) 8×10^{-4} V (2) 2×10^{-3} V

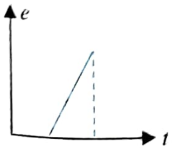
(3) 5×10^{-3} V (4) 8×10^{-2} V

61. Radius of a circular ring is changing with time and the coil is placed in uniform magnetic field perpendicular to its plane. The variation of 'r' with time 't' is shown in figure.

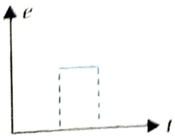


Then induced emf e with time t will be best represented by

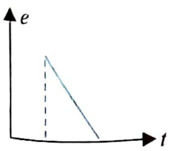
(1)



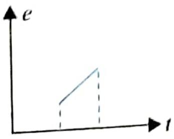
(2)



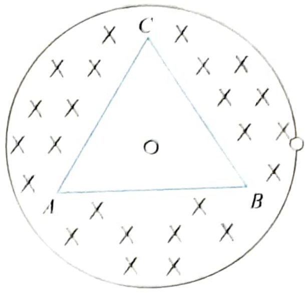
(3)



(4)



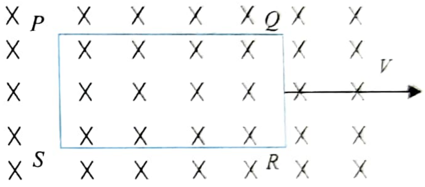
62. A triangular wire frame (each side = 2 m) is placed in a region of time variant magnetic field $dB/dt = \sqrt{3}$ T/s. The magnetic field is perpendicular to the plane of the triangle and its centre coincides with the centre of triangle. The base of the triangle AB has a resistance 1 Ω while the other two sides have resistance 2 Ω each. The magnitude of potential difference between the points A and B will be



(1) 0.4 V (2) 0.6 V

(3) 1.2 V (4) None

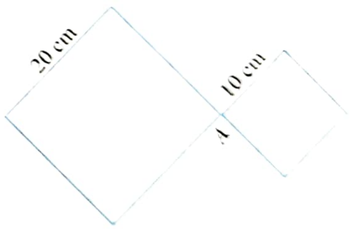
63. A metallic square loop $PQRS$ is moving in its own plane with velocity v in a uniform magnetic field perpendicular to its plane as shown in figure. If V_P, V_Q, V_R and V_S are the potentials of points P, Q, R and S, then which of the following is an incorrect statement?



(1) $V_P = V_Q$ (2) $V_P > V_S$

(3) $V_P > V_R$ (4) $V_S > V_R$

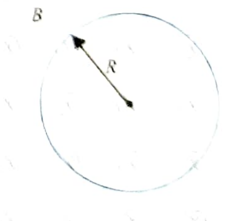
64. A plane loop is shaped as two squares (figure) and placed in a uniform magnetic field at right angle to the loop's plane. The magnetic induction varies with time as $B = B_0 \sin \omega t$, where $B_0 = 10$ mT and $\omega = 100$ rads^{-1}. The wires do not touch at point A. If resistance per unit length of the loop is 50 mΩ/m, then amplitude of current induced in the loop is



(1) 1.5 A (2) 1.0 A

(3) 0.5 A (4) 2.0 A

65. A conducting loop of radius R is present in a uniform magnetic field B perpendicular to the plane of the ring. If radius R varies as a function of time t, as $R = R_0 + t$. The emf induced in the loop is



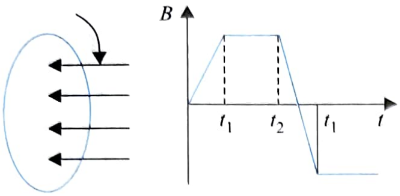
(1) $2\pi(R_0 + t)$B clockwise

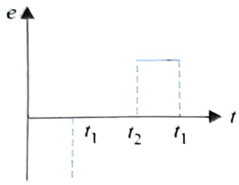
(2) $\pi(R_0 + t)$B clockwise

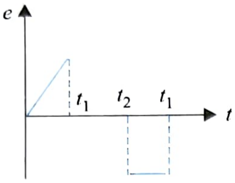
(3) $2\pi(R_0 + t)$B anticlockwise

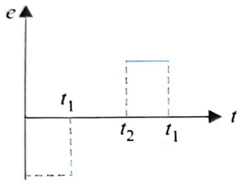
(4) zero.

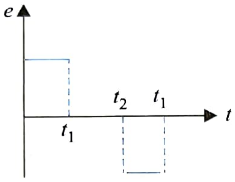
66. A wire loop is placed in a region of time varying magnetic field which is oriented orthogonally to the plane of the loop as shown in figure. The graph shows the magnetic field variation as the function of time. Assume the positive emf is the one which drives a current in the clockwise direction and seen by the observer in the direction of B. Which of the following graphs best represents the induced emf as a function of time ?



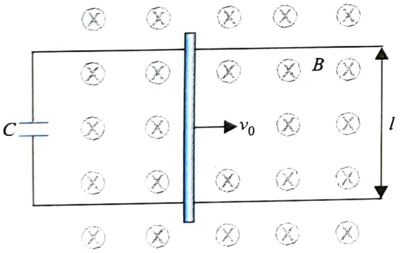
(1)


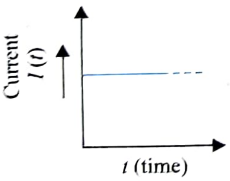
(2)


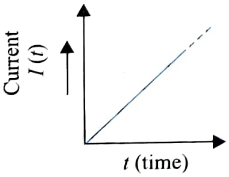
(3)


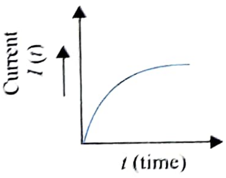
(4)


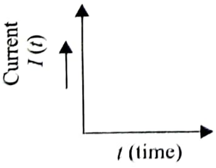
67. Two infinitely long conducting parallel rails are connected through a capacitor C as shown in figure. A conductor of length l is moved with constant speed v_0. Which of the following graph truly depicts the variation of current through the conductor with time ?



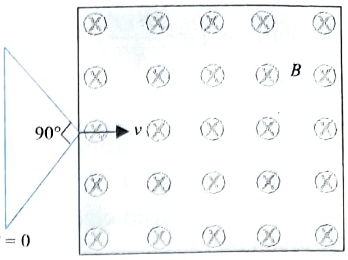
(1)


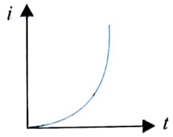
(2)


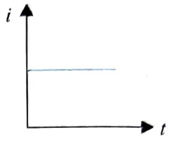
(3)


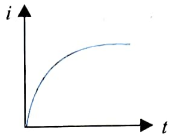
(4)


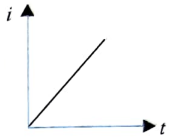
68. Figure shows an isosceles triangle wire frame with apex angle equal to $\pi/2$. The frame starts entering into the region of uniform magnetic field B with constant velocity v at $t = 0$. The longest side of the frame is perpendicular to the direction of velocity. If i is the instantaneous current through the frame then choose the alternative showing the correct variation of i with time.



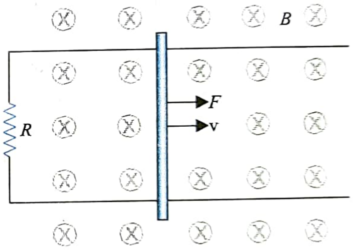
(1)


(2)


(3)


(4)


69. A rod closing the circuit shown in figure moves along a U shaped wire at a constant speed v under the action of the force F. The circuit is in a uniform magnetic field perpendicular to the plane. Calculate F if the rate of heat generation in the circuit is Q.



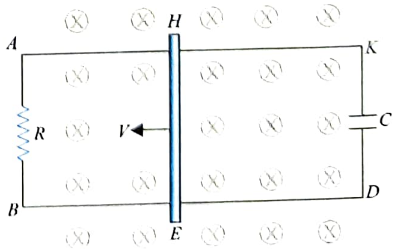
(1) $F = Qv$

(2) $F = \dfrac{Q}{v}$

(3) $F = \dfrac{v}{Q}$

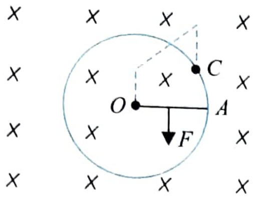
(4) $F = \sqrt{Qv}$

70. In the circuit shown in figure, a conducting wire HE is moved with a constant speed v towards left. The complete circuit is placed in a uniform magnetic field $\vec{B}$ perpendicular to the plane of circuit inwards. The current in $HKDE$ is



(1) clockwise
(2) anticlockwise
(3) direction will change with time
(4) zero

71. Figure shows a conducting circular loop of radius a placed in a uniform, perpendicular magnetic field B. A thick metal rod OA is pivoted at the centre O. The other end of the rod touches the loop at A. The centre O and a fixed point C on the loop are connected by a wire OC of resistance R. A force is applied at the middle point of the rod OA perpendicularly, so that the rod rotates clockwise at a uniform angular velocity ω. Find the force.



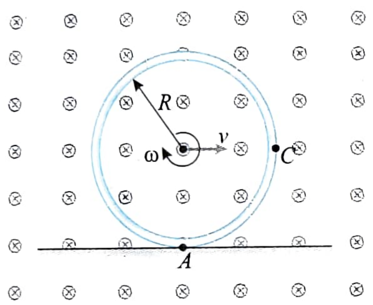
(1) $\frac{\omega a^3 B^2}{R}$ to the right of OA in the figure

(2) $\frac{\omega a^3 B^2}{2R}$ to the right of OA in the figure

(3) $\frac{\omega a^3 B^2}{2R}$ to the left of OA in the figure

(4) None of these

72. A ring of radius R is rolling on a horizontal plane with constant velocity v. There is a constant and uniform magnetic field B which is perpendicular to the plane of the ring. Emf across the lowest point A and right-most point C as shown in the figure will



(1) increase with time

(2) decrease with time

(3) remains constant and equal to $\sqrt{2}BvR$

(4) remains constant and equal to BvR

73. A current $I = 3.36\ (1 + 2t) \times 10^{-2}$ A increases at a steady state in a long straight wire. A small circular loop of radius 10^{-3} m has its plane parallel to the wire and is placed at a distance of 1 m from the wire. The resistance of loop is $8.4 \times 10^{-4}\ \Omega$. Find the approximate value of induced current in the loop.

(1) 5.024×10^{-11} A (2) 3.8×10^{-11} A

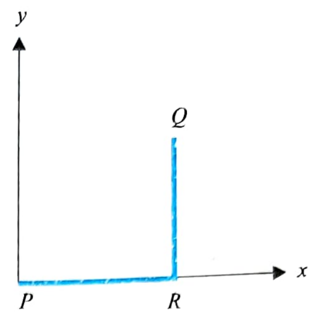
(3) 2.75×10^{-11} A (4) 1.23×10^{-11} A

74. The magnetic field in a region is given by $\vec{B} = \hat{k}\frac{B_0}{L}y$ where L is a fixed length. A conducting rod of length L lies along the Y-axis between the origin and the point $(0, L, 0)$. If the rod moves with a velocity $\vec{v} = v_0\hat{i}$, find the emf induced between the ends of the rod.

(1) $2B_0v_0l$ (2) B_0v_0l

(3) $\frac{B_0v_0l}{2}$ (4) None of these

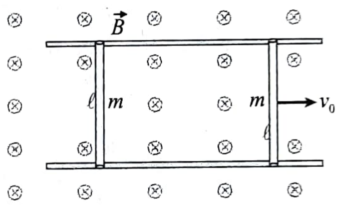
75. An inverted L shaped conductor PRQ is made by joining two perpendicular conducting rods, each of length $1.5\ L$, at end R. This structure is moving in x–y plane containing variable magnetic field, $\vec{B} = -3x\hat{k}$ with a velocity $v\hat{i} + v\hat{j}$. If potential of P is V_P and that of Q is V_Q, then value of V_P–V_Q at the instant when P is at origin as shown in figure, will be



(1) $\frac{9vL^2}{8}$ (2) $\frac{27vL^2}{8}$

(3) $-\frac{9vL^2}{8}$ (4) $-\frac{27vL^2}{8}$

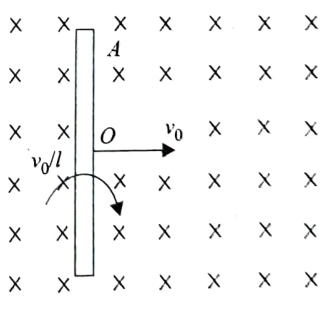
76. Consider parallel conducting rails separated by a distance l. There exists a uniform magnetic field B perpendicular to the plane of the rails as shown in figure. Two conducting wires each of length l are placed so as to slide on parallel conducting rails. One of the wires is given a velocity v_0 parallel to the rails. Till steady state is achieved, loss in kinetic energy of the system is



(1) zero (2) $\frac{3}{4}mv_0^2$

(3) $\frac{1}{4}mv_0^2$ (4) $\frac{3}{8}mv_0^2$

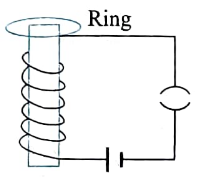
77. A conducting rod of length l is moving on a horizontal smooth surface. Magnetic field in the region is vertically downward and of magnitude B_0. If centre of mass (COM) of the rod is translating with velocity v_0 and rod rotates about COM with angular velocity v_0/l, then potential difference between points O and A will be



(1) $\frac{5}{8}B_0v_0l$ (2) $\frac{3}{8}B_0v_0l$

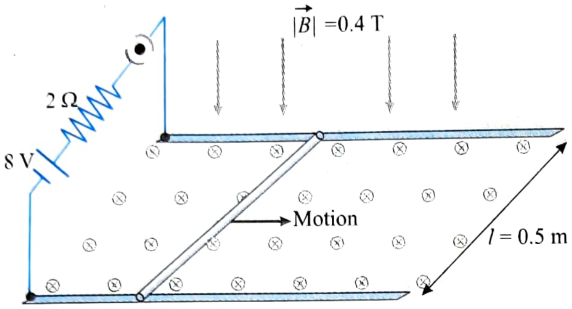
(3) $\frac{1}{8}B_0v_0l$ (4) $\frac{1}{2}B_0v_0l$

78. Figure shows a powerful electromagnet arrangement. A copper ring, which is free to move, is placed on the projecting part of the core as shown. When the key is inserted, the ring



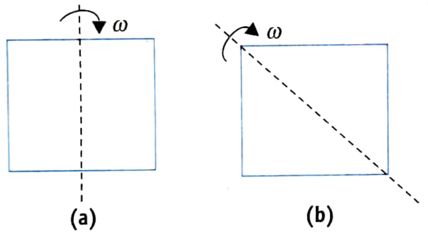
(1) is thrown up
(2) remains stationary
(3) sticks to the core
(4) slips down along the core

79. Figure shows a conducting frame having battery and a resistance on which a movable conductor of length 0.5 m can slide. The whole arrangement is placed in a uniform magnetic field of $B = 0.4$ T directed perpendicular and into the plane of frame. Initially the circuit is open. When the key is inserted, the conductor begins to move. It is found that a force 0.5 N has to be applied on the conductor to the left to keep it moving at constant speed to the right. Current flowing in the conductor is:



(1) 5 A
(2) 2.5 A
(3) 1.25 A
(4) 1 A

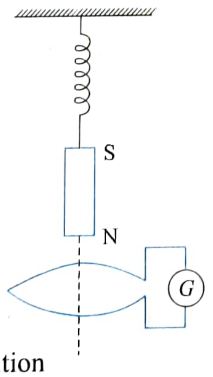
80. Figures (a) and (b) show a square loop of side a rotating about the given axes in a uniform magnetic field such that the field is perpendicular to the axis in both cases. Angular speed of rotation is both cases being the same,



(a) **(b)**

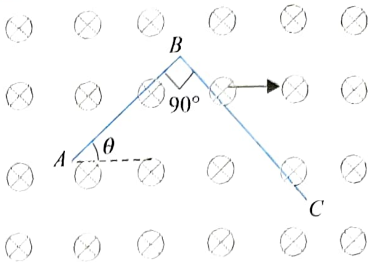
(1) induced emf is more in (a) than in (b)
(2) induced emf is more in (b) than in (a)
(3) induced emf in the two cases are equal and non-zero
(4) induced emf in the two cases are equal and zero.

81. Figure shows a magnet suspended at the lower end of a spring while its length lies along the axis of a fixed circular conducting coil. The magnet is made to oscillate. Deflection in the galvanometer is



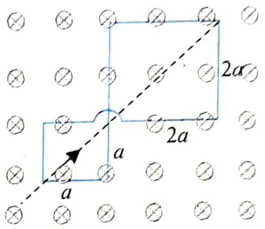
(1) minimum when the magnet is at mean position
(2) maximum when the magnet is at mean position
(3) zero when the magnet is at mean position
(4) maximum when the magnet is at extreme position

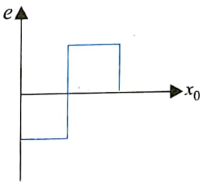
82. A conducting wire ABC (as shown) is moving with a constant velocity along horizontal direction. The magnetic field is perpendicular to the wire and directed into the page. $AB = BC$. Find the range of angle θ made by rod AB with horizontal at A so that value of induced emf at A is greater than at C.

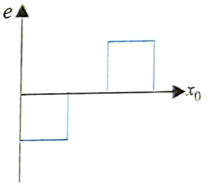


(1) $\theta > 45°$
(2) $\theta < 45°$
(3) $\theta > 60°$
(4) $\theta < 75°$

83. A uniform magnetic field exists in a square of side $2a$ (as shown in figure). A square loop of side a enters the field along a diagonal and leave it at a constant speed. Draw the curve between induced emf e and distance along the diagonal, say x_0.



(1)


(2)


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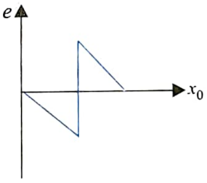

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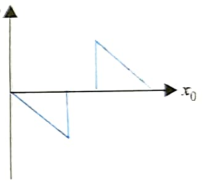
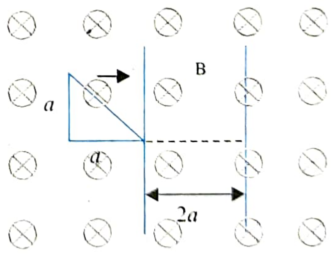
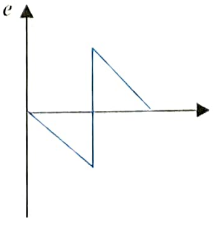
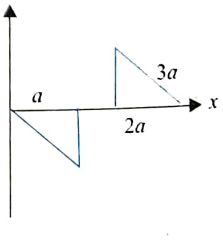
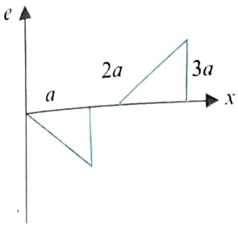


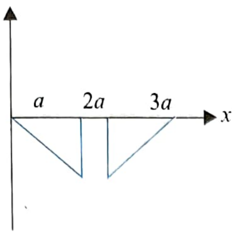
84. A right angled triangular loop as shown below enters uniform magnetic field (at right angle to the boundary of the field) directed into the paper. Draw the graph between induced emf e and the distance along the perpendicular to the boundary of the field, (say x) along which loop moves.



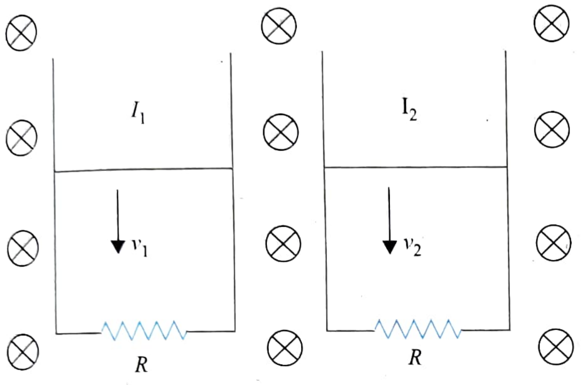
(1)


(2)


(3) 

(4) 

85. In a magnetic field as shown, in figure two horizontal wires of same mass and lengths l_1 and l_2 are free to slide on different vertical rails with velocities v_1 and v_2 respectively. If the resistances of two circuits are same and a_1 and a_2 are accelerations of two horizontal wires respectively, the condition for $a_1 > a_2$ is



(1) $\frac{l_1}{l_2} = \frac{v_2}{v_1}$ (2) $\frac{l_1}{l_2} > \left(\frac{v_2}{v_1}\right)^{1/2}$

(3) $\frac{l_1}{l_2} < \left(\frac{v_1}{v_2}\right)^{1/2}$ (4) $\frac{l_1}{l_2} < \frac{v_1}{v_2}$

86. In the previous problem, if two falling conductors attain velocities v_1 and v_2 respectively after falling through same height h, ratio of energy dissipated as heat to the energy dissipated in resistor per unit time for two conductors is same. Find which of the following conditions will be satisfied

(1) $gh + \frac{1}{2}v_1v_2 = 0$ (2) $gh + v_1v_2 = 0$

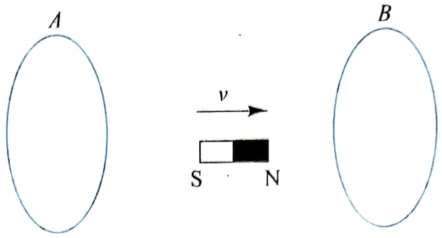
(3) $\frac{1}{2}gh + v_1v_2 = 0$ (4) $gh + \frac{v_1^2 v_2}{v_1 + v_2} = 0$

Multiple Correct Answers Type

1. An infinite current-carrying conductor is placed along the z-axis and a wire loop is kept in the x–y plane. The current in the conductor is increasing with time. Then the
(1) emf induced in the wire loop is zero
(2) magnetic flux passing through the wire loop is zero
(3) emf induced is zero but magnetic flux is not zero
(4) emf induced is not zero but magnetic flux is zero

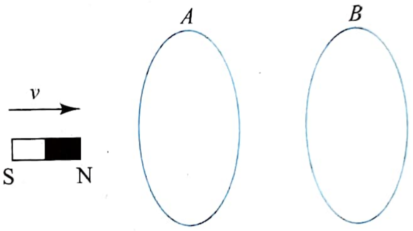
2. A conducting loop rotates with constant angular velocity about its fixed diameter in a uniform magnetic field, whose direction is perpendicular to that fixed diameter.
(1) The emf will be maximum at the moment when flux is zero
(2) The emf will be 0 at the moment when flux is maximum
(3) The emf will be maximum at the moment when plane of the loop is parallel to the magnetic field
(4) The phase difference between the flux and the emf is $\pi/2$

3. A bar magnet is moved between two parallel circular loops A and B with a constant velocity v as shown in figure.



(1) The current in each loop flows in the same direction
(2) The current in each loop flows in opposite directions
(3) The loops will repel each other
(4) The loops will attract each other

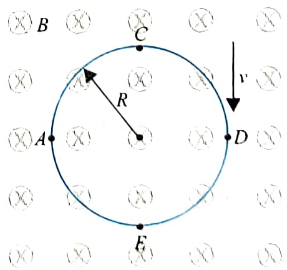
4. A bar magnet moves toward two identical parallel circular loops with a constant velocity v as shown in figure.



(1) Both the loops will attract each other
(2) Both the loops will repel each other
(3) The induced current in A is more than that in B
(4) The induced current is same in both the loops

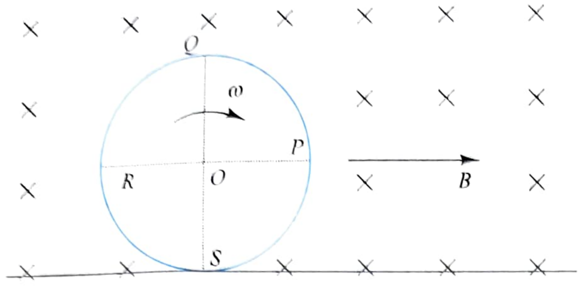
5. A small magnet M is allowed to fall through a fixed horizontal conducting ring R. Let g be the acceleration due to gravity. The acceleration of M will be
(1) $<g$ when it is above R and moving toward R
(2) $>g$ when it is above R and moving toward R
(3) $<g$ when it is below R and moving away from R
(4) $>g$ when it is below R and moving away from R

6. A vertical conducting ring of radius R falls vertically in a horizontal magnetic field of magnitude B. The direction of B is perpendicular to the plane of the ring. When the speed of the ring is v,



(1) no current flows in the ring
(2) A and D are at the same potential
(3) C and E are at the same potential
(4) the potential difference between A and D is $2BRv$, with D at a higher potential

7. A disc of radius R is rolling without sliding on a horizontal surface with a velocity of center of mass v and angular velocity ω in a uniform magnetic field B which is perpendicular to the plane of the disc as shown in figure. O is the center of the disc and P, Q, R, and S are the four points on the disc. Which of the following statements is true?



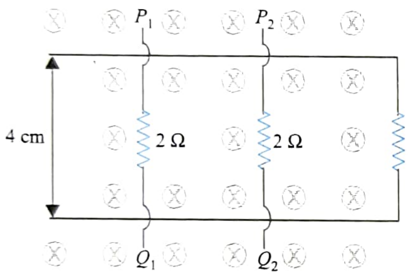
(1) Due to translation, induced emf across $PS = Bvr$

(2) Due to rotation, induced emf across $QS = 0$

(3) Due to translation, induced emf across $RO = 0$

(4) Due to rotation, induced emf across $OQ = Bvr$

8. In figure, the wires P_1Q_1 and P_2Q_2 are made to slide on the rails with same speed of 5 cm s^{-1}. In this region, a magnetic field of 1 T exists. The electric current in the 9 Ω resistance is



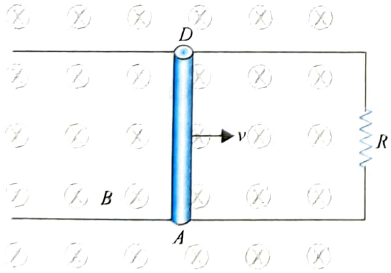
(1) zero if both wires slide toward left

(2) zero if both wires slide in opposite directions

(3) 0.2 mA if both wires move toward left

(4) 0.2 mA if both wires move in opposite directions

9. The conductor AD moves to the right in a uniform magnetic field directed into the plane of the paper.



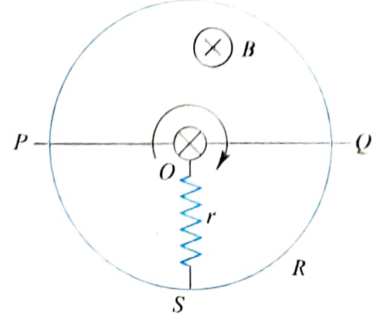
(1) The free electron in AD will move toward A

(2) D will acquire a positive potential with respect to A

(3) A current will flow from A to D in AD in closed loop

(4) The current in AD flows from lower to higher potential

10. In figure, R is a fixed conducting ring of negligible resistance and radius a. PQ is a uniform rod of resistance r. It is hinged at the center of the ring and rotated about this point in clockwise direction with a uniform angular velocity ω. There is a uniform magnetic field of strength B pointing inward and r is a stationary resistance. Then



(1) current through r is zero

(2) current through r is $(2B\omega a^2)/5r$

(3) direction of current in external resistance r is from center to circumference

(4) direction of current in external resistance r is from circumference to center

11. The magnetic flux ϕ linked with a conducting coil depends on time as $\phi = 4t^n + 6$, where n is a positive constant. The induced emf in the coil is e.

(1) If $0 < n < 1$, $e \neq 0$ and $|e|$ decreases with time.

(2) If $n = 1$, e is constant.

(3) If $n > 1$, $|e|$ increases with time.

(4) If $n > 1$, $|e|$ decreases with time.

12. A circular loop of radius r, having N turns of a wire, is placed in a uniform and constant magnetic field B. The normal of the loop makes an angle θ with the magnetic field. Its normal rotates with an angular velocity ω such that the angle θ is constant. Choose the correct statement from the following.

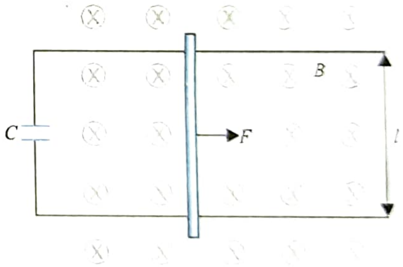
(1) emf in the loop is $NB\omega r^2/2 \cos\theta$.

(2) emf induced in the loop is zero.

(3) emf must be induced as the loop crosses magnetic lines.

(4) emf must not be induced as flux does not change with time.

13. A conducting wire of length l and mass m can slide without friction on two parallel rails and is connected to capacitance C. The whole system lies in a magnetic field B and a constant force F is applied to the rod. Then



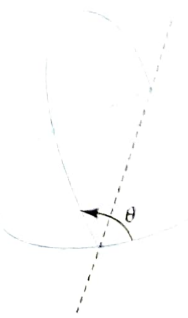
(1) the rod moves with constant velocity.

(2) the rod moves with an acceleration of $(F)/(m + B^2l^2C)$.

(3) there is constant charge on the capacitor.

(4) charge on the capacitor increases with time.

14. A uniform circular loop of radius a and resistance R is placed perpendicular to a uniform magnetic field B. One half of the loop is rotated about the diameter with angular velocity ω as shown in figure. Then, the current in the loop is



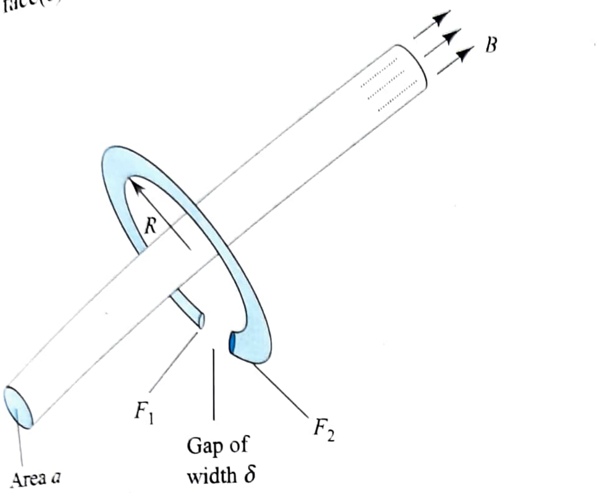
(1) zero, when θ is zero

(2) $\dfrac{\pi a^2 B\omega}{2R}$, when θ is zero

(3) zero, when $\theta = \pi/2$

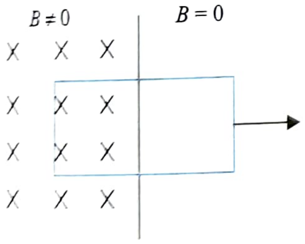
(4) $\dfrac{\pi a^2 B\omega}{2R}$, when $\theta = \pi/2$

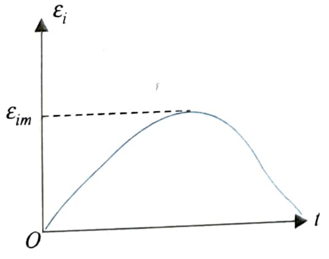
15. A highly conducting ring of radius R is perpendicular to and concentric with the axis of a long solenoid as shown in figure. The ring has a narrow gap of width δ in its circumference. The cross-sectional area of the solenoid is a. The solenoid has a uniform internal field of magnitude $B(t) = B_0 + \beta t$, where $\beta > 0$. Assume that no charge can flow across the gap, the face(s) accumulating an excess of positive charge is/are



(1) F_1
(2) F_2
(3) F_1 and F_2 both
(4) difficult to conclude as data given are insufficient

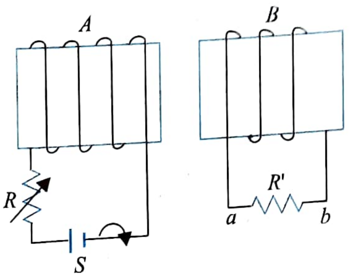
16. A plane rectangular loop is placed in a magnetic field. The emf induced in the loop due to this field is ε_i whose maximum value is ε_{im}. The loop was pulled out of the magnetic field at a variable velocity. Assume that $\vec{B}$ is uniform and constant. ε_i is plotted against time t as shown in the graph. Which of the following are/is correct statement(s):





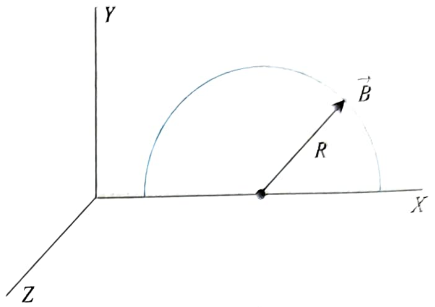
(1) ε_{im} is independent of rate of removal of coil from the field.
(2) The total charge that passes through any point of the loop in the process of complete removal of the loop does not depend on velocity of removal.
(3) The total area under the curve (ε_i vs t) is independent of rate of removal of coil from the field.
(4) The area under the curve is dependent on the rate of removal of the coil.

17. For the given electromagnetically coupled circuits: (S is initially in closed state)



(1) When switch S is opened, current in R' flows from a to b
(2) When switch S is opened, current in R' flows from b to a
(3) When coil B is brought closer to coil A (with S closed) current in R' flows from b to a
(4) When R is decreased (with S closed) then current in R' flows from b to a

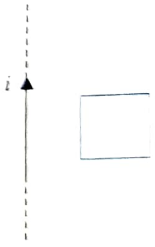
18. A semicircle conducting ring of radius R is placed in the xy plane, as shown in figure. A uniform magnetic field is set up along the x-axis. No emf, will be induced in the ring if



(1) it moves along the x-axis
(2) it moves along the y-axis
(3) it moves along the z-axis
(4) it remains stationary

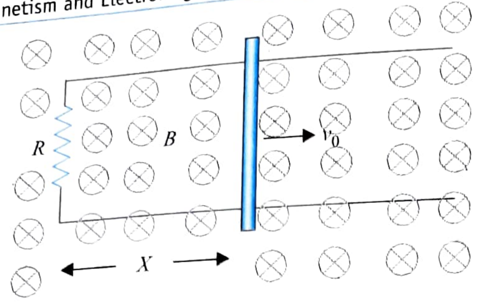
19. A bar magnet is moved along the axis of copper ring placed far away from the magnet. Looking from the side of the magnet, an anticlockwise current is found to be induced in the ring. Which of the following may be true?
(1) The south pole faces the ring and the magnet moves towards it.
(2) The north pole faces the ring and the magnet moves towards it.
(3) The south pole faces the ring and the magnet moves away from it.
(4) The north pole faces the ring and the magnet moves away from it.

20. A square conducting loop is placed in the neighbourhood of a coplanar long straight wire carrying a current i.



(1) If $\frac{di}{dt} = 0$, no current is induced in the loop
(2) If $\frac{di}{dt} > 0$, current in the loop is clockwise
(3) If $\frac{di}{dt} < 0$, current in the loop is anticlockwise
(4) If $\frac{di}{dt} > 0$, current in the loop is anticlockwise

21. A conducting rod of length is moved at constant velocity 'v_0' on two parallel, conducting, smooth, fixed rails, that are placed in a uniform constant magnetic field B perpendicular to the plane of the rails as shown in figure. A resistance R is connected between the two ends of the rail. Then which of the following is/are correct:



(1) The thermal power dissipated in the resistor is equal to rate of work done by external person pulling the rod.

(2) If applied external force is doubled then a part of external power increases the velocity of rod.

(3) Lenz's Law is not satisfied if the rod is accelerated by external force

(4) If resistance R is doubled then power required to maintain the constant velocity v_0 becomes half.

22. A rectangular coil 20 cm × 10 cm having 500 turns rotates in a magnetic field of 5×10^{-3} T with a frequency of 1200 rev min^{-1} about an axis perpendicular to the field.

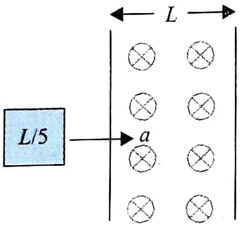
(1) The maximum value of the induced emf is $2\pi/5$ volt.

(2) The instantaneous emf when the plane of the coil is perpendicular to the field is zero.

(3) The instantaneous emf when the plane of the coil makes an angle of 60° with the field is $\pi/5$ volt .

(4) The instantaneous emf when the plane of the coil makes an angle of 30° with the field is $\pi/10$ volt.

23. Uniform magnetic field $B = 5$ T is acting in the region of length $L = 5$ m as shown in figure. A square loop of side $L/5$ enters in it with constant acceleration $a = 1$ m s^{-2}. Resistance per unit length of the square frame is 1 Ω m^{-1}. At $t = 1$ s



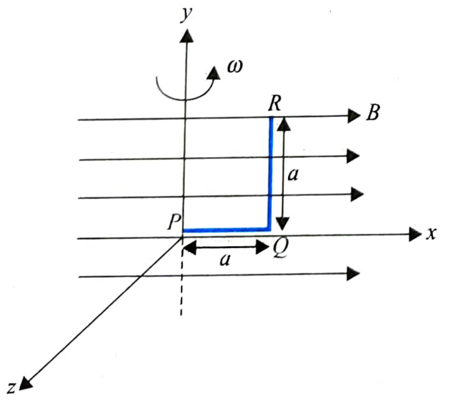
(1) Induced current in the square frame is anti-clockwise.

(2) Induced current in the frame is 1.25 A.

(3) Magnetic force on the frame is 6.25 N.

(4) Magnetic torque on the frame is zero

24. In a region there exists a magnetic field B_0 along positive x-axis. A metallic wire of length $2a$, one side along x-axis and one side parallel of y-axis is rotates about y-axis with a angular velocity. Then at the instant shown



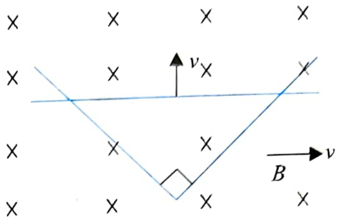
(1) Potential difference across PQ is 0

(2) Potential difference across PQ is $\frac{1}{2}B_0\omega a^2$

(3) Potential difference across QR is $\frac{1}{2}B_0\omega a^2$

(4) Potential difference across QR is $B_0\omega a^2$

25. Two straight conducting rails form a right angle where their ends are joined. A conducting bar in contact with the rails starts at the vertex at time $t = 0$ and moves with constant velocity v along them as shown in figure. A magnetic field $\vec{B}$ is directed into the page. The induced emf in the circuit at any time t is proportional to



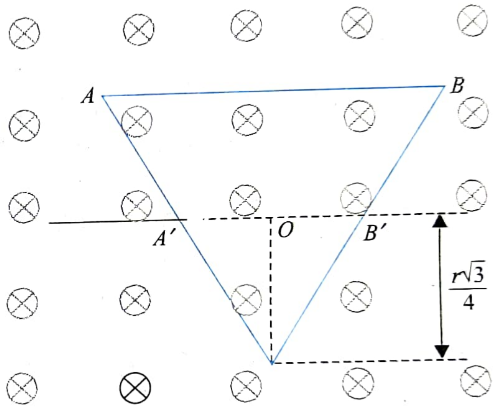
(1) t_0

(2) t

(3) v

(4) v^2

26. ABC is an equilateral triangular frame of mass m and side r. It is at rest under the action of horizontal magnetic field B (as shown) and the gravitational field.



(1) The frame remains at rest if the current in the frame is $\frac{2mg}{rB}$.

(2) The frame remains at rest if the current in the frame is $\frac{2mg}{rB\sqrt{3}}$.

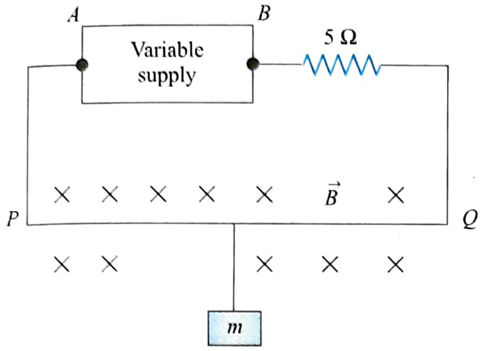
(3) The frame is in simple harmonic motion when frame is slightly displaced in its plane perpendicular to AB. The period of oscillation is $\pi\left[\frac{r\sqrt{3}}{g}\right]^{1/2}$.

(4) For same as in above option, the period of oscillation is $\pi\left[\frac{3r}{2g}\right]^{1/2}$.

Linked Comprehension Type

For Problems 1–2

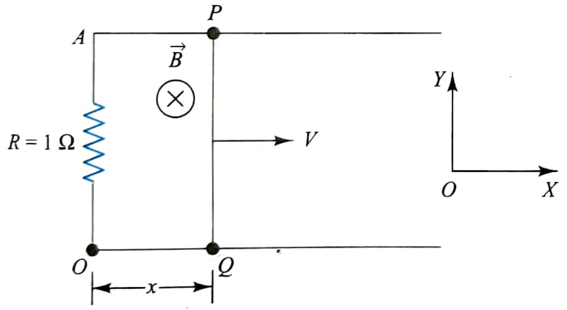
A brillant student of physics developed a magnetic balance to weigh objects. The mass m to be measured is hung from the center of the bar. Bar is kept in a uniform magnetic field of 1.5 T directed into the plane of the figure. Battery voltage can be adjusted to vary the current in the circuit. The horizontal bar shown is 60 cm long and is made of extremely light weight material. It is connected to the battery via a resistance. There is no tension in the supporting wires. The magnetic force only supports the hanging weight.



1. Which point of battery terminal is positive?
   (1) A (2) B
   (3) either A or B (4) cannot be found
2. If $V = 150$ V, what is the maximum mass m?
   (1) 1.3 kg (2) 1.8 kg
   (3) 2.2 kg (4) 2.7 kg

For Problems 3–5

Consider two parallel, conducting frictionless tracks kept in a gravity-free space as shown in figure. A movable conductor PQ, initially kept at OA, is given a velocity 10 m s^{-1} toward right. The space contains a magnetic field which depends upon the distance moved by conductor PQ from the OA line and given by



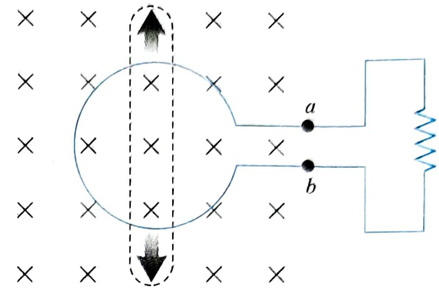
$$\vec{B} = cx(-\hat{k}) \quad [c = \text{constant} = 1 \text{ SI unit}]$$

The mass of the conductor PQ is 1 kg and length of PQ is 1 m. Answer the following questions based on the above passage.

3. The distance travelled by the conductor when its speed is 5 m s^{-1} is
   (1) $\left(\frac{15}{2}\right)^{1/3}$ (2) $\left(\frac{10}{3}\right)^{1/3}$
   (3) $(10)^{1/3}$ (4) none of the above
4. The heat loss during the time interval $t = 0$ to time t seconds, when the speed of the conductor is 5 m s^{-1} is
   (1) 50 J (2) 30 J
   (3) 10 J (4) none of the above
5. The work done by magnetic force acting on the conductor PQ during its motion in the time interval $t = 0$ to $t = t$ seconds when the speed of conductor is 5 m s^{-1} is
   (1) zero (2) 50 J
   (3) 10 J (4) 30 J

For Problems 6–7

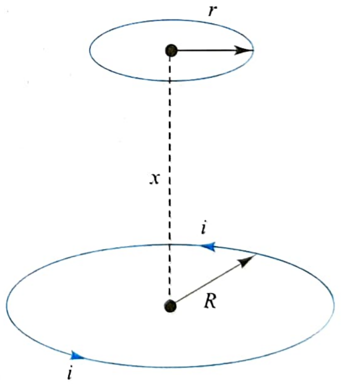
A flexible circular loop 20 cm in diameter lies in a magnetic field with magnitude 1.0 T, directed into the plane of the page as shown in figure. The loop is pulled at the points indicated by the arrows, forming a loop of zero area in 0.314 s.



6. The average induced emf in the circuit is
   (1) 0.2 V (2) 0.1 V
   (3) 1 V (4) 10 V
7. If $R = 0.01\ \Omega$, the magnitude and direction of current flowing in the loop are
   (1) 1 A, clockwise (2) 1 A, anticlockwise
   (3) 10 A, clockwise (4) 10 A, anticlockwise

For Problems 8–9

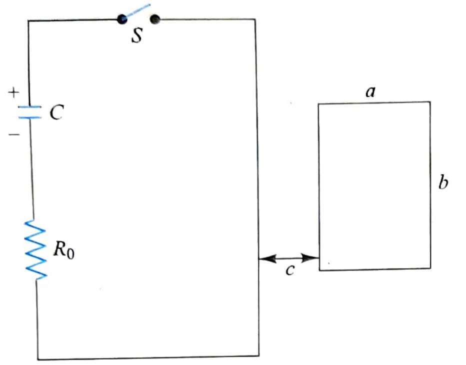
Figure shows two parallel and coaxial loops. The smaller loop (radius r) is above the larger loop (radius R), by distance $x \gg R$. The magnetic field due to current i in the larger loop is nearly constant throughout the smaller loop. Suppose that x is increasing at a constant rate of $dx/dt = v$.



8. Determine the magnetic flux through the smaller loop as a function of x.
   (1) $\frac{\mu_0 i R^2 \pi r^2}{x^3}$ (2) $\frac{\mu_0 i R^2 \pi r^2}{2x^3}$
   (3) $\frac{2\mu_0 i R^2 \pi r^2}{x^3}$ (4) $\frac{\sqrt{2}\mu_0 i R^2 \pi r^2}{x^3}$
9. The induced emf in the smaller loop is
   (1) $\frac{\mu_0 \pi i R^2 r^2}{x^4} v$ (2) $\frac{\mu_0 \pi i R^2 r^2}{2x^4} v$
   (3) $\frac{3}{2}\frac{\mu_0 \pi i R^2 r^2}{x^4} v$ (4) $\frac{\mu_0 \pi i R^2 r^2}{3x^4} v$

For Problems 10–12

In the circuit shown in figure, the capacitor has capacitance $C = 20\ \mu F$ and is initially charged to 100 V with the polarity shown. The resistor R_0 has resistance 10 Ω. At time $t = 0$, the switch is closed. The smaller circuit is not connected in any way to the larger one. The wire of the smaller circuit has a resistance of 1.0 Ω m^{-1} and contains 25 loops. The larger circuit is a rectangle 2.0 m by 4.0 m, while the smaller one has dimensions $a = 10.0$ cm and $b = 20.0$ cm. The distance c is 5.0 cm. (The figure is not drawn to scale.) Both circuits are held stationary. Assume that only the wire nearest to the smaller circuit produces an appreciable magnetic field through it.



10. The current in the larger circuit 200 ms after closing S is

(1) $\frac{5}{e}$ A (2) $\frac{2}{e}$ A

(3) $\frac{15}{e}$ A (4) $\frac{10}{e}$ A

11. The current in the smaller circuit 200 μs after closing S is

(1) 54 μA (2) 10 μA

(3) 15 μA (4) 36 μA

12. The direction of current in the smaller circuit is

(1) clockwise

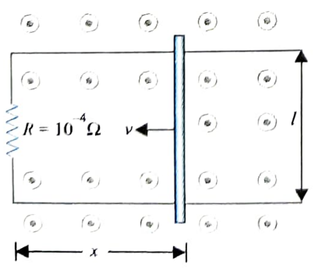
(2) anticlockwise

(3) always changes with time

(4) cannot be calculated

For Problems 13–14

A metal bar is moving with a velocity of $v = 5\ cms^{-1}$ over a U-shaped conductor. At $t = 0$, the external magnetic field is 0.1 T directed out of the page and is increasing at a rate of 0.2 T s^{-1}. Take $l = 5$ cm, and at $t = 0$, $x = 5$ cm.



13. The emf induced in the circuit is

(1) 125 μV (2) 250 μV

(3) 100 μV (4) 300 μV

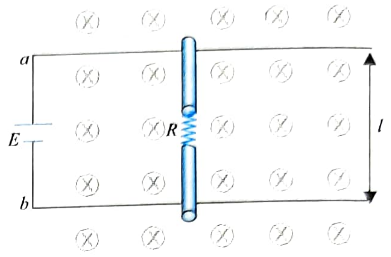
14. The current flowing in the circuit is

(1) 2.5 A (2) 5 A

(3) 1 A (4) 2 A

For Problems 15–17

In figure shown, the rod has a resistance R, the horizontal rails have negligible friction. A battery of emf E and negligible internal resistance is connected between points a and b. The rod is released from rest.



15. The velocity of the rod as function of time is

(1) $\frac{E}{Bl}(1 - e^{-t/\tau})$ (2) $\frac{E}{Bl}(1 + e^{-t/\tau})$

(3) $\frac{3}{2}\frac{E}{Bl}(1 - e^{-t/\tau})$ (4) $\frac{E}{2Bl}(1 - e^{-t/\tau})$

16. After some time, the rod will approach a terminal speed. Find an expression for it.

(1) $\frac{3}{2}\frac{E}{Bl}$ (2) $\frac{E}{2Bl}$

(3) $\frac{E}{Bl}$ (4) $\frac{2E}{Bl}$

17. The current when the rod attains its terminal speed is

(1) $\frac{2E}{R}$ (2) zero

(3) $\frac{3}{2}\frac{E}{R}$ (4) $\frac{E}{2R}$

For Problems 18–20

A fan operates at 200 V (dc) consuming 1000 W when running at full speed. Its internal wiring has resistance 1 Ω. When the fan runs at full speed, its speed becomes constant. This is because the torque due to magnetic field inside the fan is balanced by the torque due to air resistance on the blades of the fan and torque due to friction between the fixed part and the shaft of the fan. The electrical power going into the fan is spent (i) in the internal resistance as heat, call it P_1, (ii) in doing work against internal friction and air resistance producing heat, sound, etc., call it P_2. When the coil of fan rotates, an emf is also induced in the coil. This opposes the external emf applied to send the current into the fan. This emf is called back emf, call it e. Answer the following questions when the fan is running at full speed.

18. The current flowing into the fan and the value of back emf e is

(1) 200 A, 5 V (2) 5 A, 200 V

(3) 5 A, 195 V (4) 1 A, 0 V

19. The value of power P_1 is

(1) 1000 W (2) 975 W

(3) 25 W (4) 200 W

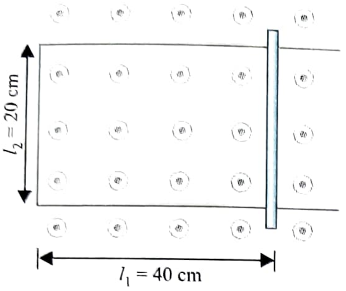
20. The value of power P_2 is

(1) 10000 W (2) 975 W

(3) 25 W (4) 200 W

For Problems 21–23

Figure shows a conducting rod of negligible resistance that can slide on a smooth U-shaped rail made of wire of resistance $1\ \Omega\ \text{m}^{-1}$. Position of the conducting rod at $t = 0$ is shown. A time-dependent magnetic field $B = 2T$ (tesla) is switched on at $t = 0$.



21. The current in the loop at $t = 0$ due to induced emf is

(1) 0.16 A, clockwise (2) 0.08 A, clockwise

(3) 0.08 A, anticlockwise (4) zero

22. At $t = 0$, when the magnetic field is switched on, the conducting rod is moved to the left at a constant speed of 5 cm s^{-1} by some external means. The rod moves perpendicular to the rail. At $t = 2$ s, induced emf has magnitude

(1) 0.12 V (2) 0.08 V

(3) 0.04 V (4) 0.02 V

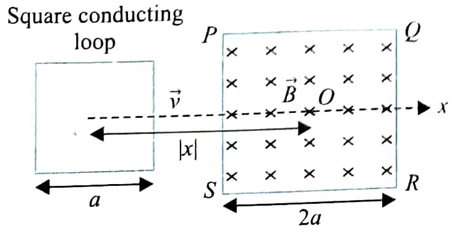
23. Following situation of the previous question, the magnitude of the force required to move the conducting rod at a constant speed of 5 cm s^{-1} at the same instant $t = 2$ s is equal to

(1) 0.16 N (2) 0.12 N

(3) 0.08 N (4) 0.06 N

For Problems 24–26

PQRS is a square region of side $2a$ in the plane of paper. A uniform magnetic field B, directed perpendicular to the plane of paper and into its plane is confined within this square region. A square loop of side 'a' and made of a conducting wire of resistance R is moved at a constant velocity $\vec{v}$ from left to right in the plane of paper as shown. Obviously, the square loop will enter the magnetic field at some time and then leave it after some time. During the motion of loop, whenever magnetic flux through it changes, emf will be induced resulting in induced current. Let the motion of the square loop be along x-axis and let us measure x coordinate of the centre of square loop from the centre of the square magnetic field region (taken as origin). Thus, x coordinate will be positive if the centre of square loop is to the right of the origin O (centre of magnetic field) and negative if centre is to the left.



24. For $x = -9a/5$, magnitude of induced current and its direction as seen from above will be:

(1) Bav, clockwise (2) Bav/R, clockwise

(3) zero (4) Bav/R, anticlockwise

25. External force required to maintain constant velocity of the loop for $x = -\frac{9}{5}a$ will be

(1) $B^2a^2v^2$ to the right (2) $\frac{B^2a^2v^2}{R}$ to the right

(3) $\frac{B^2a^2v^2}{R}$ to the left (4) zero

26. For $x = -a/4$,

(i) magnetic flux through the loop,

(ii) induced current in the loop and

(iii) external force required to maintain constant velocity of the loop, will be

(1) (i) Ba^2 (ii) $\frac{Bav}{2R}$ (iii) $\frac{B^2a^2v^2}{4R^2}$

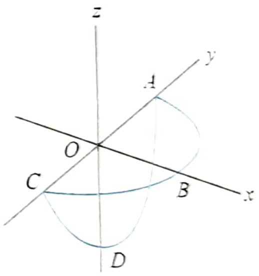
(2) (i) Ba^2 (ii) zero (iii) zero

(3) (i) Ba^2 (ii) $\frac{Bav}{2R}$ (iii) zero

(4) (i) zero (ii) zero (iii) zero

For Problems 27–29

ABCDA is a closed loop of conducting wire consisting of two semicircular sections, the part *ABC* lying in the *XY* plane and the part *CDA* lying in the *YZ* plane, both the parts having the centre at origin *O* (see the diagram). This loop is placed in a uniform field $\vec{B}$ which varies with time. Radius of the semicircular section is 0.5 m.



27. If $\vec{B}$ be directed along $(+\vec{i} - \vec{k})$ and decreases at the rate of 10^{-2} Tesla/second, the magnitude and the sense of the induced emf in the loop, as seen along the magnetic field direction will be

(1) zero

(2) $\frac{5\pi}{4\sqrt{2}}$ mV in the counterclockwise sense

(3) $\frac{5\pi}{4\sqrt{2}}$ mV in the clockwise sense

(4) $\frac{5\pi}{2\sqrt{2}}$ mV in the clockwise sense

28. If $\vec{B}$ be directed along the vector $(\vec{i} - \vec{j})$ and decreases at the rate of 10^{-2} Tesla/second. The magnitude and sense of the induced emf in the loop as seen along the positive x-axis will be

(1) zero

(2) $\frac{5\pi}{4\sqrt{2}}$ mV in the counterclockwise sense

(3) $\frac{5\pi}{4\sqrt{2}}$ mV in the clockwise sense

(4) $\frac{5\pi}{2\sqrt{2}}$ mV in the clockwise sense

29. If $\vec{B}$ be directed along the $+z$ axis and increases at the rate of 10^{-2} Tesla/second, the magnitude and sense of the induced emf in the loop as seen along the field direction will be

(1) zero

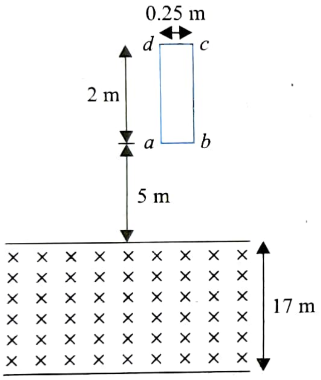
(2) $\frac{5\pi}{4}$ mV in the counterclockwise sense

(3) $\frac{5\pi}{4\sqrt{2}}$ mV in the clockwise sense

(4) $\frac{5\pi}{2\sqrt{2}}$ mV in the counter clockwise sense

For Problems 30–31

A rectangular wire frame of dimensions (0.25 m × 2.0 m) and mass 0.5 kg falls from a height 5m above a region occupied by uniform magnetic field of magnetic induction 1 T. The resistance of the wire frame is 1/8 Ω. Find



30. time taken to completely enter into the field is

(1) 0.2 s (2) 1 s

(3) 2.2 s (4) $\sqrt{\frac{1}{5}}$ s

31. time taken by the wire frame when it just starts coming out of the magnetic field.

(1) 0.2 s (2) 1 s

(3) 2.2 s (4) $\sqrt{\frac{1}{5}}$ s

For Problems 32–33

A stationary circular loop of radius a is located in a magnetic field which varies with time from $t = 0$ to $t = T$ according to law $B = B_0 t(T - t)$. If plane of loop is normal to the direction of field and resistance of the loop is R, calculate

32. amount of heat generated in the loop during this inter-val.

(1) $\frac{\pi^2 a^4 B_0^2 T^3}{3R}$ (2) $\frac{\pi^2 a^4 B_0^2 T^3}{R}$

(3) $\frac{3\pi^2 a^4 B_0^2 T^3}{R}$ (4) None of these

33. magnitude of charge flown through the loop from instant $t = 0$ to the instant when current reverses its direction.

(1) $\frac{\pi a^2 B_0 T^2}{R}$ (2) $\frac{\pi a^2 B_0 T^2}{4R}$

(3) $\frac{4\pi a^2 B_0 T^2}{R}$ (4) None of these

For Problems 34–35

A circular ring of radius a is made from a wire having resistance λ per unit length. The ring is mounted on a car such that ring remains vertical. The car moves along a horizontal circle of radius R and completes n revolutions per minute. The horizontal component of earth's magnetic field be H.

34. The emf induced in the ring is

(1) $\frac{\pi^2 a^2 nH}{10}\cos\left(\frac{\pi nt}{30}\right)$ (2) $\frac{\pi^2 a^2 nH}{30}\cos\left(\frac{\pi nt}{30}\right)$

(3) $\frac{\pi^2 a^2 nH}{30}\sin\left(\frac{\pi nt}{30}\right)$ (4) None of these

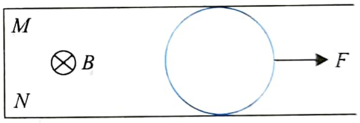
35. Calculate average rate at which heat is produced in the ring.

(1) $\frac{\pi^3 a^3 H^2 n^2}{1800\lambda}$ J s^{-1} (2) $\frac{\pi^3 a^3 H^2 n^2}{3600\lambda}$ J s^{-1}

(3) $\frac{\pi^3 a^3 H^2 n^2}{1200\lambda}$ J s^{-1} (4) None of these

For Problems 36–37

Two long parallel conducting horizontal rails are connected by a conducting wire at one end. A uniform magnetic field B (directed vertically downwards) exists in the region of space.



A light uniform ring of dia-meter d which is practically equal to separation between the rails is placed over the rails as shown in figure. If resistance of ring be λ per unit length,

36. The current in the wire MN is

(1) $\frac{2Bv}{\pi\lambda}$ from M to N (2) $\frac{Bv}{\pi\lambda}$ from M to N

(3) $\frac{2Bv}{\pi\lambda}$ from N to M (4) None of these

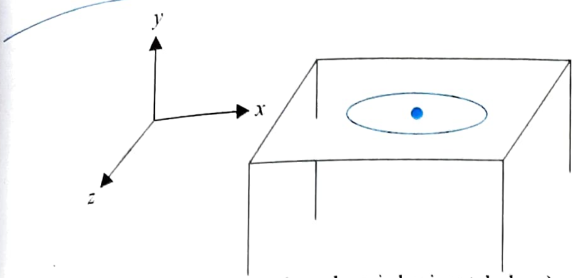
37. The force required to pull the ring with uniform velocity v is

(1) $\frac{4B^2vd}{3\pi\lambda}$ (2) $\frac{3B^2vd}{4\pi\lambda}$

(3) $\frac{2B^2vd}{3\pi\lambda}$ (4) $\frac{4B^2vd}{\pi\lambda}$

For Problems 38–41

A uniform conducting ring of mass π kg and radius 1 m is kept on smooth horizontal table. A uniform but time varying magnetic field $B = (\hat{i} + t^2\hat{j})$ T is present in the region, where t is time in seconds. Resistance of ring is 2 Ω. Then,



(x-z plane is horizontal plane)

38. Net magnetic field (in Newton) on conducting ring as function of time is

(1) $2\pi^2 t$ (2) $2\pi^2 t^2$

(3) $2\pi^2 t^3$ (4) zero

39. Time (in second) at which ring start toppling is

(1) $\frac{10}{\pi}$ (2) $\frac{20}{\pi}$

(3) $\frac{5}{\pi}$ (4) $\frac{25}{\pi}$

40. Heat generated (in kJ) through the ring till the instant when ring start toppling is

(1) $\frac{1}{3\pi}$ (2) $\frac{2}{\pi}$

(3) $\frac{2}{3\pi}$ (4) $\frac{1}{\pi}$

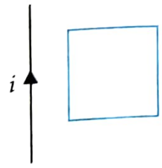
41. Induced electric field (in volt/metre) at the circumference of ring at the instant ring start toppling is

(1) $\frac{10}{\pi}$ (2) $\frac{20}{\pi}$

(3) $\frac{5}{\pi}$ (4) $\frac{25}{\pi}$

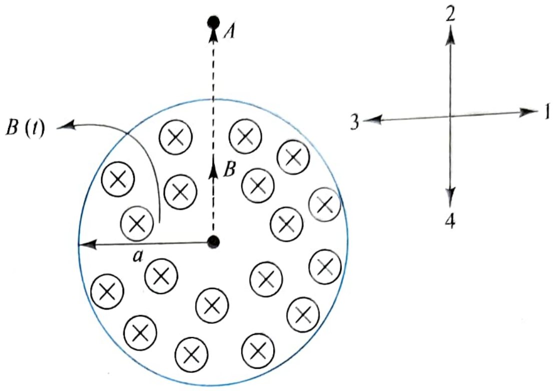
Matrix Match Type

1. A square loop is placed near a long straight current carrying wire as shown in figure. Match the following table.



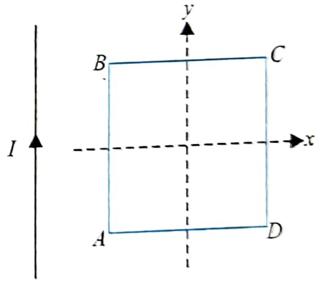
Column I	Column II
i. If current is increased,	**a.** induced current in loop is clockwise
ii. If current is decreased,	**b.** induced current in loop is anticlockwise
iii. If loop is moved away from the wire,	**c.** wire will attract the loop
iv. If loop is moved towards the wire,	**d.** wire will repel the loop

2. A uniform but time varying magnetic field $B(t)$ exists in a cylindrical region of radius a and is directed into the plane of the paper, as shown in figure. The magnetic field decreases at a constant rate inside the region. If r is the distance from the axis of the cylindrical region, then match column I with column II.



Column I	Column II
i. Induced electric field at point A	**a.** Directed along 3
ii. Induced electric field at point B	**b.** Directed along 1
iii. Force on an electron placed at point A	**c.** Increases as r
iv. Force on an electron placed at point B	**d.** Decreases as $1/r$

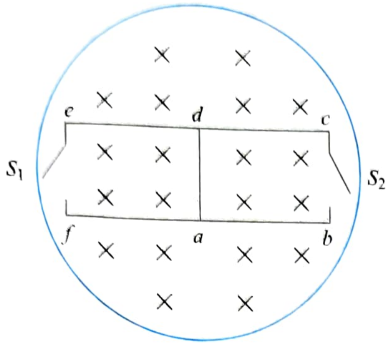
3. A long current carrying wire and a loop made of conducting wire are placed in x–y plane, such that the long wire is parallel to y-axis. Column I is regarding some changes made in the position of loop and Column II indicates the resulting effects.



Match the columns.

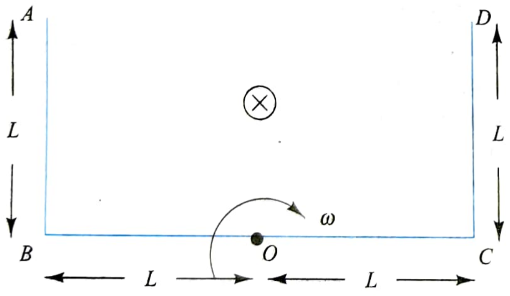
Column I	Column II
i. If loop is moved away from the wire while keeping in x-y plane,	**a.** current is induced in the loop in anticlockwise direction.
ii. If loop is moved toward the wire while keeping in x-y plane	**b.** current is induced in the loop in clockwise direction.
iii. If loop is rotated about x-axis, then just after this	**c.** no emf is induced in the loop.
iv. If loop is rotated about y-axis, then just after this	**d.** the wire will attract or repel the loop.

4. The magnetic field in the cylindrical region shown in figure increases at a constant rate of 10.0 mT s^{-1}. Each side of the square loop *abcd* and *defa* has a length of 2.00 cm and a resistance of 2.00 Ω. Correctly match the current in the wire *ad* in four different situations as listed in column I with the values given in column II.



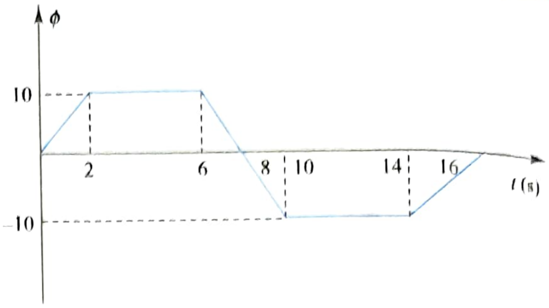
Column I		Column II	
i.	The switch S_1 is closed but S_2 is open	**a.**	5×10^{-7} A, *d* to *a*
ii.	S_1 is open but S_2 is closed	**b.**	5×10^{-7} A, *a* to *d*
iii.	both S_1 and S_2 are open	**c.**	2.5×10^{-8} A, *d* to *a*
iv.	both S_1 and S_2 are closed	**d.**	no current flows

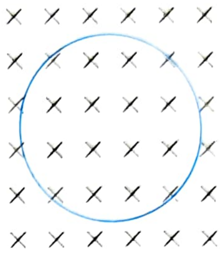
5. A frame *ABCD* is rotating with an angular velocity *ω* about an axis passing through point *O* perpendicular to the plane of paper as shown in the figure. A uniform magnetic field $\vec{B}$ is applied into the plane of the paper in the region as in the figure. Match the following.



Column I		Column II	
i.	Potential difference between *A* and *O* is	**a.**	zero
ii.	Potential difference between *O* and *D* is	**b.**	$\frac{B\omega L^2}{2}$
iii.	Potential difference between *C* and *D* is	**c.**	$B\omega L^2$
iv.	Potential difference between *A* and *D* is	**d.**	constant

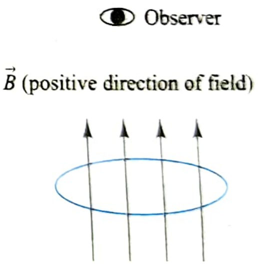
6. Magnetic flux in a circular coil of resistance 10 Ω changes with time as shown in figure. Cross indicates a direction perpendicular to paper inward. Match the following.



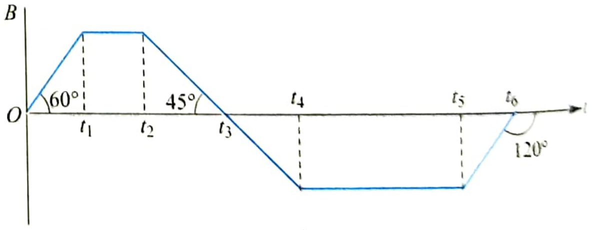


Column I		Column II	
i.	At 1 s, induced current is	**a.**	Clockwise
ii.	At 5 s, induced current is	**b.**	Anticlockwise
iii.	At 9 s, induced current is	**c.**	Zero
iv.	At 15 s, induced current is	**d.**	2 A

7. A conducting loop is held in a magnetic field such that the field is oriented perpendicular to the area of the loop as shown in Fig. (a). At any instant, magnetic flux density over the entire area has the same value but it varies with time as shown in Fig. (b).



(a)



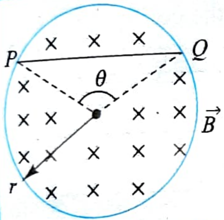
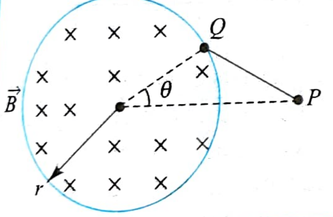
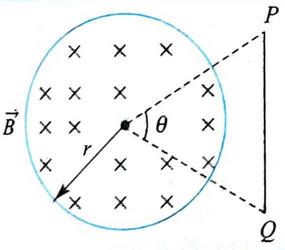
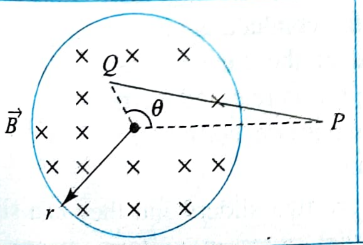
(b)

Column I		Column II	
i.	Induced current in the coil is in the clockwise sense	**a.**	For $t_2 < t < t_3$
ii.	Induced current in the coil is in the anticlockwise sense	**b.**	For $t_3 < t < t_4$
iii.	Induced current is zero	**c.**	For $t_5 < t < t_6$
iv.	Induced current is maximum	**d.**	For $t_4 < t < t_5$

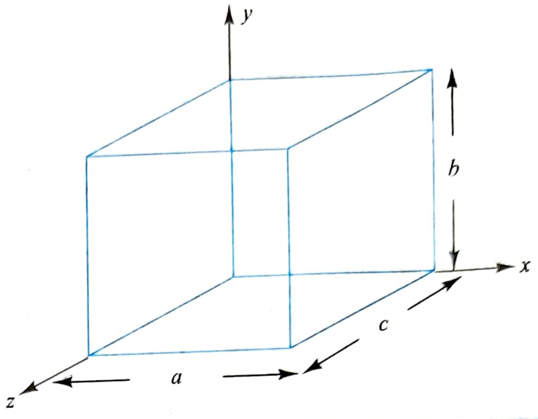
8. Column I gives situations involving a charged particle which may be realized under the condition given in column II. Match the situations in column I with the conditions in column II.

Column I	Column II
i. Increase in speed of a charged particle	**a.** Electric field uniform in space and constant in time
ii. Exert a force on an electron initially at rest	**b.** Magnetic field uniform in space and constant in time
iii. Move a charged particle in a circle with uniform speed	**c.** Magnetic field uniform in space but varying with time
iv. Accelerate a moving charged particle	**d.** Magnetic field non-uniform in space but constant with time

9. Column I shows the cylindrical region of radius r where a downward magnetic field $\vec{B}$ exists, where $\vec{B}$ is increasing at the rate of dB/dt. A rod PQ is placed in different citation as shown. Match the column I with the correct statement in column II regarding the induced emf in rod.

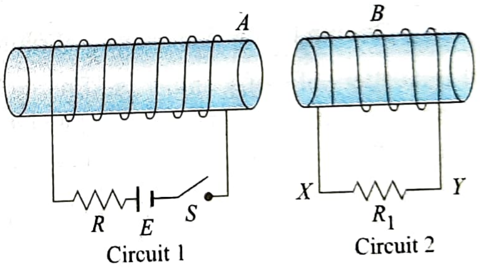
Column I	Column II
i. 	**a.** Induced emf in rod PQ is $\frac{1}{2}r^2\theta\frac{dB}{dt}$.
ii. 	**b.** Induced emf in rod PQ is less than $\frac{1}{2}r^2\theta\frac{dB}{dt}$.
iii. 	**c.** End P is positive with respect to point Q.
iv. 	**d.** End Q is positive with respect to point P.

10. Figure shows a metallic solid block, placed in a way so that its faces are parallel to the coordinate axes. Edge lengths along axes x, y, and z are a, b, and c, respectively. The block is in a region of uniform magnetic field of magnitude 30 mT. One of the edge lengths of the block is 25 cm. The block is moved at 4 ms^{-1} parallel to each axis and in turn, the resulting potential difference V that appears across the block is measured. When the motion is parallel to the y-axis, V = 24 mV; with the motion parallel to the z-axis, V = 36 mV; with the motion parallel to the x-axis, V = 0. Using the given information, correctly match the dimensions of the block with the values given.



Column I	Column II
i. a	**a.** 20 cm
ii. b	**b.** 24 cm
iii. c	**c.** 25 cm
iv. $(bc)/a$	**d.** 30 cm

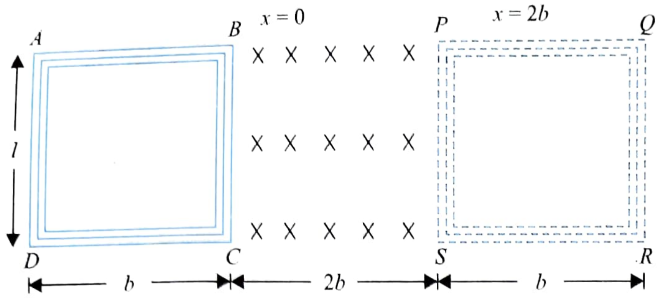
11. In the circuit shown in figure, two coils are arranged as shown. In **Column I** some operations which are carried out in circuit 1 are mentioned and in **Column II** its effects. Match the entries of **Column I** with **Column II**. Take heating effect of current also into the consideration.



Column I	Column II
i. Switch S is closed [exclude small switching time]	**a.** current in R_1 is from X to Y
ii. Switch S is closed for long time then it is opened. For this the transition time is	**b.** current in R_1 is from Y to X
iii. If coil A is brought nearer to B while switch S is kept closed, then	**c.** no current is flowing through R_1.
iv. The battery of constant emf is replaced by a varying emf, switch S is closed.	**d.** current in R_1 can be from X to Y or from Y to X.

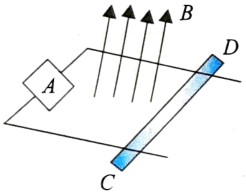
12. A rectangular loop of wire with dimensions l and b has N turns and a total resistance R. The loop is moved with constant velocity such that one side shifts from AB to PQ

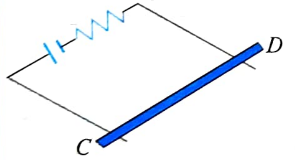
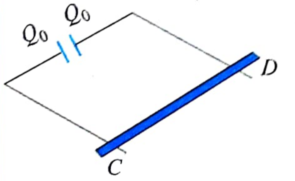
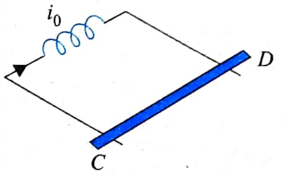
in a region of uniform magnetic field as shown. Let x be the distance of AB from PQ.



	Column I		Column II
i.	$0 < x < b$, $BNlx$	**a.**	$BvlN$
ii.	$b < x < 2b$, $BNlb$	**b.**	0
iii.	$2b < x < 3b$, $BNl(3b - x)$	**c.**	$-BvlN$
iv.	$x > 3b$	**d.**	BvN

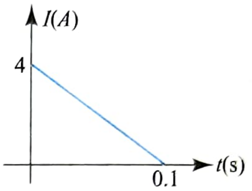
13. A frictionless conducting resistance less rod of mass m straddles the two rails of a track as shown in figure. The rails are also resistanceless. Width of the track is l and it is sufficiently long. Entire arrangement lies in horizontal plane. A homogeneous magnetic field B is present perpendicular to the plane of track. For the situation suggested in column I match the appropriate entries in column II.



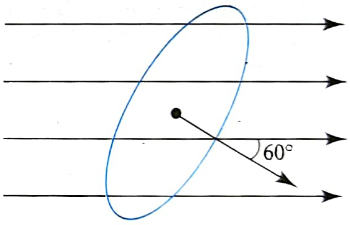
	Column I		Column II
i.	A is a battery of emf V and internal resistance R. The rod is initially at rest.  	**a.**	Energy is dissipated during the motion of the rod.
ii.	A is a charged capacitor. The system has no resistance. The rod is initially at rest.  	**b.**	The rod moves with a constant velocity after a long time.
iii.	A is an inductor with initial current i_0. It is having no resistance.  	**c.**	After a certain time interval, rod will change its direction of motion.
iv.	A is a resistance. The rod is projected to the right with a velocity V_0.	**d.**	If a constant force is applied on the rod to the right, it can immediately move with a constant velocity.

Numerical Value Type

1. Some magnetic flux is changed from a coil of resistance 10 Ω. As a result, an induced current is developed in it, which varies with time as shown in figure. Find the magnitude of the change in flux through the coil in weber.

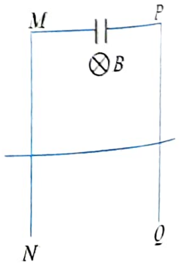


2. A circular coil of wire consists of exactly 100 turns with a total resistance 0.20 Ω. The area of the coil is 100 cm^2. The coil is kept in a uniform magnetic field B as shown in figure. The magnetic field is increased at a constant rate of 2 T s^{-1}. Find the induced current in the coil in A.

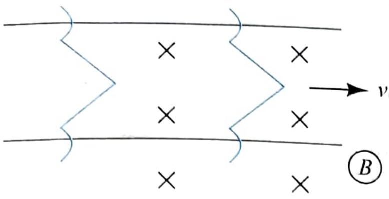


3. A long coaxial cable consists of two thin-walled conducting cylinders with inner radius 2 cm and outer radius 8 cm. The inner cylinder carries a steady current 0.1 A, and the outer cylinder provides the return path for that current. The current produces a magnetic field between the two cylinders. Find the energy stored in the magnetic field for length 5 m of the cable. Express answer in nJ (use ln 2 = 0.7).

4. Figure shows a conducting rod of length $l = 10$ cm, resistance R and mass $m = 100$ mg moving vertically downward due to gravity. Other parts are kept fixed. Magnetic field is $B = 1$ T. MN and PQ are vertical, smooth, conducting rails. The capacitance of the capacitor is $C = 10$ mF. The rod is released from rest. Find the maximum current (in mA) in the circuit.

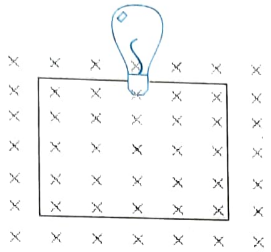


5. In figure, there are two sliders and they can slide on two frictionless parallel wires in uniform magnetic field B, which is present everywhere. The mass of each slider is m, resistance R and initially these are at rest. Now, if one slider is given a velocity $v_0 = 16$ m s^{-1}, what will be the velocity (in m s^{-1}) of other slider after long time? (neglect the self-induction)

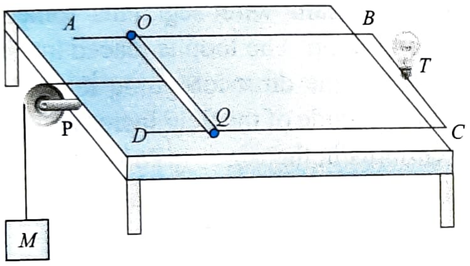


6. A square wire loop of 10.0 cm side lies at right angles to a uniform magnetic field of 7 T.

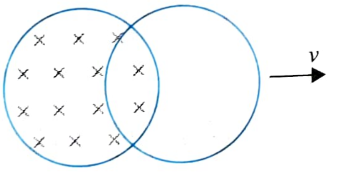
A 10 V light bulb is in a series with the loop as shown in figure. The magnetic field is decreasing steadily to zero over a time interval Δt. For what value of Δt (in ms), the bulb will shine with full brightness?



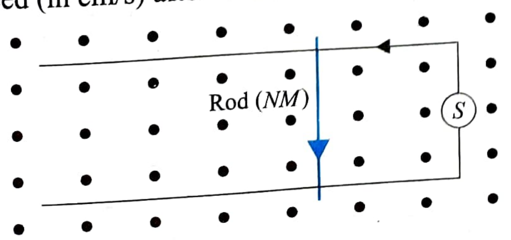
7. In figure, $ABCD$ is a fixed smooth conducting frame in horizontal plane. T is a bulb of power 100 W, P is a smooth pulley and OQ is a conducting rod. Neglect the self-inductance of the loop and resistance of any part other than the bulb. The mass M is moving down with constant velocity 10 m s^{-1}. Bulb lights at its rated power due to induced emf in the loop due to earth's magnetic field. Find the mass M (in kg) of the block. ($g = 10$ m s^{-2})



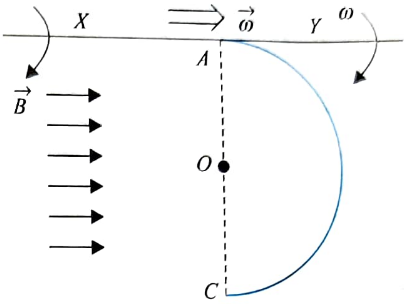
8. A uniform magnetic field $B = 0.5$ T exists in a circular region of radius $R = 5$ m. A loop of radius $R = 5$ m encloses the magnetic field at $t = 0$ and then pulled at uniform speed $v = 2$ ms^{-1} in the plane of the paper. Find the induced emf (in V) in the loop at time $t = 3$ s.



9. A conducting rod of mass 200 g and length 10 cm can slide without friction on two long, horizontal rails. A uniform magnetic field of magnitude 5 mT exists in the region as shown. A source S is used to maintain a constant current 2 A through the rod. If motion of the rod starts from the rest, find its speed (in cm/s) after 10 s from the start of the motion.



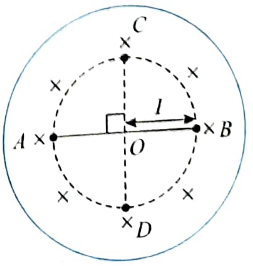
10. A thin wire AC shaped as a semi-circle of diameter d rotates with a constant angular frequency ω in a uniform magnetic field of induction $\vec{B}$. The vector $\vec{\omega}$ is parallel to $\vec{B}$ and the rotation axis XY passes through the end A of the wire and is perpendicular to the diameter AC (see figure). $\vec{B}$ is directed left to right in the plane of the paper. The value of the line integral $I = \int \vec{E} \cdot d\vec{r}$ taken along the wire from the point A to the point C will be $\frac{\omega B d^2}{?}$. What integer should be inserted in place of question mark.



11. A copper wire of length 2 m placed perpendicular to the plane of magnetic field $\vec{B} = (2\hat{i} + 4\hat{j})T$. If it moves with velocity $(4\hat{i} + 6\hat{j} + 8\hat{k})$ m/sec. Calculate the magnitude of dynamic emf (in volt)across its ends.

12. Two concentric coplanar circular loops made of wire, resistance per unit length 10^{-4} Ωm^{-1}, have diameters 0.2 m and 2 m. A time-varying potential difference $(4 + 2.5t)$ volt is applied to the larger loop. Calculate the current in the smaller loop (in A).

13. A massless non-conducting rod AB of length $2l$ is placed in uniform time varying magnetic field confined in a cylindrical region of radius ($R > l$) as shown in the figure. The center of the rod coincides with the centre of the cylindrical region. The rod can freely rotate in the plane of the figure about an axis coinciding with the axis of the cylinder. Two particles, each of mass m and charge q are attached to the ends A and B of the rod. The time varying magnetic field in this cylindrical region is given by $B = B_0\left[1 - \frac{t}{2}\right]$ where B_0 is a constant. The field is switched on at time $t = 0$. Consider: $B_0 = 100$T, $l = 4$ cm, $\frac{q}{m} = \frac{4\pi}{100}$ C/kg. Calculate the time (in sec) in which the rod will reach position CD shown in the figure for the first time. Will end A be at C or D at this instant?

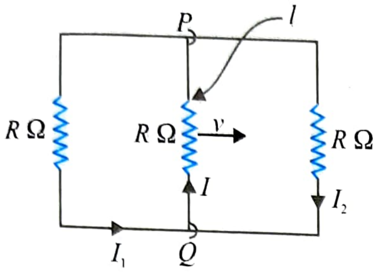


Archives

JEE MAIN

Single Correct Answer Type

1. A rectangular loop has a sliding connector PQ of length l and resistance R ohm and it is moving with a speed v as shown. The set up is placed in a uniform magnetic field going into the plane of the paper.



The three currents I_1, I_2, and I are

(1) $I_1 = -I_2 = \frac{Blv}{R}, I = \frac{2Blv}{R}$ (2) $I_1 = I_2 = \frac{Blv}{3R}, I = \frac{2Blv}{3R}$

(3) $I_1 = I_2 = I = \frac{Blv}{R}$ (4) $I_1 = I_2 = \frac{Blv}{6R}, I = \frac{Blv}{3R}$

(AIEEE 2010)

2. A boat is moving due east in a region where the earth's magnetic field is 5.0×10^{-5} NA^{-1}m^{-1} due north and horizontal. The boat carries a vertical aerial 2 m long. If the speed of the boat is 1.50 ms^{-1}, the magnitude of the induced emf in the wire of aerial is

(1) 1 mV (2) 0.75 mV

(3) 0.50 mV (4) 0.15 mV **(AIEEE 2011)**

3. A coil is suspended in a uniform magnetic field, with the plane of the coil parallel to the magnetic lines of force. When a current is passed through the coil it starts oscillating; it is very difficult to stop. But if an aluminium plate is placed near to the coil, it stops. This is due to

(1) developement of air current when the plate is placed

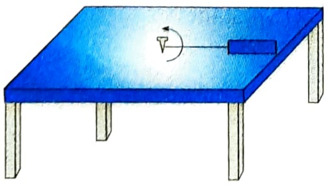
(2) induction of electrical charge on the plate

(3) shielding of magnetic lines of force as aluminium is a paramagnetic material

(4) electromagnetic induction in the aluminium plate giving rise to electromagnetic damping

(AIEEE 2012)

4. A metallic rod of length 'l' is tied to a string of length $2l$ and made to rotate with angular speed ω on a horizontal table with one end of the string fixed. If there is a vertical magnetic field 'B' in the region, the e.m.f. induced across the ends of the rod is



(1) $\frac{3B\omega l^2}{2}$ (2) $\frac{4B\omega l^2}{2}$

(3) $\frac{5B\omega l^2}{2}$ (4) $\frac{2B\omega l^2}{2}$ **(JEE Main 2013)**

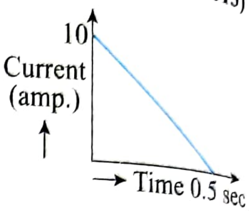
5. A circular loop of radius 0.3 cm lies parallel to a much bigger circular loop of radius 20 cm. The centre of the small loop is on the axis of the bigger loop. The distance between their centres is 15 cm. If a current of 2.0 A flows through the smaller loop, then the flux linked with bigger loop is

(1) 6×10^{-11} weber (2) 3.3×10^{-11} weber

(3) 6.6×10^{-9} weber (4) 9.1×10^{-11} weber

(JEE Main 2013)

6. In a coil of resistance 100 Ω, a current is induced by changing the magnetic flux through it as shown in the figure. The magnitude of change in flux through the coil is



(1) 250 Wb (2) 275 Wb

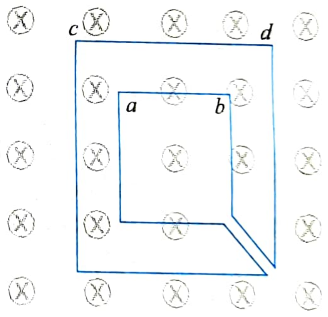
(3) 200 Wb (4) 225 Wb

(JEE Main 2017)

JEE ADVANCED

Single Correct Answer Type

1. Figure shows certain wire segments joined together to form a coplanar loop. The loop is placed in a perpendicular magnetic field in the direction going into the plane of the figure. The magnitude of the field increases with time. I_1 and I_2 are the currents in the segments ab and cd. Then,



(1) $I_1 > I_2$

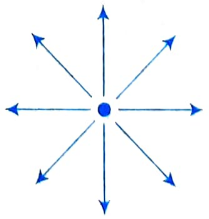
(2) $I_1 < I_2$

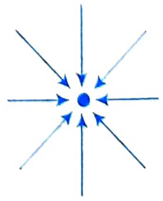
(3) I_1 is in the direction ba and I_2 is in the direction cd

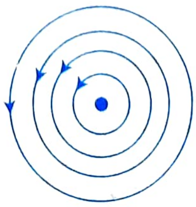
(4) I_1 is in the direction ab and I_2 is in the direction dc

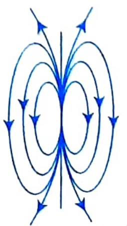
(IIT-JEE 2009)

2. Which of the field patterns given below is valid for electric field as well as for magnetic field?

(1) 

(2) 

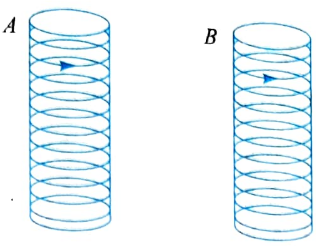
(3) 

(4) 

(IIT-JEE 2011)

Multiple Correct Answers Type

1. Two metallic rings A and B, identical in shape and size but having different resistivities ρ_A and ρ_B, are kept on top of two identical solenoids as shown in figure. When current I is switched on in both the solenoids in identical manner, the rings A and B jump to heights h_A and h_B, respectively, with $h_A > h_B$. The possible relation(s) between their resistivities and their masses m_A and m_B is(are)



(1) $\rho_A > \rho_B$ and $m_A = m_B$ (2) $\rho_A < \rho_B$ and $m_A = m_B$
(3) $\rho_A > \rho_B$ and $m_A > m_B$ (4) $\rho_A < \rho_B$ and $m_A < m_B$

(IIT-JEE 2009)

2. A current carrying infinitely long wire is kept along the diameter of a circular wire loop, without touching it. The correct statement(s) is/are

The current induced in the loop is

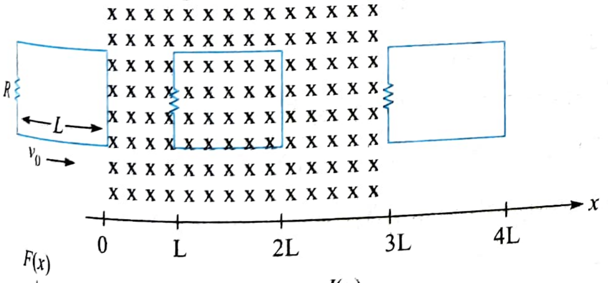
(1) The emf induced in the loop is zero if the current is constant

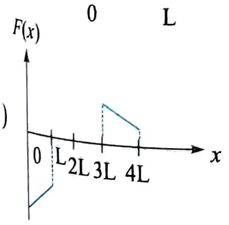
(2) The emf induced in the loop is finite if the current is constant

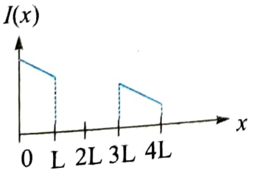
(3) The emf induced in the loop is zero if the current decreases at a steady rate.

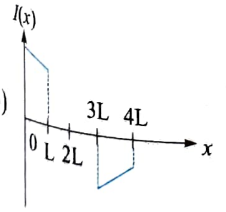
(4) The emf induced in the loop is finite if the current decreases at a steady rate. **(IIT-JEE 2012)**

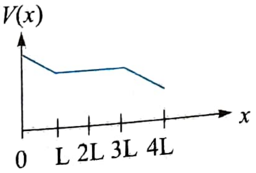
3. A rigid wire loop of square shape having side of length L and resistance R is moving along the x-axis with a constant velocity v_0 in the plane of the paper. At $t = 0$, the right edge of the loop enters a region of length $3L$ where there is a uniform magnetic field B_0 into the plane of the paper; as shown in the figure. For sufficiently large v_0, the loop eventually crosses the region. Let x be the location of the right edge of the loop. Let $v(x)$, $I(x)$ and $F(x)$ represent the velocity of the loop, current in the loop, and force on the loop, respectively, as a function of x. Counter-clockwise current is taken as positive.



(1)


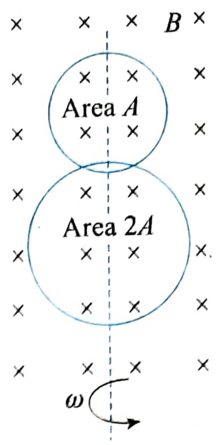
(2)


(3)


(4)


anced 2016)

4. A circular insulated copper wire loop is twisted to form two loops of area A and $2A$ as shown in the figure. At the point of crossing the wires remain electrically insulated from each other. The entire loop lies in the plane (of the paper). A uniform magnetic field $\vec{B}$ points into the plane of the paper. At $t = 0$, the loop starts rotating about the common diameter as axis with a constant angular velocity ω in the magnetic field. Which of the following options is/are correct?



(1) The amplitude of the maximum net emf induced due to both the loops is equal to the amplitude of maximum emf induced in the smaller loop alone

(2) The rate of change of the flux is maximum when the plane of the loops is perpendicular to plane of the paper

(3) The net emf induced due to both the loops is proportional to cos ωt

(4) The emf induced in the loop is proportional to the sum of the areas of the two loops

(JEE Advanced 2017)

Linked Comprehension Type

For Problems 1–2

Point Q is moving in a circular orbit of radius R in the x–y plane with an angular velocity ω. This can be considered as equivalent to a loop carrying a steady current $Q\omega/2\pi$. uniform magnetic field along the positive z-axis is now switched on, which increases at a constant rate from 0 to B in one second. Assume that the radius of the orbit remains constant. The application of the magnetic field induces an emf in the orbit. The induced emf is defined as the work done by an induced electric field in moving a unit positive charge around closed loop. It is known that, for an orbiting charge, the magnetic dipole moment is proportional to the angular momentum with a proportionality constant γ.

(JEE Advanced 2013)

1. The magnitude of the induced electric field in the orbit at any instant of time during the time interval of the magnetic field change, is

(1) $\dfrac{BR}{4}$ (2) $\dfrac{BR}{2}$

(3) BR (4) $2BR$

2. The change in the magnetic dipole moment associated with the orbit, at the end of time interval of the magnetic field change, is

(1) $-\gamma BQR^2$ (2) $-\gamma \dfrac{BQR^2}{2}$

(3) $\gamma \dfrac{BQR^2}{2}$ (4) γBQR^2

Answers Key

EXERCISES

Single Correct Answer Type

1. (1) **2.** (2) **3.** (2) **4.** (3) **5.** (2)
6. (1) **7.** (4) **8.** (4) **9.** (4) **10.** (4)
11. (4) **12.** (3) **13.** (2) **14.** (1) **15.** (2)
16. (3) **17.** (2) **18.** (1) **19.** (3) **20.** (1)
21. (3) **22.** (3) **23.** (1) **24.** (1) **25.** (2)
26. (1) **27.** (1) **28.** (2) **29.** (1) **30.** (2)
31. (1) **32.** (4) **33.** (4) **34.** (1) **35.** (3)
36. (4) **37.** (3) **38.** (1) **39.** (2) **40.** (4)
41. (4) **42.** (1) **43.** (1) **44.** (1) **45.** (4)
46. (4) **47.** (4) **48.** (4) **49.** (2) **50.** (2)
51. (3) **52.** (3) **53.** (4) **54.** (4) **55.** (4)
56. (4) **57.** (4) **58.** (3) **59.** (3) **60.** (1)
61. (4) **62.** (1) **63.** (4) **64.** (3) **65.** (3)
66. (3) **67.** (3) **68.** (4) **69.** (2) **70.** (4)
71. (2) **72.** (4) **73.** (1) **74.** (3) **75.** (4)
76. (3) **77.** (1) **78.** (1) **79.** (2) **80.** (3)
81. (2) **82.** (2) **83.** (4) **84.** (3) **85.** (2)
86. (1)

Multiple Correct Answers Type

1. (1),(2) **2.** (1),(2),(3),(4) **3.** (2),(3)
4. (1),(3) **5.** (1),(3) **6.** (1),(3),(4)
7. (1),(2),(3),(4) **8.** (2),(3) **9.** (1),(2),(3),(4)
10. (2),(4) **11.** (1),(2),(3) **12.** (2),(4)
13. (2),(4) **14.** (1),(4) **15.** (1),(4)
16. (2),(3) **17.** (1),(3),(4) **18.** (1),(2),(3),(4)
19. (2),(3) **20.** (1),(4) **21.** (1),(2),(4)
22. (1),(2),(3) **23.** (1),(2),(3),(4) **24.** (1),(4)
25. (2),(4) **26.** (1),(3)

Linked Comprehension Type

1. (1) **2.** (4) **3.** (1) **4.** (4) **5.** (1)
6. (2) **7.** (3) **8.** (2) **9.** (3) **10.** (4)
11. (1) **12.** (2) **13.** (2) **14.** (1) **15.** (1)
16. (3) **17.** (2) **18.** (3) **19.** (3) **20.** (2)
21. (1) **22.** (2) **23.** (3) **24.** (3) **25.** (4)
26. (3) **27.** (4) **28.** (3) **29.** (2) **30.** (1)
31. (3) **32.** (1) **33.** (2) **34.** (3) **35.** (2)
36. (4) **37.** (4) **38.** (4) **39.** (1) **40.** (3)
41. (1)

Matrix Match Type

1. i. → b., d.; ii. → a., c.; iii. → a., c.; iv. → b., d.
2. i. → b., d.; ii. → b., c.; iii. → a., d.; iv. → a., c.
3. i. → b., d.; ii. → a., d.; iii. → c.; iv. → c.
4. i. → b.; ii. → a.; iii. → d.; iv. → d.
5. i. → c., d.; ii. → c., d.; iii. → b., d.; iv. → a., d.
6. i. → b.; ii. → c.; iii. → a.; iv. → b.
7. i. → c.; ii. → a., b.; iii. → d.; iv. → c.
8. i. → a., c.; ii. → a., c.; iii. → b., d.; iv. → a., b., c., d.
9. i. → b., c.; ii. → a., d.; iii. → a., c.; iv. → b., d.
10. i. → c.; ii. → d.; iii. → a.; iv. → b.
11. i. → a., d.; ii. → a., d.; iii. → d.; iv. → d.
12. i. → c.; ii. → b.; iii. → a.; iv. → b.
13. i. → a., b.; ii. → a., b.; iii. → c.; iv. → a., d.

Numerical Value Type

1. (2) **2.** (5) **3.** (7) **4.** (5) **5.** (8)
6. (7) **7.** (1) **8.** (8) **9.** (5) **10.** (4)
11. (8) **12.** (1.25) **13.** (1)

ARCHIVES

JEE Main

Single Correct Answer Type

1. (2) **2.** (4) **3.** (4) **4.** (3) **5.** (4)
6. (1)

JEE Advanced

Single Correct Answer Type

1. (4) **2.** (3)

Multiple Correct Answers Type

1. (2),(4) **2.** (1),(3) **3.** (3),(4) **4.** (1),(2)

Linked Comprehension Type

1. (2) **2.** (2)

5

Inductance

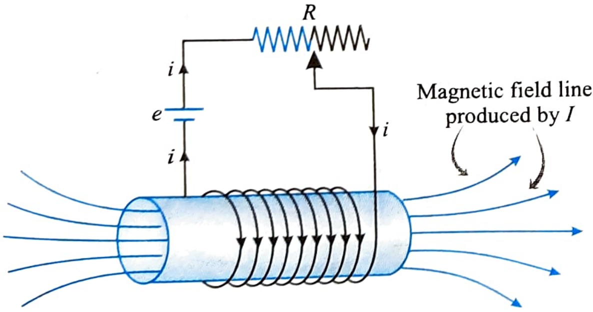
INTRODUCTION

In previous chapter we learned that an emf and a current are induced in a loop of wire when the magnetic flux through the area enclosed by the loop changes with time. In this chapter we will learn some practical applications of flux change, we first discuss two important effects of electromagnetic induction known as self-induction and mutual-induction.

Next, we will discuss the energy stored in the magnetic field of an inductor and the energy density associated with the magnetic field. Finally, we will examine the characteristics of circuits that contain inductors, resistors, and capacitors in various combinations.

SELF-INDUCTANCE

In almost all cases of induced emfs presented in previous chapter, the magnetic field has been produced by some external source, such as a permanent magnet or an electromagnet. But it is not necessary that the magnetic field always arise from an external source. The emf can be induced in a current-carrying coil by a change in the magnetic field that the current itself produces. For instance, figure shows a coil connected with a circuit where the current can be changed. The current creates magnetic field that can be changed by changing the current, which in turn creates a changing flux through the coil. The change in flux induces an induced emf in the coil, in accordance with Faraday's law. The effect in which a changing current in a circuit induces an emf in the same circuit is referred to as self-induction. The emf set up in this case is called a **self-induced emf**.



Let us consider a coil which we can now call inductor, having N turns and a current I is flowing through it, if ϕ is the magnetic flux that passes through one turn of the coil, then $N\phi$ is the net flux through a coil of N turns. Since ϕ is proportional to the magnetic field, and the magnetic field is proportional to the current I, so we have

$$N\phi \propto I \Rightarrow N\phi = LI \qquad \text{...(i)}$$

In above Eq. (i), where L is a proportionality constant—called 'Coefficient of Self Induction' or simply the inductance, of the coil or inductor and its associated circuit in which current I is flowing—that depends on the geometry of the loop and other physical characteristics.

If current changes at a rate of dI/dt then EMF induced in the circuit of coil or conductor is given as

$$e = \left|N\frac{d\phi}{dt}\right| = L\left|\frac{dI}{dt}\right| \qquad \text{...(ii)}$$

According to Lenz's law the direction of induced EMF is such that it opposes the causes of induction which is change in current thus signs of e and dI/dt are opposite in above equation which can be written without modulus as

$$e = -L\frac{dI}{dt} \qquad \text{...(iii)}$$

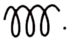
From Eq. (iii), we can also write the inductance as the ratio

$$L = -\frac{e}{di/dt} \qquad \text{...(iv)}$$

We can write resistance as $R = \dfrac{\Delta V}{i}$ ...(v)

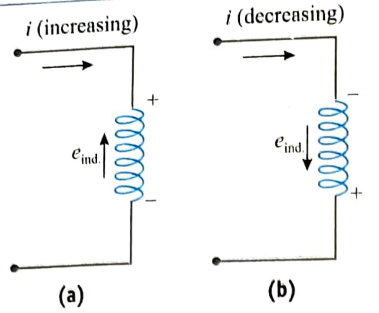
We say that resistance is a measure of the opposition to current. In comparison, Eq. (i), being of the same mathematical form as Eq. (v), shows us that inductance is a measure of the opposition to a change in current.

The SI unit of inductance is the **henry** (H), which as we can see from Eq. (iv) is 1 volt-second per ampere: 1 H = 1 V·s/A

A circuit device that is designed to have a particular inductance is called an inductor. The usual circuit symbol for an inductor is .

DIRECTION OF THE SELF-INDUCED EMF

Lenz's Law provides the direction of the self-induced potential difference. The negative sign in Eq. (iii) provides the clue that the induced potential difference always opposes any change in current. For example, Fig. (a) shows current flowing through an inductor and increasing with time. Thus, the self-induced potential difference will oppose the increase in current. In Fig. (b), the current flowing through an inductor is decreasing with time. Thus, a self-induced potential difference will oppose the decrease in current. We have assumed that these inductors are ideal inductors; that is, they have no resistance. All induced potential differences manifest themselves across the connections of the inductor.



INDUCTANCE OF A SOLENOID

Consider a long solenoid with N turns carrying a current, i. This current creates a magnetic field in the centre of the solenoid, resulting in a magnetic flux, ϕ_B. *The* same magnetic flux goes through each of the N windings of the solenoid. It is customary to define the flux **linkage** as the product of the number of windings and the magnetic flux, or $N\Phi_B$.

$$N\Phi_B = (nl)(BA) = (nl)(\mu_0 ni)(A)$$

Thus, the inductance is given by

$$L = \frac{N\Phi_B}{i} = \frac{(nl)(\mu_0 ni)(A)}{i} = \mu_0 n^2 lA \qquad \text{...(i)}$$

$$L = \mu_0 n^2 lA = \mu_0 n^2 V$$

Inductance per unit volume $= \mu_0 n^2$

This expression for the inductance of a solenoid is good for long solenoids because fringe field effects at the ends of such a solenoid are small. You can see from Eq. (i) that the inductance of a solenoid depends only on the geometry (length, area, and number of turns) of the device. This dependence of inductance on geometry alone holds for all coils and solenoids, just as the capacitance of any capacitor depends only on its geometry.

ILLUSTRATION 5.1

How many meters of a thin wire are required to manufacture a solenoid of length l_0 and inductance L, if the solenoid's cross-sectional diameter is considerably less than its length?

Sol. Self-induction coefficient of a solenoid,

$$L = \mu_0 n^2 l_0 A = \frac{\mu_0 N^2 \pi r^2}{l_0} \qquad \text{...(i)}$$

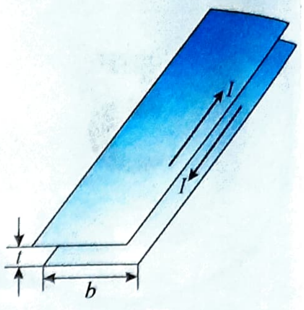
Total length of wire, $l = N \times 2\pi r \Rightarrow N = \frac{l}{2\pi r}$ ...(ii)

From (i) and (ii), we get $L = \frac{\mu_0 \left(l/2\pi r\right)^2 \pi r^2}{l_0} = \frac{\mu_0 l^2}{4\pi l_0}$

Hence length of wire used to make solenoid, $l = \sqrt{\frac{4\pi l_0 L}{\mu_0}}$

ILLUSTRATION 5.2

Find the inductance per unit length of a double tape line if the tapes are separated by a distance t which is considerably less than their width b $(t << b)$. The current is flowing in the two sheets of the tape line in opposite direction as shown in figure.



Sol. The current is uniformly distributed over the width of the tape.

The current per unit width of the sheets of tape, $j = \frac{I}{b}$

The magnetic field between the sheets is given as

$$B = \mu_0 j = \mu_0 \left(\frac{I}{b}\right)$$

The magnetic flux passing through the region between the two sheets of tape line of length l is given as

$$\phi = \frac{\mu_0 I}{b} lt \Rightarrow \frac{\phi}{I} = \frac{\mu_0}{b} lt$$

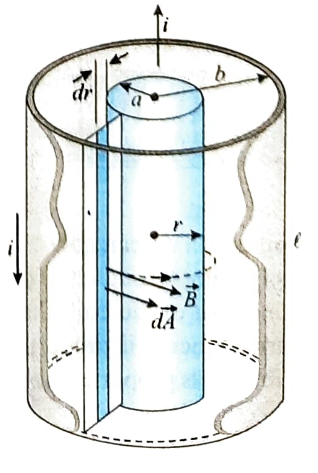
If self-inductance of the tape line, $L = \frac{\phi}{I} = \frac{\mu_0}{b} lt$

Self-inductance per unit length of the tape, $\frac{L}{l} = \frac{\mu_0 t}{b}$

ILLUSTRATION 5.3

Coaxial cables are often used to connect electrical devices, such as your video system, and in receiving signals in television cable systems. Model a long coaxial cable as a thin, cylindrical conducting shell of radius b concentric with a solid cylinder of radius a as in figure. The conductors carry the same current I in opposite directions. Calculate the inductance L of a length l of this cable.

Sol. The current in the loop sets up a magnetic field between the inner and outer conductors that passes through this loop. If the current changes, the magnetic field changes and the induced emf opposes the original change in the current in the conductors.



The magnetic field is perpendicular to the light blue rectangle of length l and width $b - a$, the cross section of interest. Because the magnetic field varies with radial position across this rectangle, we must use calculus to find the total magnetic flux.

Divide the light blue rectangle into strips of width dr such as the darker strip in figure. Evaluate the magnetic flux through such a strip:

$$d\Phi_B = BdA = Bl\,dr$$

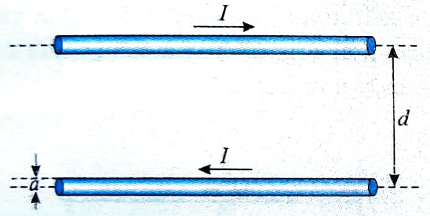
Substitute for the magnetic field and integrate over the entire light gold rectangle:

$$\Phi_B = \int_a^b \frac{\mu_0 i}{2\pi r} l\,dr = \frac{\mu_0 il}{2\pi}\int_a^b \frac{dr}{r} = \frac{\mu_0 il}{2\pi}\ln\left(\frac{b}{a}\right)$$

Hence the inductance of the cable $L = \dfrac{\Phi_B}{i} = \dfrac{\mu_0 l}{2\pi}\ln\left(\dfrac{b}{a}\right)$

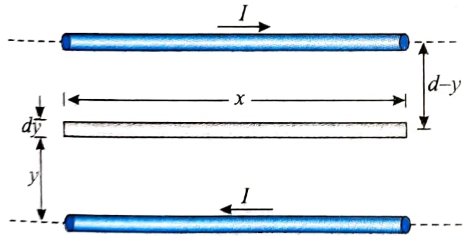
ILLUSTRATION 5.4

Two long straight cylindrical wires each of radius a are placed at a separation d. Both the wires carry equal and opposite current of magnitude I. Find the inductance (L) per unit length of this arrangement. Neglect magnetic flux inside the wires.



Sol. Let us calculate the coefficient of self-induction for this arrangement, we need to find the flux between the wires.

Let us consider a strip of width dy and length x as shown in figure.



Magnetic field at the strip due to current in two wires,

$$B = \frac{\mu_0 I}{2\pi}\left(\frac{1}{y} + \frac{1}{d-y}\right)\otimes$$

Area of the strip, $dA = x\,dy$

Hence the flux through the strip, $d\phi = \dfrac{\mu_0 I}{2\pi}\left(\dfrac{1}{y} + \dfrac{1}{d-y}\right)x\,dy$

Flux linked with the area between two wires in a length x will be

$$\phi = \frac{\mu_0 Ix}{2\pi}\left[\int_a^{d-a}\frac{dy}{y} + \int_a^{d-a}\frac{dy}{d-y}\right]$$

$$= \frac{\mu_0 Ix}{2\pi}\left[\ln\left(\frac{d-a}{a}\right) - \ln\left(\frac{d-d+a}{d-a}\right)\right]$$

$$= \frac{\mu_0 Ix}{\pi}\ln\left(\frac{d-a}{a}\right)$$

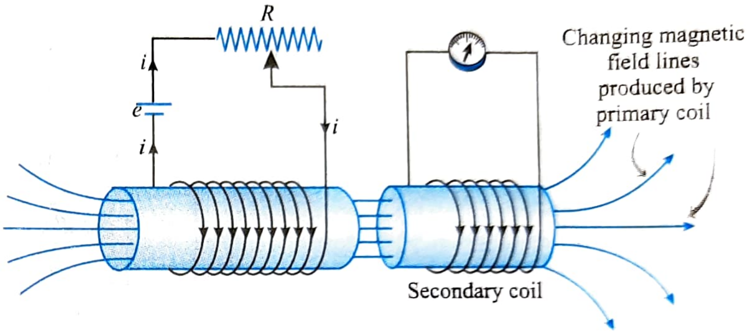
$$\therefore \quad L = \frac{\phi}{I} = \frac{\mu_0 x}{\pi}\ln\left(\frac{d-a}{a}\right)$$

Hence the inductance (L) per unit length of this arrangement,

$$\frac{L}{x} = \frac{\mu_0}{\pi}\ln\left(\frac{d-a}{a}\right)$$

MUTUAL INDUCTANCE

In this section we will analyze the case of two interacting coils, if two coils are close together, a steady current i in one coil will set up a magnetic flux ϕ through the other coil (linking the other coil). If we change i with time, this condition induces an emf through a process known as mutual induction, so named because it depends on the interaction of two circuits.



Consider the two closely wound coils of wire, called the primary coil, and the secondary coil, are placed close to each other. The current i_1 in primary coil, which has N_1 turns, creates a magnetic field. Some of the magnetic field lines pass through secondary coil, which has N_2 turns. The magnetic flux caused by the current in primary coil and passing through secondary coil is represented by Φ_{12}. In analogy to Coefficient of Self Induction, we can identify the **'Coefficient of mutual inductance'** M_{12} of coil 2 with respect to coil 1:

$$M_{12} = \frac{N_2\Phi_{12}}{i_1} \qquad \text{...(i)}$$

Mutual inductance depends on the geometry of both circuits and on their orientation with respect to each other. As the circuit separation distance increases, the mutual inductance decreases because the flux linking the circuits decreases.

If the current i_1 varies with time, we see from Faraday's law and Eq. (i) that the emf induced by primary coil in secondary coil is

$$e_{12} = -N_2\frac{d\Phi_{12}}{dt} = -N_2\frac{d}{dt}\left(\frac{M_{12}i_1}{N_2}\right) = -M_{12}\frac{di_1}{dt} \qquad \text{...(ii)}$$

In the preceding discussion, it was assumed the current is in primary coil. Let's also imagine a current i_2 in secondary coil. The preceding discussion can be repeated to show that there is a mutual inductance M_{21}. If the current i_2 varies with time, the emf induced by secondary coil in primary coil is

$$e_{21} = -M_{21}\frac{di_2}{dt} \qquad \text{...(iii)}$$

In mutual induction, the emf induced in one coil is always proportional to the rate at which the current in the other coil is changing. Although the proportionality constants M_{12} and M_{21} have been treated separately, it can be shown that they are equal.

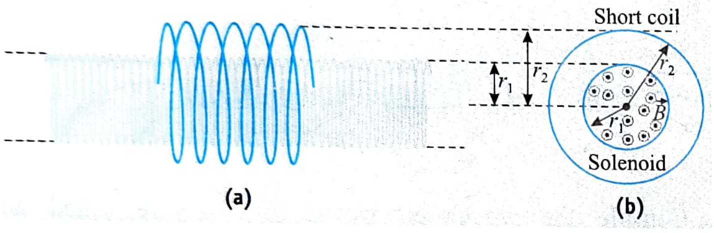
Therefore, with $M_{12} = M_{21} = M$, Eqs. (ii) and (iii) become

$$e_2 = -M\frac{di_1}{dt} \quad \text{and} \quad e_1 = -M\frac{di_2}{dt} \qquad \text{...(iv)}$$

These two equations are similar in form to equation for the self-induced emf $e = -L\left({di}/{dt}\right)$. The direction of mutually induced EMF in either coil will be such that it opposes the change in current in the other coil. The SI unit of mutual inductance is the henry. The unit of mutual inductance is the henry.

ILLUSTRATION 5.5

A long solenoid with circular cross section of radius r_1 and n turns per unit length is inside and coaxial with a short coil with circular cross section of radius r_2 and N turns (figure). The current in the solenoid is increased at a constant rate from zero to i over a time interval of 'T'. Find the mutual inductance between the two coils and the potential difference induced in the short coil while the current is changing.



(a) (b)

Sol. The potential difference induced in the short coil is due to the changing current flowing in the solenoid.

The mutual inductance between the coil and the solenoid,

$$M = \frac{N_c\Phi_{s\to c}}{i}, \qquad \text{...(i)}$$

where N_c is the number of turns in the short coil, $N_c\Phi_{s\to c}$ is the flux linkage in the coil resulting from the magnetic field of the solenoid, and i is the current in the solenoid. The flux can be expressed as

$$\Phi_{s\to c} = BA = (\mu_0 ni).\left(\pi r_1^2\right) \qquad \text{...(ii)}$$

where B is the magnitude of the magnetic field inside the solenoid and A is its cross-sectional area.

For a solenoid, the magnitude of the magnetic field is $B = \mu_0 ni$, where n is the number of turns per unit length. The cross-sectional area of the solenoid is $A = \pi r_1^2$

We can combine Eqs. (i) and (ii), to obtain the mutual inductance between the two coils:

$$M = \frac{NBA}{i} = \frac{N\left(\mu_0 ni\right)\left(\pi r_1^2\right)}{i} = N\pi\mu_0 nr_1^2.$$

The potential difference induced in the short coil is ,

$$e_c = -M\left(\frac{di}{dt}\right) = -\left(N\pi\mu_0 nr_1^2\right)\frac{di}{dt} \qquad \text{...(iii)}$$

The rate of change in the current is constant,

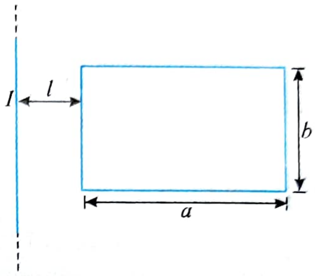
$$\frac{di}{dt} = \frac{(i-0)}{T} = \frac{i}{T}. \qquad \text{...(iv)}$$

From (iii) and (iv), the magnitude of the potential difference induced in the short coil,

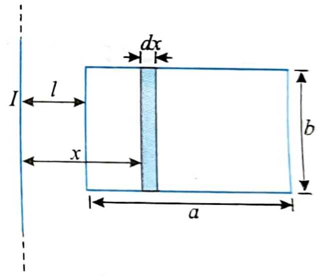
$$\left|e_c\right| = \left(N\pi\mu_0 nr_1^2\right)\frac{i}{T}$$

ILLUSTRATION 5.6

A rectangular wire frame with side a and b is placed near a long straight wire and a rectangular frame with side a and b as shown in figure. The straight wire and the wire frame lie in the same plane. Calculate the mutual inductance of a long straight wire and a rectangular frame.



Sol. For finding the mutual inductance between the wire and the frame, let us supply a current I through the wire and calculate the magnetic flux through the frame. For flux calculation, we consider an elemental strip in the frame of width dx at a distance x from wire as shown in figure.



The magnetic flux through the elemental strip is given as

$$d\phi = \left(\frac{\mu_0 I}{2\pi x}\right)bdx$$

$$\Rightarrow \quad \phi = \int_l^{a+l}\left(\frac{\mu_0 I}{2\pi x}\right)bdx = \frac{\mu_0 Ib}{2\pi}\int_l^{a+l}\frac{dx}{x}$$

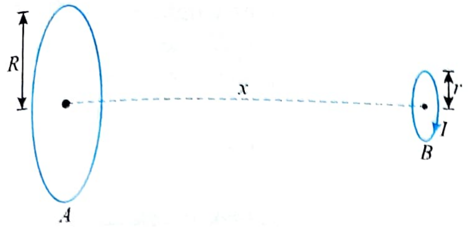
$$\Rightarrow \quad \phi = \frac{\mu_0 Ib}{2\pi}\left[\ln x\right]_a^{a+l} = \frac{\mu_0 Ib}{2\pi}\ln\left(1+\frac{a}{l}\right)$$

Hence the mutual inductance between the two is given as

$$M = \frac{\phi}{I} = \frac{\mu_0 b}{2\pi}\ln\left(1+\frac{a}{l}\right)$$

ILLUSTRATION 5.7

Two coaxial coils of radii r and R ($R >> r$) placed at large separation x ($x >> R$). Calculate the magnetic flux passing through coil L_1 due to a current I in coil L_2.



Sol. Let M be the coefficient of mutual induction between the two coils then magnetic flux through coil L_1 due to current in coil L_2 is given as

$$\phi_1 = MI \qquad \text{...(i)}$$

Let us calculate the coefficient of mutual induction between the two coils and for this we find the flux through coil L_2 if a current I_1 flow in coil L_1 as shown in figure.

The magnetic field at the center of coil L_2 due to current in coil L_1 is given as

$$B_2 = \frac{\mu_0 I_1 R^2}{2\left(x^2 + R^2\right)^{3/2}}$$

As the size of the coil is small, we can assume uniform magnetic field inside the loop. The magnetic flux through coil L_2 due to above magnetic induction is given as

$$\phi_2 = B_2 \cdot \pi r^2 \Rightarrow \phi_2 = \frac{\mu_0 \pi r^2 R^2}{2\left(x^2 + R^2\right)^{3/2}} \cdot I_1$$

The coefficient of mutual inductance, $M = \dfrac{\phi_2}{I_1} = \dfrac{\mu_0 \pi r^2 R^2}{2\left(x^2 + R^2\right)^{3/2}}$

Above is the mutual inductance coefficient for the pair of these coil which can be used in calculation of flux from either coil due to current in other coil so from reciprocity property, we have

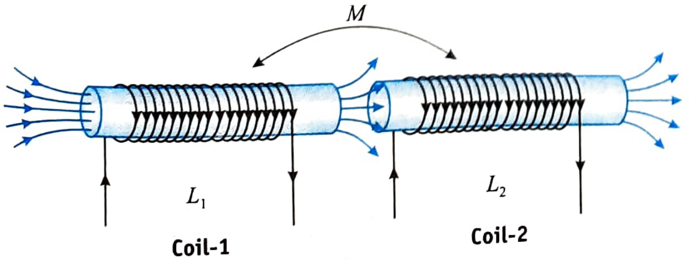
$$\phi_1 = MI_2 = \frac{\mu_0 \pi r^2 R^2 I}{2\left(x^2 + R^2\right)^{3/2}}$$

TOTAL FLUX LINKED TO A PAIR OF COILS

If two current carrying coils are placed near to each other, for calculation of total flux, we need to consider self-induction of individual coils and mutual induction between the coils. Let two coils coils having their self-induction coefficients L_1 and L_2 and mutual induction coefficient for the pair M are kept close to each other, two coils carry currents i_1 and i_2 respectively.

The total magnetic flux linked with either coil will be the sum of magnetic flux linked with the coil due to both self-induction and mutual induction. Here we consider two cases

(i) **When flux of the two inductors are in same direction**



If the magnetic flux due to both coils are in same direction as shown in figure, then the net flux linked with coil-1 and coil-2 are given as

$$\phi_1 = L_1 i_1 + M i_2 \qquad \text{...(i)}$$

$$\phi_2 = L_2 i_2 + M i_1 \qquad \text{...(ii)}$$

If currents in the coils start changing, then EMF induced in the two coils are given as

$$e_1 = \frac{d\phi_1}{dt} = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \qquad \text{...(iii)}$$

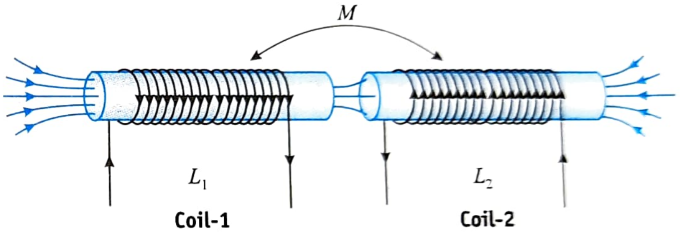
and $$e_2 = \frac{d\phi_2}{dt} = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} \qquad \text{...(iv)}$$

(ii) **When the flux of the two inductors are in opposite direction**

In this case the net magnetic flux linked with the two coils are given as

$$\phi_1 = L_1 i_1 - M i_2 \qquad \text{...(v)}$$

$$\phi_2 = L_2 i_2 - M i_1 \qquad \text{...(vi)}$$



If current in above coils starts changing then EMF induced in the two coils are given as

$$e_1 = \frac{d\phi_1}{dt} = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} \qquad \text{...(vii)}$$

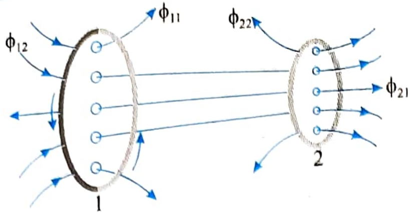
and $$e_2 = \frac{d\phi_2}{dt} = L_2 \frac{di_2}{dt} - M \frac{di_1}{dt} \qquad \text{...(viii)}$$

CALCULATION OF MUTUAL INDUCTANCE BETWEEN TWO COILS FROM THEIR SELF-INDUCTANCE

Suppose, two coils carry currents i_1 and i_2 respectively. Then, their self-inductances can be given as $L_1 = \dfrac{\phi_{11}}{i_1}$ and $L_2 = \dfrac{\phi_{22}}{i_2}$ respectively. Similarly, their mutual inductance $M = \dfrac{\phi_{12}}{i_2} = \dfrac{\phi_{21}}{i_1}$.

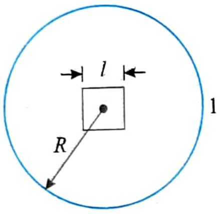
When the coils are placed very close side by side, they can interlink a maximum flux. In that case, there is no difference between self-flux and mutual flux; then $\phi_{11} = \phi_{12} = \phi_{21} = \phi_{22}$. Hence, $L_1 L_2$ will be equal to M^2. In other words, maximum mutual inductance is equal to the geometric mean of the self-inductances of the coils, that is, $M = \sqrt{L_1 L_2}$ more generally, $M \le \sqrt{L_1 L_2}$ in most practical cases because a fraction of the (self) flux produced by one coil is linked with the other. We call this fraction K "coefficient of flux linkage," which is given as $K = \dfrac{\text{mutual flux}}{\text{self flux}}$; $0 \le K \le 1$.

Then we have $M = K\sqrt{L_1 L_2}$, where K can also be termed as "coefficient of coupling" of magnetic flux between any two circuits.

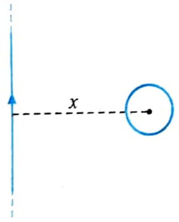


CONCEPT APPLICATION EXERCISE 5.1

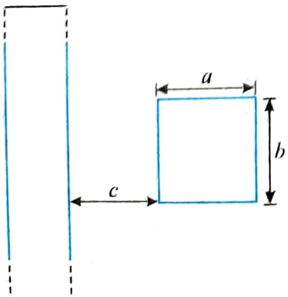
1. An inductor coil of negligible resistance drops a voltage of 6 V when the current increases at a rate of 600 A/s. Find the inductance of the coil.
2. A coil has inductance of 4 mH. If the current through the coil changes from 0.25 amp to 2 amp during 0.5 s, find the average voltage dropped across the inductor.
3. A square loop of side $l = 1$ cm is placed at the center of a circular loop of radius $R = 0.25$ m as shown in figure. Find the mutual inductance between the circular loop and square loop.



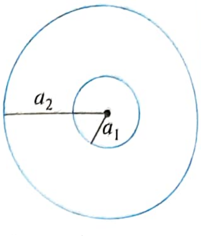
4. Find the mutual inductance between the long straight conductor and a small loop of area 10^{-8} m² placed at a distance of separation $x = 0.5$ m as shown in figure.



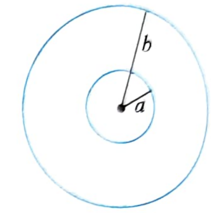
5. A small square loop of wire of side l is placed inside a large square loop of wire of side L (>>l). The loops are coplanar and their centers coincide. What is the mutual inductance of the system?
6. A finite rectangular loop is placed near an infinite loop as shown in figure. Find the mutual inductance between two rectangular loops.



7. Two insulated wires are wound on the same hollow cylinder, so as to form two solenoids sharing a common air-filled core. Let l be the length of the core, A the cross-sectional area of the core, N_1 the number of times the first wire is wound around the core, and N_2 the number of times the second wire is wound around the core. Find the mutual inductance of the two solenoids, neglecting the end effects.
8. Find the mutual inductance of two concentric coils of radii a_1 and a_2 ($a_1 << a_2$) if the planes of the coils are same.



9. Solve problem 8 if the planes of the coils are perpendicular.
10. Solve problem 8 if the planes of the coils make an angle θ with each other.
11. Figure shows two concentric coplanar coils with radii a and b ($a << b$). A current $i = 2t$ flows in the smaller loop. Neglecting self-inductance of the larger loop,
    - **(a)** find the mutual inductance of the two coils;
    - **(b)** find the emf induced in the larger coil;
    - **(c)** if the resistance of the larger loop is R, find the current in it as a function of time.



ANSWERS

1. 10^{-2} mH **2.** 14 mV **3.** $8\pi \times 10^{-11}$ H **4.** 4×10^{-15} H

5. $\frac{2\sqrt{2}\,\mu_0 l^2}{\pi l}$ **6.** $\frac{\mu_0 b}{2\pi}\ln\frac{(c+b)(a+c)}{c(a+b+c)}$ **7.** $\frac{\mu_0 N_1 N_2 A}{l}$

8. $\frac{\mu_0 \pi a_1^2}{2a_2}$ **9.** 0 **10.** $\frac{\mu_0 \pi a_1^2 \cos\theta}{2a_2}$

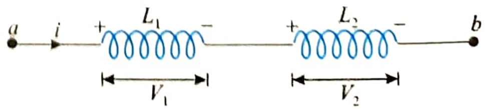
11. (a) $\frac{\mu_0}{2b}\pi a^2$ (b) $\frac{\mu_0 \pi a^2}{b}$ (c) $\frac{\mu_0 \pi a^2}{6R}$

COMBINATION OF INDUCTORS

SERIES COMBINATION

Non-Interacting Coils

When the coils are electromagnetically isolated and connected in series, only the self-induced voltages are taken into account. For an increasing current, a voltage is induced across each coil. Then the total voltage drop V_{ab} is equal to the summation of induced voltages due to each coil.



$$V_{ab} = V_1 + V_2 + \cdots + V_n. \quad \text{...(i)}$$

If we choose a coil of inductance L so as to induce same voltage V_{ab} for same variation of current, we can call it as equivalent inductance.

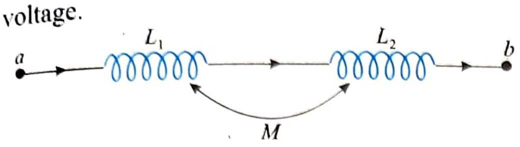
Substituting $V_{ab} = L\frac{di}{dt}, V_1 = L_1\frac{di}{dt}, V_2 = L_2\frac{di}{dt}, \ldots\ldots, V_n = L_n\frac{di}{dt}$, in Eq. (i) etc., we get

$$L\frac{di}{dt} = L_1\frac{di}{dt} + L_2\frac{di}{dt} + \ldots\ldots + L_n\frac{di}{dt}$$

$$L = L_1 + L_2 + \cdots + L_n$$

Interacting Coils

If inductor coils in series with flux of one coil favors the other. We have to take the mutually induced voltage in addition to self-induced voltage.



The total induced voltage across the coils of inductances L_1 and L_2 are given as

$$V_1 = L_1\frac{di}{dt} + M\frac{di}{dt}, \quad \ldots\text{(i)}$$

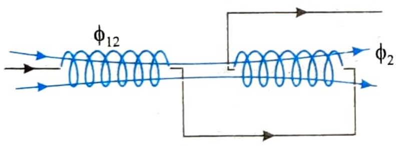
$$V_2 = L_2\frac{di}{dt} + M\frac{di}{dt} \quad \ldots\text{(ii)}$$

Then, for getting total voltage drop across the combination, we need to add Eqs. (i) and (ii)

$$V_{ab} = V_1 + V_2 = (L_1 + L_2 + 2M)\frac{di}{dt}$$

It gives the equivalent inductance $L = \dfrac{V_{ab}}{di/dt} = L_1 + L_2 + 2M$

When the flux of one coil opposes that of the other,



$V_1 = L_1\dfrac{di}{dt} - M\dfrac{di}{dt}$ and $V_2 = L_2\dfrac{di}{dt} - M\dfrac{di}{dt}$. Then following the above argument, the equivalent inductance can be given as $L = L_1 + L_2 - 2M$. Generalizing the above results we can write the equivalent inductance of two interacting coils connected in series as,

$$L = L_1 + L_2 \pm 2M$$

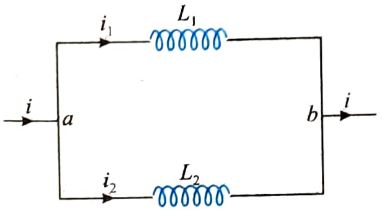
When there is no interaction between the coils, M is equal to zero. Then the equivalent inductance is given as,

$$L = L_1 + L_2$$

PARALLEL COMBINATION

For parallel combination of the inductors, voltage between the common terminals a and b of the inductors is given as

$$V_{ab} = L_1\frac{di_1}{dt} = L_2\frac{di_2}{dt}.$$



Application of Kirchhoff's first law yields $i = i_1 + i_2$ ...(i)

Differentiating Eq. (i) both sides with respect to time,

we obtain $\dfrac{di}{dt} = \dfrac{di_1}{dt} + \dfrac{di_2}{dt}$; ...(ii)

Substituting $\dfrac{di_1}{dt} = \dfrac{V_{ab}}{L_1}$ and $\dfrac{di_2}{dt} = \dfrac{V_{ab}}{L_2}$ in Eq. (ii) we have

$$\frac{di}{dt} = \frac{V_{ab}}{L_1} + \frac{V_{ab}}{L_2} = V_{ab}\left(\frac{1}{L_1} + \frac{1}{L_2}\right) \quad \ldots\text{(iii)}$$

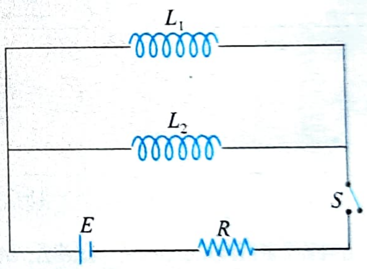
If L is the equivalent inductance connected between a and b, for same variation $\dfrac{di}{dt}$, it induces same voltage V_{ab}. Then

$$V_{ab} = L\left(\frac{di}{dt}\right) \Rightarrow \frac{1}{L} = \frac{di/dt}{V_{ab}} \quad \ldots\text{(iv)}$$

From (iii) and (iv), we get $\dfrac{1}{L} = \dfrac{1}{L_1} + \dfrac{1}{L_2}$ hence, $\dfrac{1}{L} = \displaystyle\sum_{i=1}^{i=n}\frac{1}{L_i}$

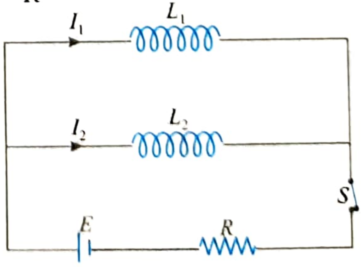
ILLUSTRATION 5.8

Two inductor coils h inductances L_1 and L_2 are connected in parallel with a battery of EMF E, and a resistance R. The internal resistance of the source and the coil resistances are negligible. Find the steady state currents in the coils long time after the switch Sw was shorted.



Sol. At steady state constant current flows through inductor coil. It means the flux associated with the coil remains constant. In this case no emf will induced in the coil. It means in steady state inductors behave like conducting wire or short circuit so the current through battery is given as

$$i = i_1 + i_2 = \frac{E}{R} \quad \ldots\text{(i)}$$



For two inductors connected in parallel, the potential difference across the coils should be same.

Hence, $L_1\dfrac{di_1}{dt} = L_2\dfrac{di_2}{dt}$ ...(ii)

Integrating Eq. (ii) both sides we obtain $L_1 i_1 = L_2 i_2$ ...(iii)

From Eq. (i) and (iii), we get

$$i_1 = \frac{EL_2}{R(L_1+L_2)} \text{ and } i_2 = \frac{EL_1}{R(L_1+L_2)}$$

It means the current in inductors connected in parallel combination is divided in inverse ratio of their inductances.

ILLUSTRATION 5.9

The equivalent inductance of two inductors is 2.4 H when connected in parallel and 10 H when connected in series. What is the value of inductance of the individual inductors?

Sol. As inductances obey laws similar to the "grouping of resistances,"

$$L_1 + L_2 = 10 \text{ H and } \frac{L_1L_2}{L_1+L_2} = 2.4 \text{ H}$$

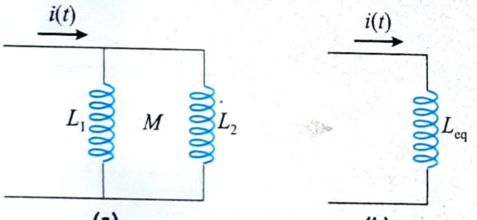
Substituting the value of $(L_1 + L_2)$ from first expression into the second, $L_1L_2 = (2.4)(L_1 + L_2) = 2.4 \times 10 = 24$

so that $(L_1 - L_2)^2 = (L_1 + L_2)^2 - 4L_1L_2$, i.e., $L_1 - L_2 = 2$ H

and as $L_1 + L_2 = 10$ H, so $L_1 = 6$ H and $L_2 = 4$ H

ILLUSTRATION 5.10

Two inductors having inductances L_1 and L_2 are connected in parallel as shown in [Fig. (a)]. The mutual inductance between the two inductors is M. Determine the equivalent inductance L_{eq} for the system [Fig. (b)].



(a) (b)

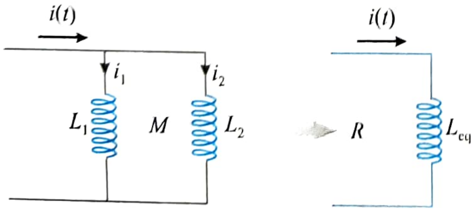
Sol. Let the currents in inductors L_1 and L_2 at any instant be i_1 and i_2 respectively.

We have $i = i_1 + i_2$, ...(i)

the voltage across the pair is:

$$\Delta V = -L_1\frac{di_1}{dt} - M\frac{di_2}{dt} = -L_{eq}\frac{di}{dt} \quad \text{...(ii)}$$

and $$\Delta V = -L_2\frac{di_2}{dt} - M\frac{di_1}{dt} = -L_{eq}\frac{di}{dt} \quad \text{...(iii)}$$



So, from (ii), we have, $-\frac{di_1}{dt} = \frac{\Delta V}{L_1} + \frac{M}{L_1}\frac{di_2}{dt}$ ...(iv)

From (iii) and (iv), $\Delta V = -L_2\frac{di_2}{dt} + M\left(\frac{(\Delta V)}{L_1} + \frac{M}{L_1}\frac{di_2}{dt}\right)$

$$\left(-L_1L_2 + M^2\right)\frac{di_2}{dt} = \Delta V(L_1 - M) \quad \text{...(v)}$$

From (iii), $-\frac{di_2}{dt} = \frac{\Delta V}{L_2} + \frac{M}{L_2}\frac{di_1}{dt}$, ...(vi)

From (ii) and (vi), $\Delta V = -L_1\frac{di_1}{dt} + M\left(\frac{\Delta V}{L_2} + \frac{M}{L_2}\frac{di_1}{dt}\right)$

$$\left(-L_1L_2 + M^2\right)\frac{di_1}{dt} = \Delta V(L_2 - M) \quad \text{...(vii)}$$

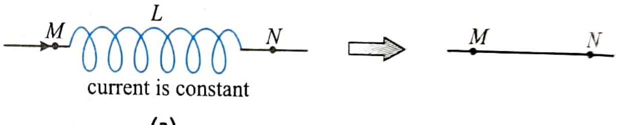
Adding (v) to (vii), we have

$$\left(-L_1L_2 + M^2\right)\frac{di}{dt} = \Delta V(L_1 + L_2 - 2M)$$

Hence, $L_{eq} = -\frac{\Delta V}{di/dt} = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}$

INDUCTOR IN ELECTRICAL CIRCUITS

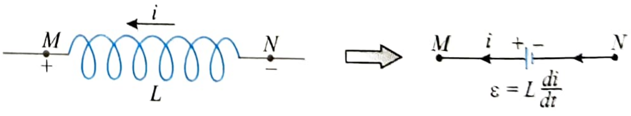
In an inductor coil a steady current flow, the magnetic flux due to the current remains constant. In this case no EMF is induced in the coil. It means in case when the current is steady, the inductor behaves like a conducting wire as shown in figure.



(a) (b)

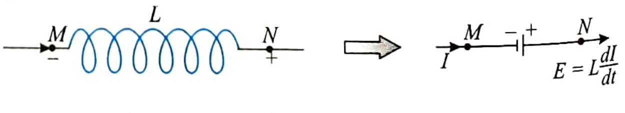
Now consider the case when current flowing through the inductor changes with time. In this case due to self-induction an EMF is induced in the inductor so as to oppose the change in current. Let us consider two cases,

Case (i): In the case when current is increasing and flowing from left to right then left end of inductor will have high potential and right end of inductor will be at low potential. In this way the EMF induced in the inductor coil tend to supply its current in direction opposite to the external current so that increasing current may be opposed. In this situation the we may consider the inductor as a battery or equivalent EMF across its terminals M and N as shown in figure.



current decreasing with time (a) (b)

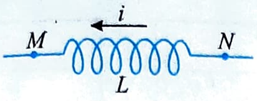
Case (ii): In the case when current is decreasing and flowing from left to right as shown in figure. In this case due to self-induction an EMF is induced so as to oppose the decrease in current. So in this case right end of inductor will be at higher potential and left end at lower potential so that the self-induced EMF in the coil will tend to supply its current in the same direction as that of the external current. In this case the inductor behaves like a battery or equivalent EMF across its terminals M and N as shown in figure.



current decreasing with time (a) (b)

ILLUSTRATION 5.11

The current in inductor shown in figure is flowing from right to left. The inductor has inductance 0.75 H and current in it is decreasing at a uniform rate, $di/dt = -0.040$ A/s.



(a) Find the magnitude of the self-induced emf.

(b) Which end of the inductor, M or N, is at a higher potential?

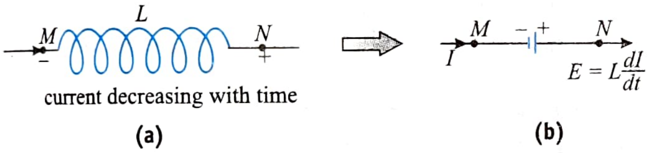
Sol.

(a) The magnitude of the EMF induced in coil due to self-induction is given as

$$e = L\left|\frac{di}{dt}\right| = 0.75\times0.04\text{ V}$$

$$\Rightarrow \quad e = 3.0\times10^{-2}\text{ V}$$

(b) Current in the shown direction is decreasing. In this case due to self-induction an EMF is induced so as to oppose the decrease in current. So, in this case left end of inductor will be at higher potential and right end at lower potential so that the self-induced EMF in the coil will tend to supply its current in the left direction as that of the external current. Hence induced battery polarity will be as shown in figure. Hence end M will be at higher potential.



(a) (b)

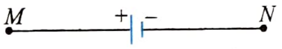
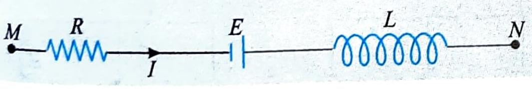
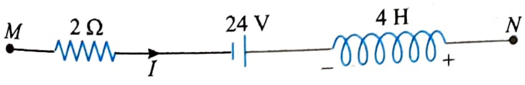


ILLUSTRATION 5.12

A section of an electrical circuit MN which contains a resistance $R = 2\ \Omega$, a battery of emf $E = 24$ V and a inductor coil of inductance $L = 4.0$ H as shown in figure. Then which carries a current of 10 A which is decreasing at the rate of 0.5 A/s. Find the potential difference $V_N - V_M$



Sol. In the above circuit section as current is decreasing with time, the polarity of self-induced EMF is shown in figure.



In this branch of circuit writing, Kirchhoff's path equation from point M to N which is given as

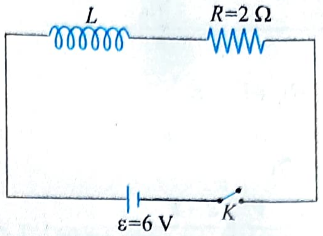
$$V_M - iR + E + L\frac{di}{dt} = V_N$$

$$\Rightarrow \quad V_M - 10\times2 + 24 + 4.0\times0.5 = V_N$$

$$\Rightarrow \quad V_N - V_M = 6.0\text{ V}$$

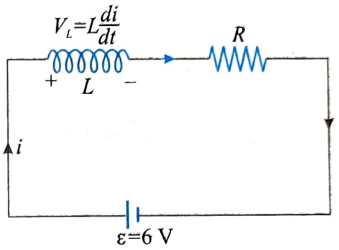
ILLUSTRATION 5.13

Find the self-inductance L of the coil whose resistance $R = 2$ ohm, a current $i = 1.5$ amp, which is increasing at a rate of 10^3 amp/s.



Sol. The current is increasing and flowing left to right in the inductor then left end of inductor will have high potential and right end of inductor will be at low potential. Applying Kirchhoff's loop law

$$\varepsilon - V_L - iR = 0 \Rightarrow \varepsilon - L\frac{di}{dt} - iR = 0$$

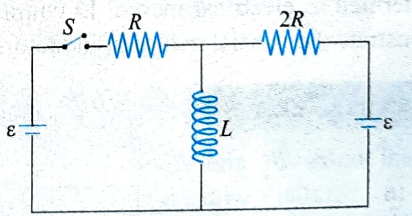


$$6.0 - L\times10^3 - 1.5\times2 = 0$$

$$\Rightarrow \quad L = \frac{6.0-3.0}{10^3} = 3.0\times10^{-3}\text{ H. Hence } L = 3\text{ mH.}$$

ILLUSTRATION 5.14

The circuit shown in the figure is in steady state. At certain instant the switch is closed, immediately after the switch S is closed. Find



(i) potential difference across the inductor coil

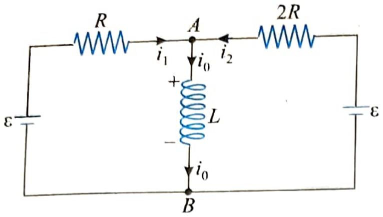
(ii) the rate of change of current through the inductor coil

(iii) potential difference across the resistance

Sol.

(i) In steady state the current through the inductor is $i = \dfrac{\varepsilon}{2R}$.

The current through the inductor will not change immediately after the switch is closed. Let current in the inductor branch is increasing, hence end A of the coil should be positive and end B should be negative and $V_{AB} = V$.



In left loop: $Ri_1 + V = \varepsilon$ ...(i)

In right loop: $2Ri_2 + V = \varepsilon$ ...(ii)

From (i) and (ii)

$$i_1 + i_2 = \frac{\varepsilon - V}{R} + \frac{\varepsilon - V}{2R}$$

Since $i_1 + i_2 = i_0 = \dfrac{\varepsilon}{2R}$ [Because $i_0 = \dfrac{\varepsilon}{2R}$]

$$\therefore \quad \frac{\varepsilon - V}{R} + \frac{\varepsilon - V}{2R} = \frac{\varepsilon}{2R} \Rightarrow V = \frac{2\varepsilon}{3}$$

(ii) The potential difference across the inductor coil,

$$V = L\frac{di}{dt} = \frac{2\varepsilon}{3} \Rightarrow \frac{di}{dt} = \frac{2\varepsilon}{3L}$$

As $\dfrac{di}{dt}$ is positive. It means current is increasing.

(iii) At this instant the potential difference across R is

$$\varepsilon - \frac{2\varepsilon}{3} = \frac{\varepsilon}{3}$$

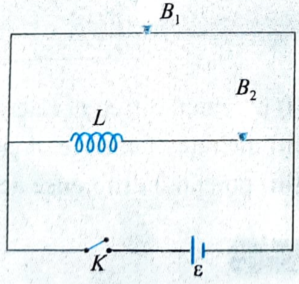
ANALOGY BETWEEN SELF-INDUCTION AND INERTIA

Newton's first law of motion define inertia. It is the property of matter by the virtue of which it opposes the change in its state. "Mass" of a matter is the measure of its inertia. Greater the mass, it is more difficult to set it in motion (or to stop it).

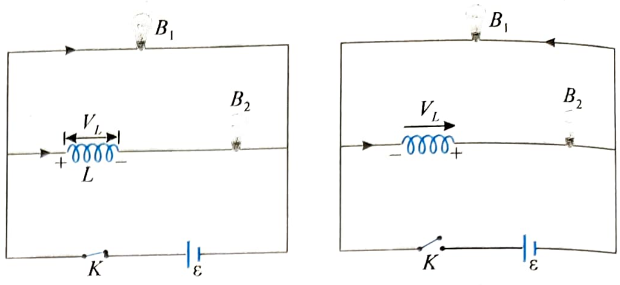
In same way, the circuit containing inductor opposes the change in its own current by inducing an emf in it owing to the induced electric field. This is known as self-induction and this opposing (inertial) nature of the electric circuit to any change in its current is termed as electrical inertia. In coming illustration, we will demonstrate the inertial nature of inductors,

ILLUSTRATION 5.15

Two identical bulbs B_1 and B_2 connected in parallel with a battery of emf ε. The bulb B_2 is connected in series with an ideal inductor L. How the brightness of the bulbs will change after closing the key K. What will happen when we open the key K after a long time?



Sol. We can consider the bulbs as simple resistors of resistance say R_1 and R_2. Without any inductive effect, as the bulbs are joined in parallel with the battery, we should expect them to glow with equal brightness. But after closing the key K as shown in Fig. (a), the presence of the inductor induces a voltage V_L across it. As the current increases, the induced voltage opposes the applied voltage ε. Then the bulb B_2 receives a voltage $\varepsilon - V_L$ to use, whereas B_1 utilizes full voltage ε for all the times. This makes B_1 glowing brighter than B_2 till B_2 attains a steady state current. That means, after sometime both the bulbs glow with equal brightness.

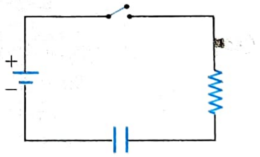


(a) (b)

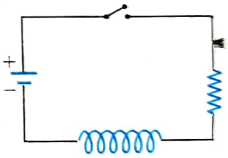
After a long time, the key is reopened. This tries to destroy the magnetic flux in the inductor rapidly. In consequence, a voltage V_L' is induced so as to retain the current. Hence, the polarity of the inductor reverses and the same current flows through both B_1 and B_2. However, current flows in a direction opposite to the initial current of B_1. The currents in both the bulbs that measure their brightness falls gradually to zero after sometime.

CONCEPT APPLICATION EXERCISE 5.2

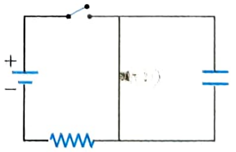
1. Consider the four circuits shown in figure, each consisting of a battery, a switch, a lightbulb, a resistor, and either a capacitor or an inductor. Assume the capacitor has a large capacitance and the inductor has a large inductance but no resistance. The light bulb has high efficiency, glowing whenever it carries electric current. (i) Describe what the lightbulb does in each of circuits (a) through (d) after the switch is thrown closed. (ii) Describe what the lightbulb does in each of circuits (a) through (d) when, having been closed for a long time interval, the switch is opened.



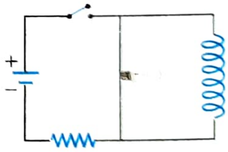
(a)



(b)

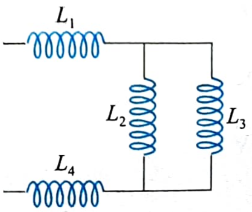


(c)

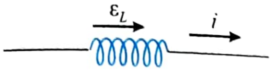


(d)

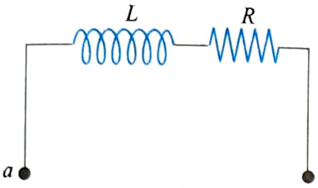
2. The inductor arrangement of figure, with L_1 = 40.0 mH, L_2 = 50.0 mH, L_3 = 10.0 mH, and L_4 = 12.0 mH, is to be connected to a varying current source. What is the equivalent inductance of the arrangement?



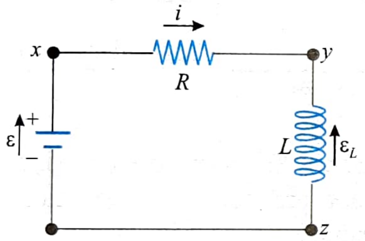
3. At a given instant the current and self-induced emf in an inductor are directed as indicated in figure. **(a)** Is the current increasing or decreasing? **(b)** The induced emf is 6.0 V, and the rate of change of the current is 1.5 kA/s; find the inductance.



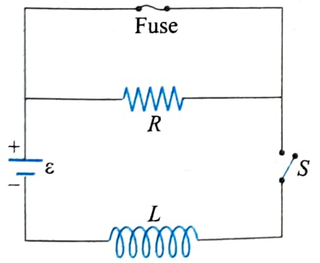
4. When the current in the portion of the circuit shown in figure is 2.00 A and increases at a rate of 0.500 A/s, the measured voltage is $\Delta V_{ab} = 9.00$ V. When the current is 2.00 A and decreases at the rate of 0.500 A/s, the measured voltage is $\Delta V_{ab} = 5.00$ V. Calculate the values of **(a)** L and **(b)** R.



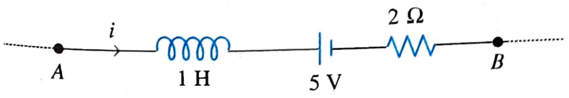
5. Suppose the emf of the battery in the circuit shown in figure varies with time t so that the current is given by $i(t) = 3.0 + 5.0t$, where i is in amperes and t is in seconds. Take $R = 4.0\ \Omega$ and $L = 6.0$ H, and find an expression for the battery emf as a function of t.



6. In given figure, $R = 15\ \Omega$, $L = 5.0$ H, the ideal battery has $\varepsilon = 10$ V, and the fuse in the upper branch is an ideal 3.0 A fuse. It has zero resistance as long as the current through it remains less than 3.0 A. If the current reaches 3.0 A, the fuse "blows" and thereafter has infinite resistance. Switch S is closed at time $t = 0$. When does the fuse blow?

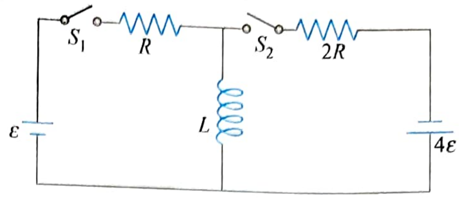


7. AB is a part of circuit. Find the potential difference $V_A - V_B$ if



(a) current $i = 2$ A and is constant;
(b) current $i = 2$ A and is increasing at the rate of 1 A s^{-1};
(c) current $i = 2$ A and is decreasing at the rate 1 A s^{-1}.

8. In a circuit S_1 remains closed for a long time and S_2 remains open. Now S_2 is closed and S_1 is opened. Find out the di/dt in the right loop just after that moment.



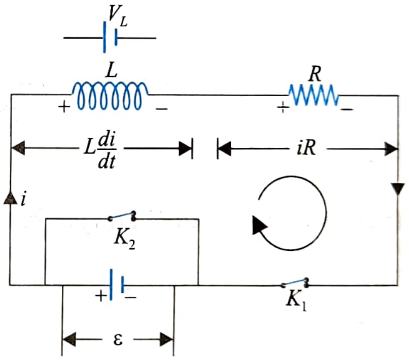
ANSWERS

2. 50 mH **3.** (a) decreasing (b) 4.0 mH
4. (a) 4.0 H (b) 3.5 Ω **5.** (42 + 20t)V **6.** 1.5 s
7. (a) 9 V (b) 10 V (c) 8 V **8.** $-\dfrac{6\varepsilon}{L}$

GROWTH AND DECAY OF CURRENT

GROWTH OF CURRENT

In the figure given a circuit with an inductor of inductance L connected in series with a resistor of resistance R across a battery of emf ε. In this circuit when the switch is closed at $t = 0$, a current start flowing and if at time 't' current flowing in circuit is considered to be i.



Applying Kirchhoff's loop Law,

$$-L\frac{di}{dt} - iR + \varepsilon = 0 \qquad \text{...(i)}$$

$$L\frac{di}{dt} + iR = \varepsilon \text{ or } L\frac{di}{dt} = \varepsilon - iR$$

or $$\frac{di}{\varepsilon - iR} = \frac{\varepsilon\, dt}{L}$$

Since, current increases from 0 to i during a time t, now integrating both sides

or $$\int_0^t \frac{di}{\varepsilon - iR} = \frac{\varepsilon}{L}\int_0^t dt$$

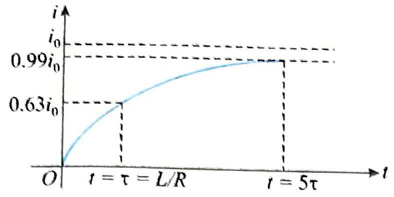
or $$i = \frac{\varepsilon}{R}\left(1 - e^{-\frac{Rt}{L}}\right) = \frac{\varepsilon}{R}\left(1 - e^{-\frac{t}{\tau}}\right) \qquad \text{...(ii)}$$

where $\tau = \dfrac{L}{R}$ is known as the time constant of L-R circuit.

Equation (ii) tells that, the current builds up exponentially to a steady value of magnitude $i_0 = \dfrac{\varepsilon}{R}$ after a long time ($t \to \infty$).

If we put $t=\tau$ in Eq. (ii) we have $i=i_0\left(1-\frac{1}{e}\right)=0.63i_0$

During a time equal to one time constant from beginning, current builds upto 63% of the steady state current.



The current after five times the time constant then we have

$$i=i_0\left(1-e^{-5}\right)\Rightarrow i=0.993i_0$$

From above equation, it can be seen that in duration equal to five times of time constant current attains 99.3% of steady state value and remaining 0.7% will be achieved theoretically in infinite time.

DECAY OF CURRENT

After reaching the current at steady state, the key K_2 is closed (say at $t=0$), let the current fall at a rate of di/dt. Hence, the inductor acts as a battery to continue in sending the current in same direction. Substituting $\varepsilon=0$ in Eq. (i), we get the differential equation

$$L\frac{di}{dt}+iR=0$$

or $$\frac{di}{i}=-\frac{R}{L}dt$$

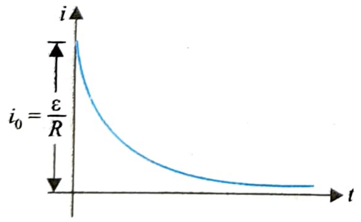
If the current falls down from i_0 to i during a time t after closing the key K_2,

$$\int_{i_0}^{i}\frac{di}{i}=-\frac{R}{L}\int_0^t dt \qquad \text{...(iii)}$$

After integrating Eq. (iii), we get

$$i=i_0e^{-\frac{Rt}{L}}=i_0e^{-\frac{t}{\tau}} \qquad \text{...(iv)}$$

It shows that the current decays exponentially, theoretically it takes infinite time ($t\to\infty$) for complete decay of the current.



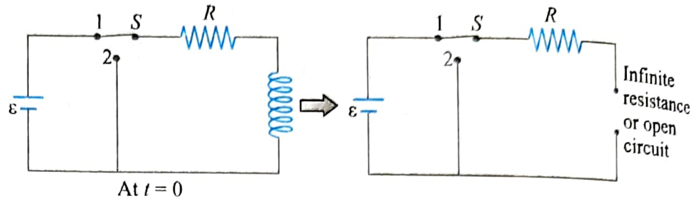
Instantaneous Behavior of an Inductor: An LR circuit is analyzed in three states:

- Initial state, i.e., just after closing the switch or just after opening the switch.
- Transient state or instantaneous state, i.e., any time after closing or opening the switch.
- Steady state, i.e., a long time after closing or opening the switch. In this state, current in the inductor does not varying with time, i.e., $\frac{di}{dt}=0$

Rise of current in inductor circuit is given by, $i_t=\frac{\varepsilon}{R}\left(1-e^{-\left(\frac{R}{L}\right)t}\right)$

At $t=0$, $i_t=\frac{\varepsilon}{R}\left(1-e^{-\left(\frac{R}{L}\right)0}\right)=0$

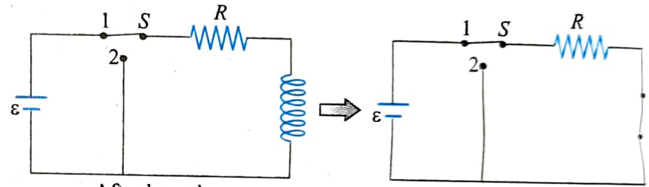
At $t=0$, opposition produced by the inductor is infinite. Hence it acts as an infinite resistance or open circuit.



At $t=0$

No current flow through the circuit at the instant of closing the switch.

After long time, at $t=\infty$, $i_t=\frac{\varepsilon}{R}\left(1-e^{-\left(\frac{R}{L}\right)\infty}\right)=\frac{\varepsilon}{R}$

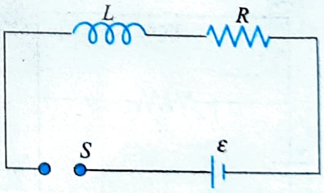


After long time

At $t=\infty$, the current has risen to its maximum value, and the inductor does not produce any opposition, it acts as zero resistance or conducting wire.

ILLUSTRATION 5.16

In an LR circuit as shown in figure, when the switch is closed, how much time will it take for the current to grow to a value n times the maximum value of current (where $n<1$)?



Sol. We know that $i=\frac{\varepsilon}{R}(1-e^{-t/\tau})$

$$i=n\frac{\varepsilon}{R} \qquad \text{(given)}$$

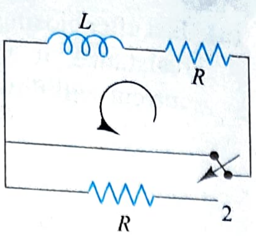
$$n\frac{\varepsilon}{R}=\frac{\varepsilon}{R}(1-e^{-t/\tau})$$

or $$e^{-t/\tau}=1-n$$

or $$t=\tau\ln\left(\frac{1}{1-n}\right)$$

ILLUSTRATION 5.17

In the circuit shown in figure, the initial current through the inductor at $t = 0$ is I_0. After a time $t = L/R$, the switch is quickly shifted to position 2.



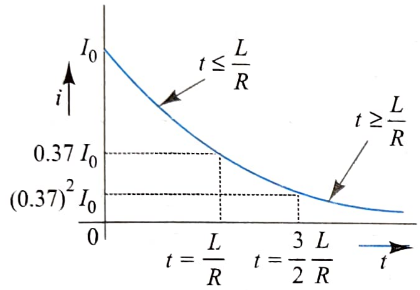
(a) Plot a graph showing the variation of current with time.

(b) Calculate the value of current in the inductor at $t = \frac{3}{2}\frac{L}{R}$.

Sol.

(a) Shifting the position of switch increases the resistance of the circuit. This decreases the value of time constant. Consequently, the rate of decrease of current increases. The variation of current with time is shown in figure.

(b) The equations of the curves are $i = I_0 e^{-Rt/L}$ (for $0 \le t \le L/R$)



For $\frac{R}{L} \le t \le \infty$ and $i = (0.37) I_0 e^{\frac{-2R}{L}\left(t-\frac{L}{R}\right)}$

At $t = \frac{3}{2}\frac{L}{R}$; hence $i = (0.37) I_0 e^{\frac{-2R}{L}\left(\frac{3L}{2R}-\frac{L}{R}\right)}$ or $i = (0.37)^2 I_0$

ILLUSTRATION 5.18

During the decay of current in an LR circuit, if the current falls to η times the initial value in time T, then determine the value of time constant.

Sol. We know that $I = I_{\max} e^{-t/T}$

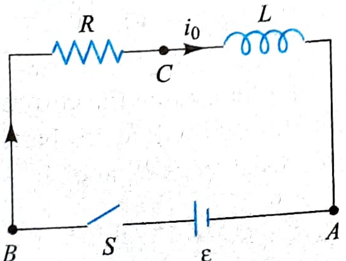
At $t = T, I = \eta I_{\max}$

$$\eta I_{\max} = I_{\max} e^{-t/T}$$

or $e^{-t/T} = \eta$; $T = \frac{t}{\ln\left|\frac{1}{\eta}\right|}$

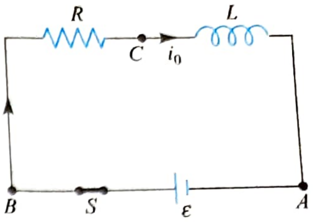
ILLUSTRATION 5.19

Figure shows a circuit consisting of an ideal cell, an inductor L, and a resistor R, connected in series. Let switch S be closed at $t = 0$. Suppose at $t = 0$, the current in the inductor is i_0, then find out the equation of current as a function of time.



Sol. Let at an instant t, the current in the circuit is i which is increasing at the rate di/dt.

Writing KVL along the circuit, we have



$$L\frac{di}{dt} = \varepsilon - iR$$

$$\Rightarrow \int_{i_0}^{i} \frac{di}{\varepsilon - iR} = \int_0^t \frac{dt}{L}$$

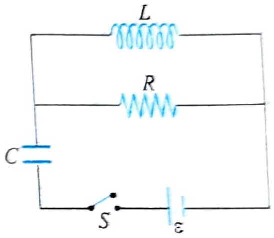
$$\Rightarrow \ln\left(\frac{\varepsilon - iR}{\varepsilon - i_0 R}\right) = -\frac{Rt}{L}$$

$$\Rightarrow \varepsilon - iR = (\varepsilon - i_0 R)\, e^{-Rt/L}$$

$$\Rightarrow i = \frac{\varepsilon - (\varepsilon - i_0 R)e^{-Rt/L}}{R}$$

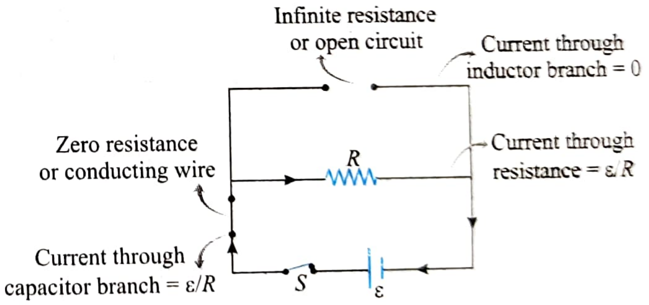
ILLUSTRATION 5.20

The open switch in figure is closed at $t = 0$, before the switch is closed the capacitor is uncharged and all currents are zero.



Determine the currents in L, C, and R, the emf across L, and the potential differences across C and R at the instant after the switch is closed and long after it is closed.

Sol. Just after closing the switch

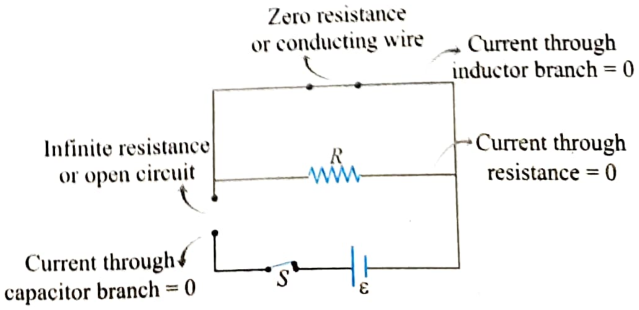

Infinite resistance or open circuit
Current through inductor branch = 0
Zero resistance or conducting wire
R
Current through resistance = ε/R
Current through capacitor branch = ε/R
S
ε

Potential difference across capacitor =0

Potential difference across resistance = ε

E.M.F. across inductor coil = ε

Long after closing the switch


Zero resistance or conducting wire
Current through inductor branch = 0
Infinite resistance or open circuit
R
Current through resistance = 0
Current through capacitor branch = 0
S
ε

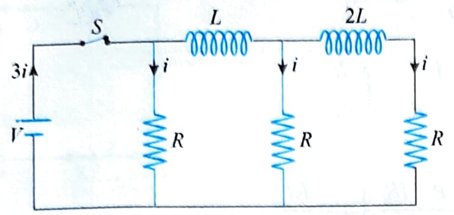
Potential difference across capacitor = ε

Potential difference across resistance = 0

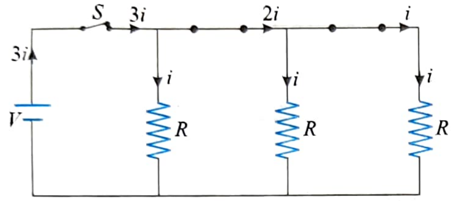
E.M.F. across inductor coil = 0

ILLUSTRATION 5.21

In the circuit shown in figure is at steady state, if $R_1 = R_2 = R_3 = R$ and the current through each resistor is i. Find the currents through the resistors immediately after the switch 'S' is opened.



Sol. In steady state the inductor coils acts as conducting wires, the currents in different branches will be as shown in figure.



Immediately after opening the switch, the current through inductors will not change. Now at this stage the current will be supplied by the inductor coils only. The distribution of currents in different branches is shown in figure.

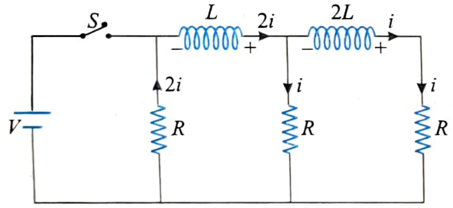
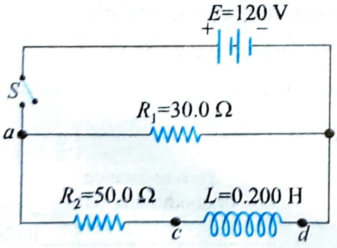


ILLUSTRATION 5.22

In the circuit shown in figure, E = 120 V, R_1 = 30.0 Ω, R_2 = 50.0 Ω, and L = 0.200 H. Switch S is closed at t = 0. Just after the switch is closed,



(a) What is the potential difference V_{ab} across the resistor R_1?

(b) Which point, c or d, is at higher potential?

(c) What is the potential difference V_{cd} across the inductor just after switch is opened?

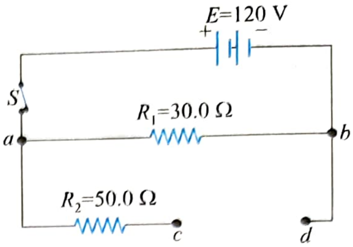
The switch is left closed a long time and then it is opened.

(d) Which point, a or b, is at higher potential? Find $V_b - V_a = V_{ba} = ?$

(e) What is the potential difference V_{cd} across the inductor?

Sol.

(a) Just after closing the switch, the inductor will act as infinite resistance, it will act as infinite resistance. Hence the current will flow through R_1 only



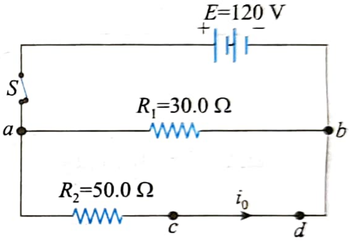
The potential difference across the resistor R_1,

$$V_{ab} = E = 120 \text{ V}$$

(b) The potential of point a = The potential of point c. Hence point c will be at higher potential.

(c) As no current in resistor R_2, it means, $V_{cd} = V_{ab} = E = 120\,\text{V}$.

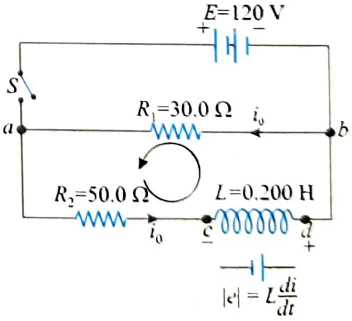
(d) The switch is left closed a long time, the inductor coil acts as conducting wire. Now the potential difference across resistor R_2 will be equal to 120 V. A constant current will flow through this branch.



Hence, just before opening the switch the current through inductor

$$I_0 = \frac{E}{R_2} = \frac{120}{50} = \frac{12}{5} \text{ A}$$

Now when the switch is opened, the inductor coil try to maintain the same current as it was flowing just before opening the switch hence end b of the coil will be at higher potential.



The potential difference across ba,

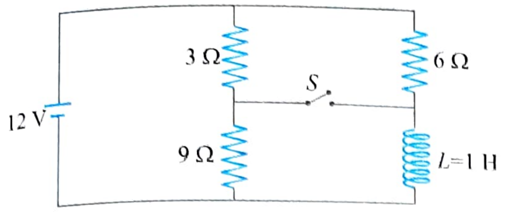
$$V_b - V_a = V_{ba} = R_1 i_0 = 30 \times \frac{12}{5} = 72\,\text{V}$$

(e) Just after opening the switch the current will flow in lower loop. The inductor coil will act as battery supplying equal current to both resistances R_1 and R_2.

$$V_{d,c} = i_0(R_1 + R_2) = \frac{12}{5}(30 + 50) = 192 \text{ V}$$

ILLUSTRATION 5.23

In the circuit shown, the switch '*S*' has been closed for a long time and then opens at $t = 0$.



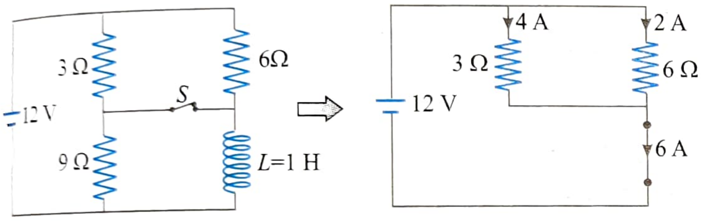
(a) Find the current through the inductor just before the switch is opened.

(b) Find the current through the inductor a long time after the switch is opened.

(c) Find current through the inductor as a function of time after the switch is opened. Also write the current through the cell as a function of time.

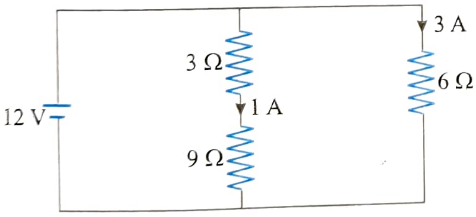
Sol.

(a) When the switch '*S*' is in closed position for a long time the circuit is in steady condition and the inductor acts as a short circuit or conducting wire.

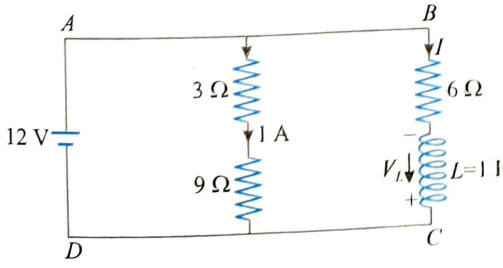


There is no potential difference across the inductor. It means no potential difference across 6 Ω resistance is no current through 6 Ω resistance. Effective circuit just before the switch is opened is as shown in figure. Required answer is 6 A.

(b) After long time(at $t \to \infty$) once again the circuit will be in steady condition $I = 3$ A



(c) At time '*t*' the circuit parameters are as shown in figure.



Applying Kirchhoff s Voltage law (KVL) in loop *ABCDA*:

$$6I + L\frac{dI}{dt} = 12$$

$$\int_6^I \frac{dI}{6I - 12} = -\int_0^t dt$$

$$= \frac{1}{2}[\ln(I-2)]_6^I = -(t-0)$$

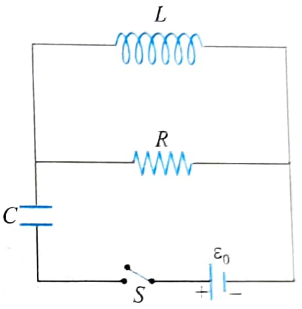
$$\ln[(I-2) - \ln(6-2)] = -2t$$

$$\ln\left(\frac{I-2}{4}\right) = -2t \Rightarrow \frac{I-2}{4} = e^{-2t}$$

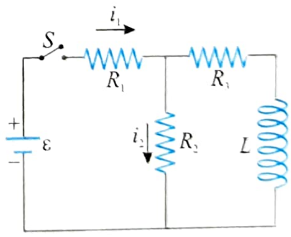
$$(I-2) = 4e^{-2t} \Rightarrow I = 2 + 4e^{-2t}$$

CONCEPT APPLICATION EXERCISE 5.3

1. The open switch in figure is thrown closed at $t = 0$. Before the switch is closed the capacitor is uncharged and all currents are zero. Determine the currents in *L*, *C*, and *R*, the emf across *L*, and the potential differences across *C* and *R*: **(a)** at the instant after the switch is closed and **(b)** long after it is closed.

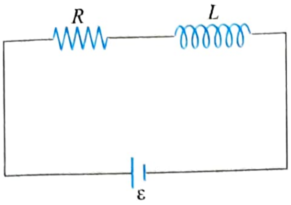


2. In figure, $\varepsilon = 110$ V, $R_1 = 10.0\ \Omega$, $R_2 = 20.0\ \Omega$, $R_3 = 30.0\ \Omega$, and $L = 2.00$ H. Immediately after switch *S* is closed, what are **(a)** i_1 and **(b)** i_2? (Let currents in the indicated directions have positive values and currents in the opposite directions have negative values.) A long time later, what are **(c)** i_1 and **(d)** i_2? The switch is then reopened. Just then, what are **(e)** i_1 and **(f)** i_2? A long time later, what are **(g)** i_1 and **(h)** i_2?

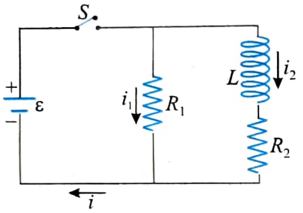


3. A circuit consists of a coil, a switch, and a battery, all in series. The internal resistance of the battery is negligible compared with that of the coil. The switch is originally open. It is thrown closed, and after a time interval Δt, the current in the circuit reaches 80.0% of its final value. The switch then remains closed for a time interval much longer than Δt. The wires connected to the terminals of the battery are then short circuited with another wire and removed from the battery, so that the current is uninterrupted. **(a)** At an instant that is a time interval Δt after the short circuit, the current is what percentage of its maximum value? **(b)** At the moment $2\Delta t$ after the coil is short-circuited, the current in the coil is what percentage of its maximum value?

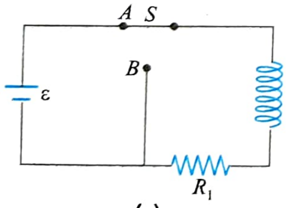
4. For the *RL* circuit shown in figure, let the inductance be 3.00 H, the resistance 8.00 Ω, and the battery emf 36.0 V. **(a)** Calculate $\Delta V_R / \varepsilon_L$, that is, the ratio of the potential difference across the resistor to the emf across the inductor when the current is 2.00 A. **(b)** Calculate the emf across the inductor when the current is 4.50 A.



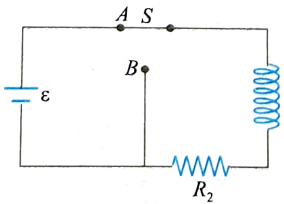
5. In figure, the battery is ideal and $\varepsilon = 10$ V, $R_1 = 5.0\ \Omega$, $R_2 = 10\ \Omega$, and $L = 5.0$ H. Switch *S* is closed at time $t = 0$. Just afterwards, what are **(a)** i_1, **(b)** i_2, **(c)** the current i_S through the switch, **(d)** the potential difference V_2 across resistor 2, **(e)** the potential difference V_L across the inductor, and **(f)** the rate of change di_2/dt? A long time later, what are **(g)** i_1, **(h)** i_2, **(i)** i_S, **(j)** V_2, **(k)** V_L, and **(*l*)** di_2/dt?



6. In Fig. (a), switch *S* has been closed on *A* long enough to establish a steady current in the inductor of inductance $L_1 = 5.00$ mH and the resistor of resistance $R_1 = 20.0\ \Omega$. Similarly, in Fig. (b), switch *S* has been closed on *A* long enough to establish a steady current in the inductor of inductance $L_2 = 6.00$ mH and the resistor of resistance $R_2 = 30.0\ \Omega$. The ratio Φ_{02}/Φ_{01} of the magnetic flux through a turn in inductor 2 to that in inductor 1 is 2.0. At time $t = 0$, the two switches are closed on *B*. At what time *t* is the flux through a turn in the two inductors equal?

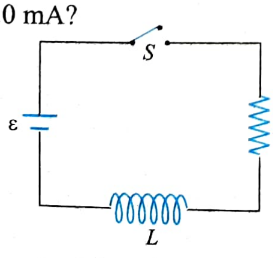


(a)

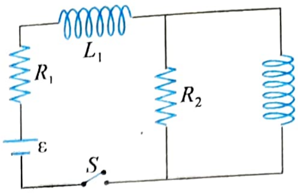


(b)

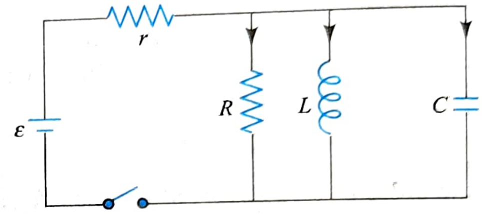
7. In given figure, $R = 4.0$ kΩ, $L = 8.0$ μH, and the ideal battery has $\varepsilon = 20$ V. How long after switch *S* is closed is the current 2.0 mA?



8. In given figure, $R_1 = 8.0\ \Omega$, $R_2 = 10\ \Omega$, $L_1 = 0.30$ H, $L_2 = 0.20$ H, and the ideal battery has $\varepsilon = 6.0$ V. **(a)** Just after switch *S* is closed, at what rate is the current in inductor 1 changing? **(b)** When the circuit is in the steady state, what is the current in inductor 1?

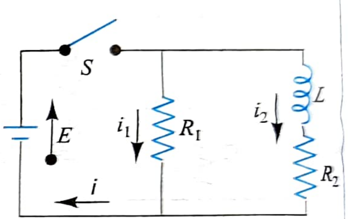


9. Figure shows an *LCR* circuit. When the switch is closed, the currents through resistor *R*, inductor *L*, and capacitor *C* are I_1, I_2, and I_3, respectively. Determine the values of I_1, I_2, and I_3.



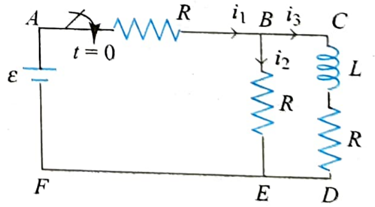
(a) at $t = 0$ **(b)** at $t = \infty$

10. In the circuit shown in figure, $E = 10$ V, $R_1 = 5\ \Omega$, $R_2 = 10\ \Omega$, and $L = 5$ H. For the two separate conditions, (i) switch *S* is just closed and (ii) switch *S* is closed for a long time, calculate

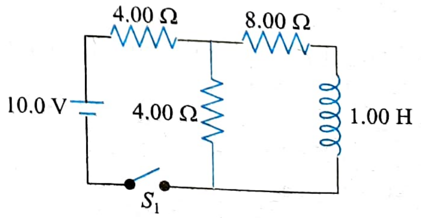


(a) current i_1 through R_1,
(b) current i_2 through R_2,
(c) current *i* through the switches,
(d) the potential difference across R_2,
(e) the potential difference across *L*,
(f) di_2/dt.

11. In the following circuit (figure), the switch is closed at $t = 0$. Find the currents i_1, i_2, i_3 and di_3/dt at $t = 0$ and at $t = \infty$. Initially, all currents are zero.



12. The switch in figure is closed at time $t = 0$. Find the current in the inductor and the current through the switch as functions of time thereafter.



ANSWERS

1. (a) $I_L = 0, I_c = \frac{\varepsilon_0}{R}, I_R = \frac{\varepsilon_0}{R}, \Delta V_L = \varepsilon_0, \Delta V_c = 0, \Delta V_R = \varepsilon_0$

(b) $I_L = 0, I_c = 0, I_R = 0, \Delta V_L = 0, \Delta V_C = \varepsilon_0, \Delta V_R = 0$

2. (a) $\frac{11}{3}$ A (b) $\frac{11}{3}$ A (c) 5.0 A (d) 3.0 A (e) 0

(f) −2.0 A (reversed) (g) 0 (h) 0

3. (a) 20% (b) 4% **4.** (a) 0.80 (b) zero

5. (a) 2.0 A (b) 0 (c) 2.0 A (d) 0 (e) 10 V (f) 2.0 A/s

(g) 2.0 A (h) 1.0 A (i) 3.0 A (j) 10 V (k) 0 (l) 0

6. $\frac{\ln(2.0)}{1000}$ s **7.** 1.0 ns **8.** (a) 20 A/s; (b) 0.75 A

9. (a) $I_1 = 0, I_2 = 0, I_3 = \frac{\varepsilon}{r}$ (b) $I_1 = 0, I_2 = \frac{\varepsilon}{r}, I_3 = 0$

10. (a) 2 A (b) 1 A (c) 3 A (d) 10 V (e) 0 V (f) 0

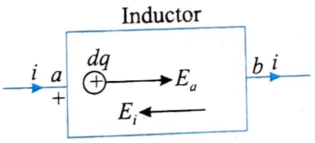
11. At $t = 0, i_1 = i_2 = \frac{\varepsilon}{2R}, i_3 = 0, \frac{di_3}{dt} = \frac{\varepsilon}{2l}$

At $t = \infty, i_2 = i_3 = \frac{\varepsilon}{3R}, i_1 = \frac{2\varepsilon}{3R}, \frac{di_3}{dt} = 0$

12. $\frac{1}{2}(1 - e^{-10t}), \frac{5}{4} + \frac{1}{4}(1 - e^{-10t})$

ENERGY STORED IN THE MAGNETIC FIELD OF AN INDUCTOR

When a current builds up at certain rate in an inductor, a self-induced voltage is developed in the inductor coil which oppose the applied emf. In this process, the source performs a positive work in pushing the charges against the induced voltage.



Let an elementary charge dq is pushed by the source from a to b against the induced voltage V_L during a time dt, constituting an electric current i. The work dW performed by the applied source during the transportation of the elementary charge dq between the terminals of the inductor is given as

$dW = V_L dq$; then substituting $V_L = L\frac{di}{dt}$ and $dq = idt$,

we have $dW = L\frac{di}{dt}idt = Lidi$

Integrating both sides, the total work done by the source in establishing a current i is given by,

$$W = \int dW = L\int_0^i i\,di = \frac{1}{2}Li^2$$

This work done by applied source will be stored as magnetic energy in inductor coil.

Hence magnetic energy stored $U = \frac{1}{2}Li^2$ ...(i)

As the flux associated with coil is given by,

$$\phi = Li \Rightarrow i = \frac{\phi}{L} \text{ or } L = \frac{\phi}{i} \quad \text{...(ii)}$$

From (i) and (ii) we can write, $U = \frac{1}{2}Li^2 = \frac{1}{2}\frac{\phi^2}{L} = \frac{1}{2}\phi i$

The energy in an inductor is actually stored in the magnetic field within the coil. We can develop magnetic energy density u (energy stored per unit volume) analogous to those we obtained in electrostatics. We will concentrate on one simple case of an ideal long cylindrical solenoid. For a long solenoid, its magnetic field can be assumed completely within the solenoid. Energy U stored in the solenoid with current i is

$$U = \frac{1}{2}Li^2 = \frac{1}{2}\left(\mu_0 n^2 V\right)i^2 \text{ as } L = \mu_0 n^2 V$$

The energy per unit volume is $u = U/V$.

$$u = \frac{U}{V} = \frac{1}{2}\mu_0 n^2 i^2 = \frac{1}{2}\frac{(\mu_0 ni)^2}{\mu_0} = \frac{1}{2}\frac{B^2}{\mu_0} \text{ as } B = \mu_0 ni$$

Thus, $u = \frac{1}{2}\frac{B^2}{\mu_0}$

This expression is similar to $u = \frac{1}{2}\varepsilon_0 E^2$ used in electrostatics.

Although we have derived it for one special situation, it turns out to be correct for any magnetic field configuration.

ILLUSTRATION 5.24

Consider the RL circuit in figure. When the switch is closed in position 1 and opens in position 2, electrical work must be performed on the inductor and on the resistor. The energy stored in the inductor is for the magnetic field inside it which increases as I increases. In the resistor energy appears as heat.

(a) What is the ratio of P_L/P_R of the rate at which energy is stored in the inductor to the rate at which energy is dissipated in the resistor?

(b) Express the ratio P_L/P_R as a function of time.

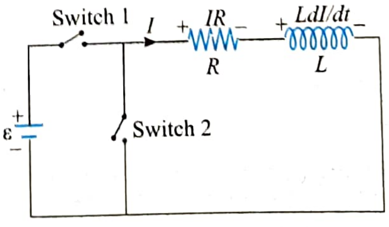
(c) If the time constant of circuit is t, what is the time at which $P_L = P_R$?

Sol.

(a) The energy stored in the inductor is $U_L = \frac{1}{2}LI^2$

$$\text{Power } P_L = \frac{dU}{dt} = \frac{d}{dt}\left(\frac{1}{2}LI^2\right) = LI\frac{dI}{dt} \quad \text{...(i)}$$

The power of a resistor $P_R = I^2R$ ...(ii)



$$\text{So the ratio is } \frac{P_L}{P_R} = \frac{LIdI/dt}{I^2R} = \frac{\tau(dI/dt)}{I} \quad \text{...(iii)}$$

(b) As $I = \frac{E}{R}\left[1 - e^{-(t/\tau)}\right]$ ...(iv)

$$\frac{dI}{dt} = \frac{d}{dt}\left[\frac{E}{R}\left(1 - e^{-(t/\tau)}\right)\right] = \frac{E}{R\tau}e^{-(t/\tau)} \quad \text{...(v)}$$

Inserting the value of dI/dt and I in Eq. (iii), we have

$$\frac{P_{\mathrm{L}}}{P_{\mathrm{R}}} = \tau\frac{\frac{E}{R\tau}e^{-t/\tau}}{\frac{E}{R}\left(1 - e^{-t/\tau}\right)} = \frac{e^{-(t/\tau)}}{1 - e^{-(t/\tau)}} = \frac{1}{e^{(t/\tau)} - 1} \quad \text{...(vi)}$$

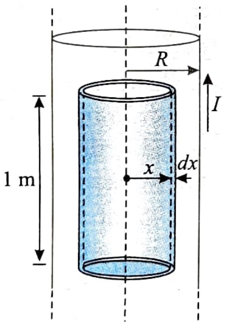
When $t = 0$, this ratio tends to infinity, due to large initial value of dI/dt and small initial value of I. When t tends to infinity, after many time constants, the current tends to zero. As a result, P_{L} vanishes and the ratio goes to zero.

(c) From Eq. (vi), $P_{\mathrm{L}} = P_{\mathrm{R}}$ when $\frac{1}{e^{(t/\tau)} - 1} = 1$ which on simplification yields $e^{(t/\tau)} = 2$, $\frac{t}{\tau} = \ln 2 \Rightarrow t = 0.693\tau$

ILLUSTRATION 5.25

A long cylindrical wire of radius R and carrying a current I. Find the total field energy of magnetic field stored per unit length inside the wire.

Sol. Let us consider an elemental shell of radius x and wall width dx inside the cylinder as shown in figure.



The magnetic field inside the wire at a distance x from axis,

$$B = \frac{\mu_0 I x}{2\pi R^2}$$

The volume of the elemental shell of length 1 m is given as

$$dV = 2\pi x dx \times 1$$

Field energy stored in elemental cylindrical sheet is given as

$$dU = \frac{B^2}{2\mu_0} \times dV = \frac{1}{2\mu_0}\left(\frac{\mu_0 I x}{2\pi R^2}\right) \times 2\pi x dx$$

$$dU = \frac{\mu_0 I^2 x^3 dx}{4\pi R^4}$$

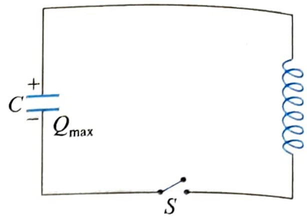
Total field energy inside the cylinder per unit length is given by integrating above expression within limits from 0 to R which is given as

$$U = \int dU = \int_0^R \frac{\mu_0 I^2}{4\pi R^4} \cdot x^3 dx$$

$$U = \frac{\mu_0 I^2}{4\pi R^4} \cdot \frac{R^2}{4} = \frac{\mu_0 I^2}{16\pi R^2}$$

OSCILLATIONS IN AN LC CIRCUIT

When a capacitor is connected to an inductor as illustrated in figure, the combination is an **LC circuit**. If the capacitor is initially charged and the switch is then closed, both the current in the circuit and the charge on the capacitor oscillate between maximum positive and negative values. If the resistance of the circuit is zero, no energy is transformed to internal energy. In the following analysis, the resistance in the circuit is neglected. We also assume an idealized situation in which energy is not radiated away from the circuit.



Assume the capacitor has an initial charge Q_{max} (the maximum charge) and the switch is open for $t < 0$ and then closed at $t = 0$.

Consider some arbitrary time t after the switch is closed so that the capacitor has a charge $q < Q_{\text{max}}$ and the current is $i < I_{\text{max}}$. At this time, both circuit elements store energy, but the sum of the two energies must equal the total initial energy U stored in the fully charged capacitor at $t = 0$:

$$U = U_E + U_B = \frac{q^2}{2C} + \frac{1}{2}Li^2 = \frac{Q_{\text{max}}^2}{2C} \quad \text{...(i)}$$

The system of the circuit is isolated: the total energy of the system must remain constant in time. We describe the constant energy of the system mathematically by setting $dU/dt = 0$. Therefore, by differentiating Eq. (i) with respect to time while noting that q and i vary with time gives

$$\frac{dU}{dt} = \frac{d}{dt}\left(\frac{q^2}{2C} + \frac{1}{2}Li^2\right) = \frac{q}{C}\frac{dq}{dt} + Li\frac{di}{dt} = 0 \quad \text{...(ii)}$$

We can reduce this result to a differential equation in one variable by remembering that the current in the circuit is equal to the rate at which the charge on the capacitor changes.

$i = -\frac{dq}{dt}$ (negative sign because the charge in capacitor is decreasing with time)

It then follows that $\frac{di}{dt} = -\frac{d^2q}{dt^2}$. Substitution of these relationships into Eq. (ii) gives

$$\frac{q}{C}(-i) + Li\left(-\frac{d^2q}{dt^2}\right) = 0 \Rightarrow \frac{q}{C} + L\frac{d^2q}{dt^2} = 0$$

$$\frac{d^2q}{dt^2} = -\frac{1}{LC}q \quad \text{...(iii)}$$

Let's solve for q by noting that this expression is of the same form as the analogous equations for a particle in simple harmonic motion:

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x = -\omega^2 x$$

where k is the spring constant, m is the mass of the block, and $\omega = \sqrt{k/m}$. The solution of this mechanical equation has the general form

$$x = A\cos(\omega t + \phi)$$

where A is the amplitude of the simple harmonic motion (the maximum value of x), ω is the angular frequency of this motion, and ϕ is the phase constant; the values of A and ϕ depend on the initial conditions. Because Eq. (iii) is of the same mathematical form as the differential equation of the simple harmonic oscillator, it has the solution

$$q = Q_{\max} \cos(\omega t + \phi) \quad \text{...(iv)}$$

The angular frequency $\omega = \frac{1}{\sqrt{LC}}$ ...(v)

Note that the angular frequency of the oscillations depends solely on the inductance and capacitance of the circuit. Equation (v) gives the natural frequency of oscillation of the LC circuit.

Because q varies sinusoidally with time, the current in the circuit also varies sinusoidally. We can show that by differentiating Eq. (iv) with respect to time:

$$i = -\frac{dq}{dt} = \omega Q_{\max} \sin(\omega t + \phi) \quad \text{...(vi)}$$

Current as a Function of Time for an Ideal LC Current

To determine the value of the phase angle ϕ, let's examine the initial conditions, which in our situation require that at $t = 0$, $i = 0$, and $q = Q_{\max}$. Setting $i = 0$ at $t = 0$ in Eq. (vi) gives

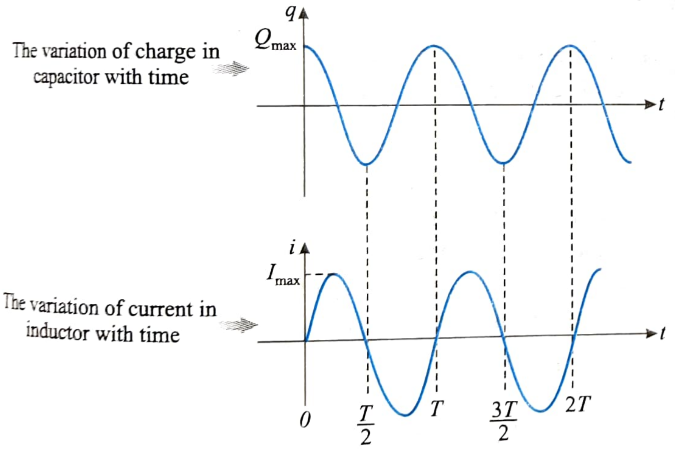
$$0 = \omega Q_{\max} \sin\phi$$

which shows that $\phi = 0$. This value for ϕ also is consistent with Eq. (iv) and the condition that $q = Q_{\max}$ at $t = 0$. Therefore, in our case, the expressions for q and i are

Charge as a function of time $q = Q_{\max} \cos\omega t$ ...(vii)

Current as a function of time,

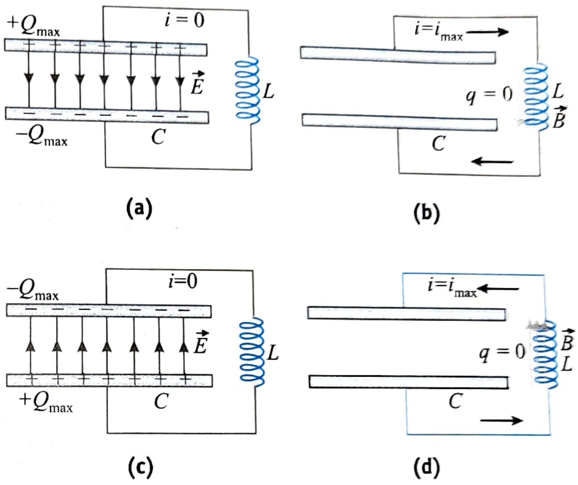
$$i = \omega Q_{\max} \sin\omega t = I_{\max} \sin\omega t \quad \text{...(viii)}$$



OSCILLATION OF ELECTRICAL AND MAGNETIC ENERGY

When the capacitor is fully charged, the energy U in the circuit is stored in the capacitor's electric field and is equal to $Q^2{}_0/2C$. At this time, the current in the circuit is zero; therefore, no energy is stored in the inductor [Fig. (a)]. After the switch is closed, the rate at which charges leave or enter the capacitor plates (which is also the rate at which the charge on the capacitor changes) is equal to the current in the circuit. After the switch is closed and the capacitor begins to discharge, the energy stored in its electric field decreases. The capacitor's discharge represents a current in the circuit, and some energy is now stored in the magnetic field of the inductor. Therefore, energy is transferred from the electric field of the capacitor to the magnetic field of the inductor. When the capacitor is fully discharged, it stores no energy. At this time, the current reaches its maximum value and all the energy in the circuit is stored in the inductor [Fig. (b)]. The current continues in the same direction, decreasing in magnitude, with the capacitor eventually becoming fully charged again but with the polarity of its plates now opposite the initial polarity [Fig. (c)]. This process is followed by another discharge until the circuit returns to its original state of maximum charge Q_0 and the plate polarity [Fig. (a)]. The energy continues to oscillate between inductor and capacitor.

The oscillations of the *LC* circuit are an electromagnetic analog to the mechanical oscillations of the particle in simple harmonic motion.



(a) **(b)**

(c) **(d)**

Electrical energy in relation with time $U_E = \frac{Q^2_{\max}}{2C} \cos^2 \omega t$

Magnetic energy in relation with time $U_B = \frac{1}{2} LI^2_{\max} \sin^2 \omega t$

The total energy, $U = U_E + U_B = \frac{Q^2_{\max}}{2C} \cos^2 \omega t + \frac{1}{2} LI^2_{\max} \sin^2 \omega t$

Plots of the time variations of U_E and U_B are shown in figure. The sum $U_E + U_B$ is a constant and is equal to the total energy $\frac{Q^2_{\max}}{2C}$, or $\frac{1}{2} LI^2_{\max}$.

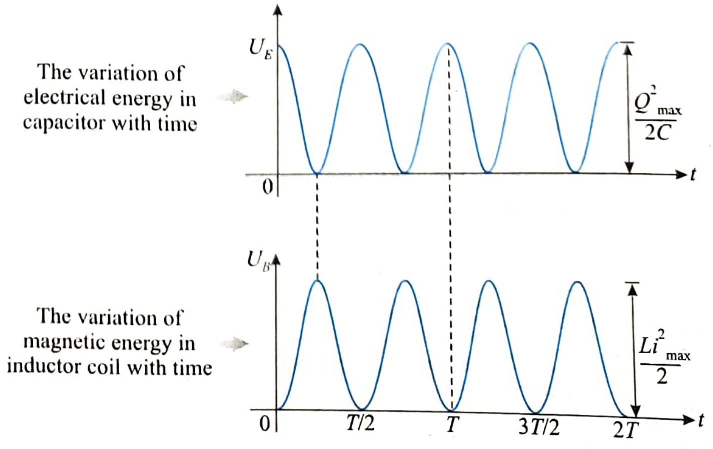
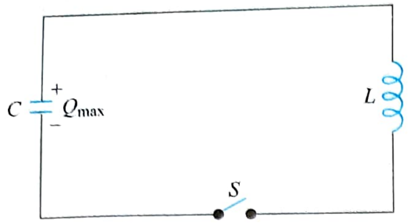


ILLUSTRATION 5.26

In an LC circuit as shown in figure, the switch is closed at $t = 0$. $Q_{\text{max}} = 100\ \mu\text{C}$; $L = 40$ mH; $C = 100\ \mu\text{F}$.



(a) Determine the equation for instantaneous charge on the capacitor.

(b) Determine the equation for instantaneous current in the circuit.

Sol.

(a) $q = Q_{\text{max}} \cos \omega t$, Here $Q_{\text{max}} = 100\ \mu\text{C}$;

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(40\times10^{-3})(100\times10^{-6})}} = 500 \text{ rad s}^{-1}$$

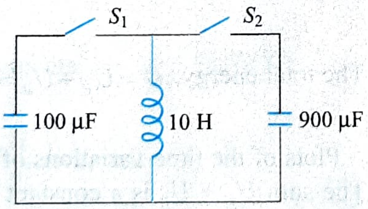
$$q = 100 \cos (500t)\ \mu\text{C}$$

(b) $i = \dfrac{-dq}{dt} = (100)\,(500) \sin (500t)\ \mu\text{A}$

or $\quad i = 50000 \sin (500t)\ \mu\text{A}$

ILLUSTRATION 5.27

Initially the 900 μF capacitor is charged to 100 V and the 100 μF capacitor is uncharged in figure. Then switch S_2 is closed for time t_1, after which it is opened and at the same instant switch S_1 is closed for time t_2 and then opened. It is now found that 100 μF capacitor is charged to 300 V. Find the minimum possible values of the time interval t_1 and t_2.



Sol. Initial energy stored in the 900 μF capacitor is

$$U_1 = (1/2) \times 900 \times 10^{-6} \times (100)^2 = 4.5 \text{ J}$$

Finally, energy stored in the 100 μF capacitor is

$$U_2 = (1/2) \times 100 \times 10^{-6} \times (300)^2 = 4.5 \text{ J}$$

The entire energy of the 900 μF capacitor has been transferred to the 100 μF capacitor. First, electrical energy of the 900 μF capacitor is converted into magnetic energy in the inductor and then this energy is converted into electrical energy once again using S_2 and S_1 appropriately.

In an LC circuit, the transfer of electrical energy into magnetic energy and vice versa takes place in time $T/4$ where $T = 2\pi\sqrt{LC}$ is the time period of the electrical oscillations. Thus,

$$T_1 = 2\pi\sqrt{10\times900\times10^{-6}} = 0.6 \text{ s}$$

$$T_2 = 2\pi\sqrt{10\times100\times10^{-6}} = 0.2 \text{ s}$$

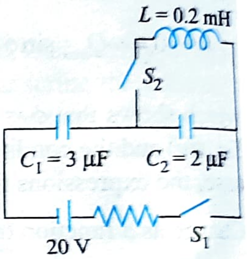
Therefore, switch S_2 is first closed for time 0.6/4 = 0.15 s, during which time the 900 μF capacitor gets fully discharged and the current in the inductor is fully established. Next switch S_2 is opened and simultaneously switch S_1 is closed for time 0.2/4 = 0.05 s during which the current in the inductor disappears and the 100 μF capacitor gets fully charged.

After this time, switch S_1 is also opened. The 100 μF capacitor is now charged to 300 V.

Thus, $t_1 = 0.15$ s

and $\quad t_2 = 0.05$ s

ILLUSTRATION 5.28



The circuit shown in figure is in the steady state with switch S_1 is closed. At $t = 0$, S_1 is opened and switch S_2 is closed.

(a) Derive an expression for the charge on capacitor C_2 as a function of time.

(b) Determine the first instant t, when the energy in the inductor becomes one-third of that in the capacitor.

Sol.

(a) In the steady state, C_1 and C_2 are in series arrangement, their equivalent is

$$C_{\text{eq}} = \frac{C_1C_2}{C_1 + C_2} = 1.2\ \mu\text{F}$$

Charge on the capacitor C_2, $Q_0 = C_{\text{eq}}V = 1.2 \times 20 = 24\ \mu\text{C}$. When S_1 is opened, S_2 is closed. Capacitor C_2 starts discharging through the inductor and let at any time t, charge on the capacitor be Q. Then we know that

$$Q = Q_0 \cos \omega t$$

$$U_E + U_B = \frac{Q_0^2}{2C_2}$$

where $\omega = \dfrac{1}{\sqrt{LC_2}} = \dfrac{1}{\sqrt{0.2\times10^{-3}\times2\times10^{-6}}} = 50{,}000 \text{ rad s}^{-1}$

(b) At the time $t = t_1$, $U_B = \dfrac{1}{3}U_E$, but $U_B + U_E = \dfrac{Q_0^2}{2C_2}$

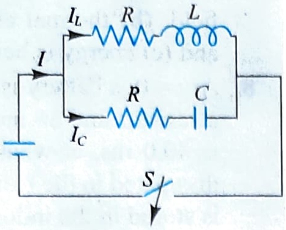
$$\text{Hence, } U_E = \frac{3}{4}\left(\frac{1}{2}\frac{Q_0^2}{C_2}\right) \Rightarrow \frac{1}{2}\frac{Q^2}{C_2} = \frac{3}{4}\left(\frac{1}{2}\frac{Q_0^2}{C_2}\right)$$

$$Q = \frac{\sqrt{3}}{2}Q_0 \quad \text{or} \quad Q_0 \cos \omega t_1 = \frac{\sqrt{3}}{2}Q_0$$

$$\omega t_1 = \frac{\pi}{6} \quad \text{or} \quad t_1 = \frac{\pi}{6\omega} = 1.05 \times 10^{-5} = 10.5\ \mu\text{s}$$

ILLUSTRATION 5.29

In the circuit shown in figure, the battery has negligible internal resistance. Show that the current in the circuit through the battery rises instantly to its steady state value E/R when the switch is closed, provided that resistance R is $\sqrt{L/C}$.



Sol. Let the currents through inductive branch and capacitive branch be I_L and I_C, respectively. Then the current through the battery, from KCL, is $I = I_L + I_C$.

Since the battery is connected in parallel to the RL and RC branches of the circuit, the current in the RL branch is unaffected by the presence of the RC branch, so

$$I_L = \frac{E}{R}\,[1 - e^{-(R/L)t}] \quad \text{and} \quad I_C = \frac{E}{R}\,e^{-(t/RC)}$$

Hence, the current through battery is $I = \dfrac{E}{R}\,[1 - e^{-(R/L)t} + e^{-(t/RC)}]$

If the current has to reach its final value E/R instantaneously, the exponential terms must cancel out, i.e.,

$e^{-t/\tau_L} = e^{-t/\tau_C}$, which is possible if $\tau_L = \tau_C$.

$$L/R = RC,\ R = \sqrt{(L/C)}$$

For all $t > 0$, this is the desired result.

ILLUSTRATION 5.30

An inductor of inductance 2.0 mH is connected across a charged capacitor of capacitance 5.0 μF and the resulting LC circuit is set oscillating at its natural frequency. Let Q denote the instantaneous charge on the capacitor and I the current in the circuit. It is found that the maximum value of charge Q is 200 μC.

(a) When $Q = 100$ μC, what is the value of $\left|\dfrac{dI}{dt}\right|$?

(b) When $Q = 200$ μC, what is the value of I?

(c) Find the maximum value of I.

(d) When I is equal to one-half its maximum value, what is the value of $|Q|$?

Sol. $L = 2.0\text{ mH} = 2.0 \times 10^{-3}\text{ H}$

$C = 5.0\ \mu\text{F} = 5.0 \times 10^{-6}\text{ F}$

$Q_{\max} = 200\ \mu\text{C} = 200 \times 10^{-6}\text{ C}$

In an LC circuit, energy transfer continues from inductance to capacitance and vice versa.

(a) By Kirchhoff's law in an LC circuit

$$\frac{Q}{C} + L\frac{dI}{dt} = 0 \;\Rightarrow\; \frac{dI}{dt} = -\frac{Q}{LC}$$

$$\therefore \quad \left|\frac{dI}{dt}\right| = \frac{Q}{LC} = \frac{100\times10^{-6}}{2.0\times10^{-3}\times5.0\times10^{-6}} = 10^4\ \text{A s}^{-1}$$

(b) When $Q = 200$ μC, the entire energy of circuit resides in capacitance. That is, no energy is stored in inductance.

$$\therefore \quad \frac{1}{2}LI^2 = 0 \Rightarrow I = 0$$

(c) Maximum value of I is given by

$$\frac{1}{2}LI_{\max}^2 = \frac{Q_{\max}^2}{2C} \;\Rightarrow\; I_{\max} = \frac{1}{\sqrt{LC}}Q_{\max}$$

$$\text{or } I_{\max} = \frac{1}{\sqrt{(2.0\times10^{-3})\times(5.0\times10^{-6})}} \times 200 \times 10^{-6} = 2\text{ A}$$

(d) Given $I = \dfrac{I_{\max}}{2} = \dfrac{2}{2} = 1\text{ A}$

Then again from the conservation of energy,

$$\frac{1}{2}LI^2 + \frac{Q^2}{2C} = \frac{1}{2}LI_{\max}^2;\ \frac{Q^2}{2C} = \frac{1}{2}L\left(I_{\max}^2 - I^2\right)$$

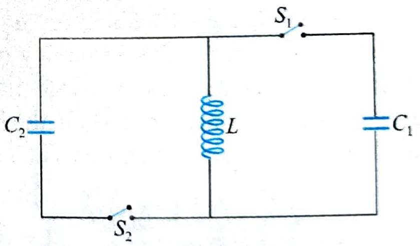
$$Q = \sqrt{LC\left(I_{\max}^2 - I^2\right)}$$

$$= \sqrt{(2.0\times10^{-3}\times5.0\times10^{-6})(2^2 - 1^2)}$$

$$= 10^{-4}\sqrt{3}\text{ C} = 1.732 \times 10^{-4}\text{ C} = 1.732\ \mu\text{C}$$

ILLUSTRATION 5.31

In the circuit shown in the figure a capacitor C_1 is used to transfer energy to other capacitor C_2. Initially capacitor of capacitance $C_1 = C_0$ is charged to a potential difference of V_0. Switch S_1 is closed at time $t = 0$. After some time S_1 is opened and S_2 is closed simultaneously. At time $t = T$, S_2 was opened and it was found that the potential difference across capacitor of capacitance $C_2 = \dfrac{C_0}{9}$ was $3V_0$. What is the smallest possible value of time T? The coil has inductance L. Assume no resistance.



Sol. Initial energy stored in C_1 is $U_1 = \dfrac{1}{2}C_0V_0^2$

Final energy stored in C_2 is $U_2 = \dfrac{1}{2}\dfrac{C_0}{9}(3V_0)^2 = \dfrac{1}{2}C_0V_0^2$

It means, complete energy of C_x gets transferred to C_2.

The time period of LC circuit consisting of L and C_1 is

$$T_1 = 2\pi\sqrt{LC_0}$$

The entire energy of the capacitor can get transferred to the inductor in smallest interval of $t_1 = \dfrac{T_1}{4} = \dfrac{\pi}{2}\sqrt{LC_0}$

At time t_1 the current through L is maximum and switch S_1 is opened and S_2 is closed.

Now the time period of LC circuit having L and C_2 is

$$T_2 = 2\pi\sqrt{\frac{LC_0}{9}} = \frac{2\pi}{3}\sqrt{LC_0}$$

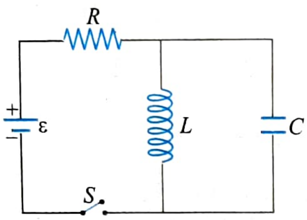
The inductor will transfer its entire energy to C_2 in smallest time given by

$$t_2 = \frac{T_2}{4} = \frac{\pi}{6}\sqrt{LC_0}$$

$$\therefore \quad T_{\min} = t_1 + t_2 = \frac{2\pi}{3}\sqrt{LC_0}$$

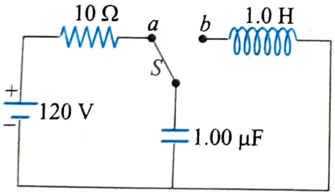
CONCEPT APPLICATION EXERCISE 5.4

1. A 1.0 μF capacitor is charged by a 40.0 V power supply. The fully charged capacitor is then discharged through a 10.0 mH inductor. Find the maximum current in the resulting oscillations.
2. In the circuit of figure, the battery emf is 50 V, the resistance is 200 Ω, and the capacitance is 1.0 μF. The switch S is closed for a long time interval, and zero potential difference is measured across the capacitor.



After the switch is opened, the potential difference across the capacitor reaches a maximum value of 100 V. What is the value of the inductance?

3. An LC circuit consists of a 20.0 mH inductor and a 0.500 μF capacitor. If the maximum instantaneous current is 0.100 A, what is the greatest potential difference across the capacitor?
4. The switch in figure is connected to position a for a long time interval. At $t = 0$, the switch is thrown to position b. After this time, what are (a) the frequency of oscillation of the LC circuit, (b) the maximum charge that appears on the capacitor, (c) the maximum current in the inductor, and (d) the total energy the circuit possesses at $t = 3.00$ s?



5. An inductor having inductance L and a capacitor having capacitance C are connected in series. The current in the circuit increases linearly in time as described by $i = Kt$, where K is a constant. The capacitor is initially uncharged. Determine (a) the voltage across the inductor as a function of time, (b) the voltage across the capacitor as a function of time, and (c) the time when the energy stored in the capacitor first exceeds that in the inductor.
6. A capacitor in a series LC circuit has an initial charge Q and is being discharged. When the charge on the capacitor is $Q/2$, find the flux through each of the N turns in the coil of the inductor in terms of Q, N, L, and C.
7. A coil with an inductance of 2.0 H and a resistance of 10 Ω is suddenly connected to an ideal battery with $\varepsilon = 100$ V. At 0.10 s after the connection is made, what is the rate at which **(a)** energy is being stored in the magnetic field, **(b)** thermal energy is appearing in the resistance, and **(c)** energy is being delivered by the battery?
8. At $t = 0$, a battery is connected to a series arrangement of a resistor and an inductor. If the inductive time constant is 40.0 ms, at what time is the rate at which energy is dissipated in the resistor equal to the rate at which energy is stored in the inductor's magnetic field?

ANSWERS

1. 40.0 mA **2.** 160 mH **3.** 20.0 V

4. (a) $\frac{500}{\pi}$ Hz (b) 12.0 μC (c) 12.0 mA (d) 72.0 μC

5. (a) $-LK$ (b) $-\frac{Kt^2}{2C}$ (c) $2\sqrt{LC}$

6. $\frac{q}{2N}\sqrt{\frac{3L}{C}}$

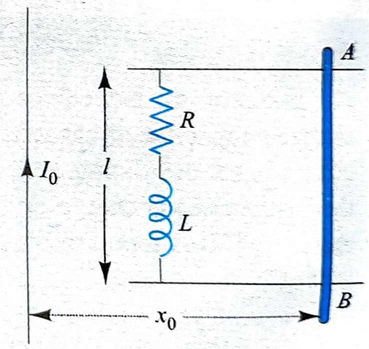
7. (a) 2.4×10^2 W; (b) 1.5×10^2 W; (c) 3.9×10^2 W

8. 40.0 ln 2 ms

Solved Examples

EXAMPLE 5.1

A metal bar AB can slide on two parallel thick metallic rails separated by a distance l. A resistance R and an inductance L are connected to the rails as shown in figure. A long straight wire carrying a constant current I_0 is placed in the plane of rails and perpendicular to them as shown. The bar AB is made to slide on the rails away from the wire. Answer the following questions:



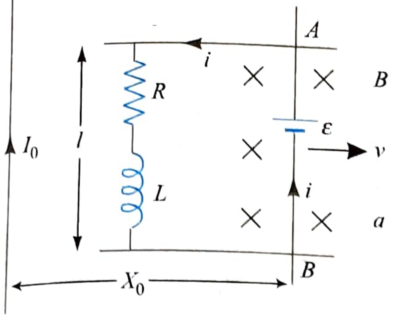
(a) Find a relation among i, di/dt and $d\phi/dt$, where i is the current in the curcuit and ϕ is the flux of the magnetic field due to the long wire through the circuit.

(b) It is observed that at time $t = T$, the metal bar AB is at a distance of $2x_0$ from the long wire and the resistance R carries a current i_1. Obtain an expression for the net charge that has flown through resistance R from $t = 0$ to $t = T$.

(c) The bar is suddenly stopped at time T. The current through resistance R is found to be $i_1/4$ at time $2T$. Find the value of L/R in terms of the other given quantities.

Sol.

(a) When bar AB slides away, the magnetic force acts on free electrons along negative Y-axis, so electrons move from end A to end B, making end A at positive potential relative to end B. So current in the circuit will flow from B to A as shown.



Let ε be the induced emf according to Kirchhoff's second law,

$$-Ri - L\frac{di}{dt} + \varepsilon = 0 \quad \Rightarrow \quad \varepsilon = Ri + L\frac{di}{dt}$$

But $\varepsilon = \dfrac{d\phi}{dt}$ (numerically)

$$\therefore \quad \frac{d\phi}{dt} = Ri + L\frac{di}{dt} \qquad \text{...(i)}$$

This is the required relation.

(b) Equation (i) can be expressed as

$$d\phi = Ridt + L\,di \quad \Rightarrow \quad d\phi = R\,dq + Ldi$$

Integrating, we get

$$\int_0^T d\phi = R\int_0^q dq + L\int_0^{i_1} di$$

$$[\phi]^T_{t=0} = Rq + Li_1$$

∴ Charge flown from $t = 0$ to $t = T$ will be

$$q = \frac{1}{R}[\phi(T) - \phi(0)] - \frac{Li_1}{R} \qquad \text{...(ii)}$$

Change in flux during the displacement of bar from $x = x_0$ to $x = 2x_0$ in time T is

$$\phi(T) - \phi(0) = \int_{x_0}^{2x_0} BdA = \int_{x_0}^{2x_0} \frac{\mu_0 I_0}{2\pi x} l dx$$

$$= \frac{\mu_0 I_0 l}{2\pi}\log_e 2 \qquad \text{...(iii)}$$

∴ From Eq. (ii), charge flown

$$q = \frac{\mu_0 I_0 l}{2\pi R}\log_e 2 - \frac{Li_1}{R} \qquad \text{...(iv)}$$

(c) The equation of decay of current in an LR circuit is given by

$$i = i_0 e^{-Rt/L} \qquad \text{...(v)}$$

Given $i = \dfrac{i_1}{4}$, $i_0 = i_1$, $t = 2T - T = T$

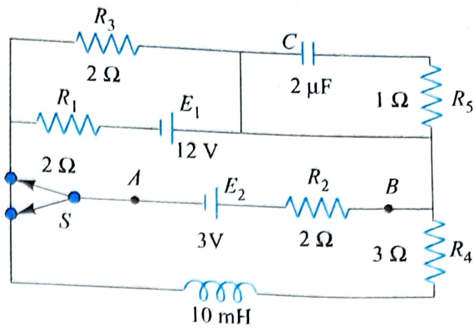
∴ Substituting these values in Eq. (v), we get

$$\frac{i_1}{4} = i_1 e^{-RT/L} \quad \Rightarrow \quad e^{-RT/L} = \frac{1}{4}$$

or $$\frac{RT}{L} = \log_e 4 \quad \therefore \quad \frac{L}{R} = \frac{T}{\log_e 4}$$

EXAMPLE 5.2

A circuit containing a two position switch S is shown in figure.



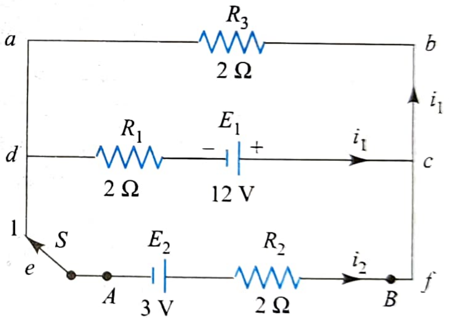
(a) Switch S is in position 1. Find the potential difference $V_A - V_B$ and the rate of production of heat in R_1.

(b) If now switch S is put in position 2 at $t = 0$, find the steady current in R_4 and the time when current in R_4 is half the steady value. Also calculate the energy stored in inductor L at that time.

Sol.

(a) The capacitor offers infinite resistance to dc in the steady state, therefore, the current in capacitor branch is zero. The equivalent circuit, in steady state, when switch is in position (1) is given in figure.

The distribution of current according to Kirchhoff's first law is shown in figure. Applying Kirchhoff's second law to mesh $abcda$,



$$i_1 \times 2 + (i_1 + i_2) \times 2 = 12$$

or $\quad 4i_1 + 2i_2 = 12$ or $2i_1 + i_2 = 6 \qquad$ ...(i)

Applying Kirchhoff's second law to mesh $abfea$,

$$i_2 \times 2 + (i_1 + i_2)\,2 = 3 \text{ or } 2i_1 + 4i_2 = 3 \qquad \text{...(ii)}$$

Solving Eqs (i) and (ii), we get, $i_1 = 3.5$ A, $i_2 = -1$ A

∴ Potential difference between A and B is

$$V_B - V_A = V = E_2 - i_2 R_2 = 3 - (-1) \times 2 = 5 \text{ V}$$

$$\therefore \quad V_A - V_B = -5 \text{ V}$$

(b) When switch S is put in position 2, the equivalent circuit takes the form as shown in figure.

Total resistance of the circuit,

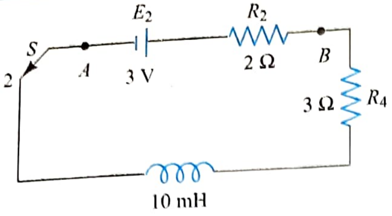
$$R = R_2 + R_4 = 2 + 3 = 5\ \Omega$$

$$L = 10 \text{ mH} = 10 \times 10^{-3} \text{ H}$$

In steady state, there is no role of inductor L.

∴ Steady current

$$i_0 = \frac{E}{R} = \frac{3}{5} = 0.6 \text{ A}$$



The growth of current in RL circuit is given by

$$i = i_0\left(1 - e^{-\frac{R}{L}t}\right) \quad \text{i.e.,} \quad \frac{i}{i_0} = \left(1 - e^{-\frac{R}{L}t}\right)$$

Given $i = \frac{i_0}{2}$, i.e., $\frac{i}{i_0} = \frac{1}{2}$ after time t.

$$\therefore \quad \frac{1}{2} = 1 - e^{-\frac{R}{L}t} \quad \text{or} \quad e^{-\frac{R}{L}t} = \frac{1}{2}$$

i.e., $\frac{R}{L}t = \log_e 2$

i.e., $t = \frac{L}{R}\log_e 2 = \frac{10\times10^{-3}}{5}\log_e 2 = 1.38\times10^{-3}$ s

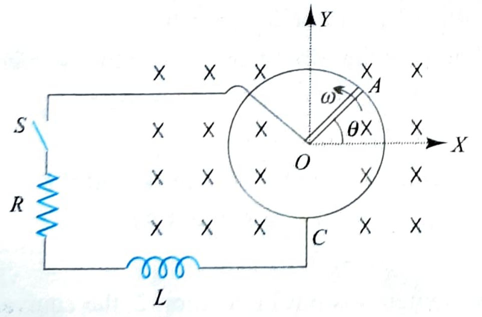
The current at the instant, $i = i_0/2 = 0.3$ A

$\therefore$ Energy stored,

$$U = \frac{1}{2}Li^2 = \frac{1}{2}\times10\times10^{-3}\times(0.3)^2 = 4.5\times10^{-4}\text{ J}$$

EXAMPLE 5.3

A metal rod OA of mass m and length r is kept rotating with a constant angular speed in a vertical plane about a horizontal axis at the end O. The free end A is arranged to slide without friction along a fixed conducting circular ring in the same plane as that of rotation. A uniform and constant magnetic induction is applied perpendicular and into the plane of rotation as shown in figure. An inductor L and an external resistance R are connected through a switch S between point O and point C on the ring to form an electrical circuit. Neglect the resistance of the ring and the rod. Initially, the switch is open.



(a) What is the induced emf across the terminals of the switch?

(b) Switch S is closed at time $t = 0$.

(i) Obtain an expression for the current as a function of time.

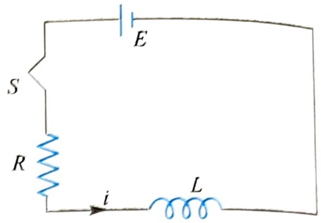
(ii) In the steady state, obtain the time dependence of the torque required to maintain the constant angular speed, given that rod OA was along the positive X-axis at $t = 0$.

Sol.

(a) The emf induced across the whole rod is given by:

$$E = \frac{1}{2}B\omega r^2 = B\omega\left[\frac{x^2}{2}\right]_0^r = \frac{1}{2}B\omega r^2 \qquad \text{...(i)}$$

(b) (i) When the rod rotates anticlockwise (as shown), the end O becomes positive and A negative. As resistance of ring and rod is negligible, therefore, the equivalent circuit is shown in figure. When switch S is closed, let i be the current and di/dt the rate of change of current in the circuit, then from Kirchhoff's second law, the equation of emf is

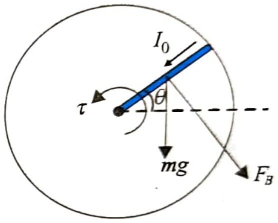


$$E = Ri + L\frac{di}{dt}$$

which upon integration gives $i = \frac{E}{R}(1 - e^{-Rt/L})$

Using Eq. (i), $i = \frac{Br^2\omega}{2R}(1 - e^{-Rt/L})$

(ii) In steady state, current is $I_0 = E/R$



Magnetic force, $F_B = I_0 rB = \frac{ErB}{R}$, $\theta = \omega t$

Required torque

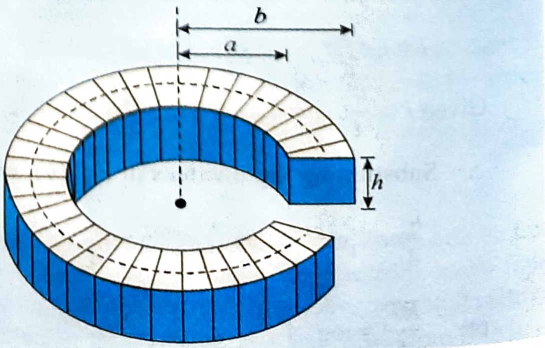
$$\tau = \tau_{\text{due to } F_B} + \tau_{\text{due to mg}}$$

$$\Rightarrow \quad \tau = F_B\frac{r}{2} + mg\frac{r}{2}\cos\theta = \frac{EBr^2}{2R} + \frac{mgr}{2}\cos\omega t$$

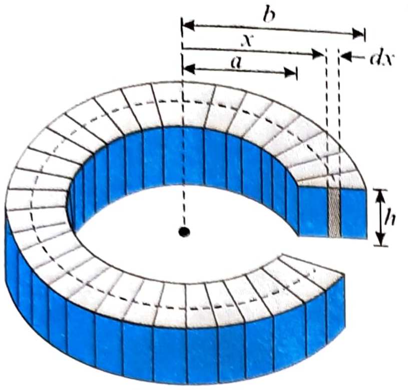
$$= \frac{B^2\omega r^4}{4R} + \frac{mgr\cos\omega t}{2}$$

EXAMPLE 5.4

Figure shows a toroidal solenoid whose cross-section is rectangular in shape. The current through the toroidal winding is I, total number of turns in winding is N, the inside and outside radii of the toroid are a and b respectively and the height of toroid is equal to h. Calculate the inductance of the toroid.



Sol. To find the magnetic flux through the cross section shown in figure, we consider an elemental strip of width dx at a distance x from the axis of toroid as shown in figure.



The magnetic field at a distance x from the axis of the toroid is given as

$$B = \mu_0\left(\frac{N}{2\pi x}\right)I = \frac{\mu_0 NI}{2\pi x}$$

Magnetic flux through an elemental strip of radial thickness dx and height h as shown in figure is given as

$$d\phi = \vec{B}\cdot d\vec{A} = BdA$$

$$\Rightarrow \quad d\phi = \left(\frac{\mu_0 NI}{2\pi x}\right)hdx$$

Total magnetic flux through the cross section of the toroid is given by integrating the above elemental flux within limits from a to b which is given as

$$\phi = \int_a^b \frac{\mu_0 NIh}{2\pi x}\,dx = \frac{\mu_0 NIh}{2\pi}\int_a^b \frac{1}{x}\,dx = \frac{\mu_0 NIh}{2\pi}[\ln x]_a^b$$

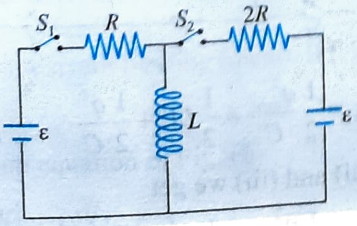
$$\Rightarrow \quad \phi = \frac{\mu_0 NIh}{2\pi}[\ln b - \ln a] = \frac{\mu_0 NIh}{2\pi}\ln\left(\frac{b}{a}\right)$$

Self-inductance of the toroid is given as

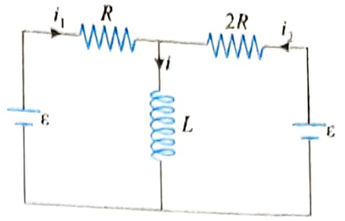
$$\Rightarrow \quad L = \frac{N\phi}{I} = \frac{\mu_0 N^2 h}{2\pi}\ln\left(\frac{b}{a}\right)$$

EXAMPLE 5.5

In the circuits shown in figure, S_1 and S_2 are switches. S_2 remains closed for a long time and S_1 is opened. Now S_1 is also closed. Just after S_1 is closed, find the potential difference (V) across R and di/dt (with sign) in L.



Sol. Before closing S_1, current in the inductor is $i = \varepsilon/2R$. Just after closing S_1, current in inductor will remain same, and other currents are as shown.



For left loop: $\varepsilon - i_1R - L\dfrac{di}{dt} = 0$

or $$\varepsilon = i_1R + L\frac{di}{dt} \qquad \text{...(i)}$$

For right loop: $\varepsilon - i_2(2R) - L\dfrac{di}{dt} = 0$

or $$\varepsilon = i_2(2R) + L\frac{di}{dt} \qquad \text{...(ii)}$$

and $$i = i_1 + i_2 \qquad \text{...(iii)}$$

From (i), (ii) and (iii), $$\frac{di}{dt} = \frac{3\varepsilon - 2Ri}{3L} \qquad \text{...(iv)}$$

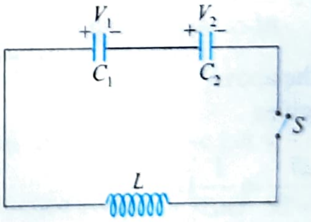
But $i = \dfrac{\varepsilon}{2R}$, hence, $\dfrac{di}{dt} = \dfrac{3\varepsilon - 2R(\varepsilon/2R)}{3L} = \dfrac{2\varepsilon}{3L}$

The potential difference across resistance,

$$V_R = i_1R = \varepsilon - L\left(\frac{di}{dt}\right) = \varepsilon - L\left(\frac{2\varepsilon}{3L}\right) = \frac{\varepsilon}{3}$$

EXAMPLE 5.6

The capacitors shown in the circuit have capacitance $C_1 = C$ and $C_2 = 3C$ and they have been changed to potentials $V_1 = 2V_0$ and $V_2 = 3V_0$ respectively. Switch S is closed to connect them to the inductor L.



(a) Find the maximum current through the inductor.

(b) Find potential difference across C_1 and C_2 when the current in the inductor is maximum.

Sol. **(a)** When current through L is maximum, $\dfrac{dI}{dt} = 0$

(b) Emf induced in L is zero at that instant. At this time a total charge $Q = 9CV_0 - 2CV_0 = 7CV_0$ is distributed on the two capacitors so that potential difference across them is same (V).

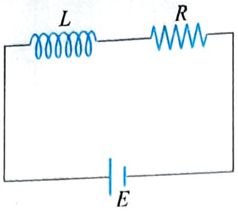
$$CV + 3CV = 7CV_0$$

Energy conservation

$$\frac{1}{2}LI_{\max}^2 = \left[\frac{1}{2}C(2V_0)^2 + \frac{1}{2}3C(3V_0)^2\right] - \left[\frac{1}{2}CV^2 + \frac{1}{3}(3C)V^2\right]$$

Solving $I_{\max} = \dfrac{5V_0}{2}\sqrt{\dfrac{3C}{L}}$

EXAMPLE 5.7



LR circuit in the figure is given. The battery of emf ε has negligible internal resistance. The inductor coil has a piece of soft iron inside it. After reaching steady state the piece of soft iron is abruptly pulled out suddenly so that the inductance of the inductor decreases to nL with $n < 1$ with battery remaining connected. Find:

(a) Current as a function of time if at $t = 0$ is the instant when the soft iron piece is pulled out.

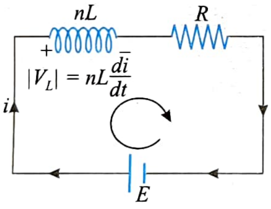
(b) The work done in pulling out the piece.

Sol.

(a) Initially the circuit is at steady state. At $t = 0$, steady state current in the circuit is $i_0 = \dfrac{E}{R}$. When the piece of iron is pulled out abruptly. The inductance will reduce to $nL\,(n<1)$. The induced coil will try to keep its flux constant.

Hence $L \cdot i_0 = nLi_0'$ which gives $i_0' = \dfrac{i_0}{n} = \dfrac{E}{nR}$.

It means the current through inductor coil will increase. Let i be the current at time t in circuit as shown in figure then by using Kirchhoff's law in the loop we have,



$$E - nL\left(\frac{di}{dt}\right) - iR = 0$$

$$\Rightarrow \quad \frac{di}{E - iR} = \frac{1}{nL}dt$$

Integrating the above expression, gives

$$\int_{i_0/n}^{i} \frac{di}{E - iR} = \frac{1}{nL}\int_0^t dt$$

$$i = \frac{E}{R} - \frac{E}{R}\left(1 - \frac{1}{n}\right)e^{-Rt/nL}$$

(b) Work done in pulling out the piece is given as

$$W = U_f - U_i = \frac{1}{2}L_f i_f^2 - \frac{1}{2}L_i i_i^2$$

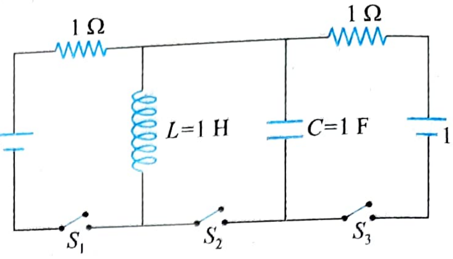
$$\Rightarrow \quad W = \frac{1}{2}(nL)\left(\frac{E}{nR}\right)^2 - \frac{1}{2}(L)\left(\frac{E}{R}\right)^2$$

$$\Rightarrow \quad W = \frac{1}{2}L\left(\frac{E}{R}\right)^2\left(\frac{1}{n} - 1\right)$$

$$\Rightarrow \quad W = \frac{1}{2}L\left(\frac{E}{R}\right)^2\left(\frac{1-n}{n}\right)$$

EXAMPLE 5.8

In the circuit shown switches S_1 and S_3 have been closed for 1 s and S_2 remained open. Just after 1 s switch S_2 is closed, and S_1 and S_3 are opened.



Find after that instant ($t = 0$):

(a) Calculate maximum current in the inductor coil and maximum charge in capacitor.

(b) The charge on the upper plate of the capacitor as a function of time.

Sol.

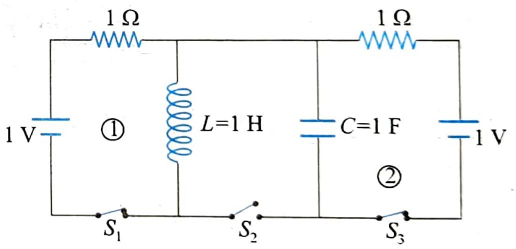
(a) When the switches S_1 and S_3 are closed and S_2 is open. The loop (1) from an ideal L-R circuit while the loop (2) form an ideal CR circuit.

Charge in capacitor, $q = CE\left(1 - e^{-t/RC}\right)$

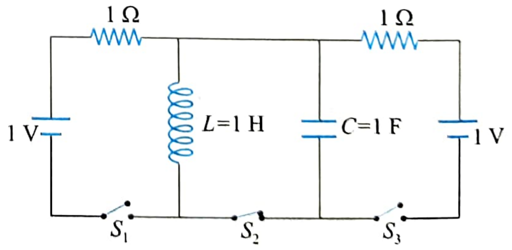
$$\Rightarrow \quad q = 1 \times 1\left(1 - e^{-1/1\times1}\right) \Rightarrow q = \left(1 - \frac{1}{e}\right) \qquad \ldots(\text{i})$$

The current in inductor,

$$i = \frac{E}{R}\left(1 - e^{-tR/L}\right) \Rightarrow i = \left(1 - \frac{1}{e}\right) \qquad \ldots(\text{ii})$$



Now the switches S_1 and S_3 are opened and the switch S_2 is closed. Total energy (magnetic + electrical) energy will remain constant.



$$\frac{1}{2}Li_{\max}^2 = \frac{1}{2}\frac{q_{\max}^2}{C} = \frac{1}{2}Li^2 + \frac{1}{2}\frac{q^2}{C} \qquad \ldots(\text{iii})$$

From (i), (ii) and (iii) we get

$$\frac{1}{2}Li_{\max}^2 = \frac{1}{2}\frac{q_{\max}^2}{C} = \frac{1}{2}L\left(1 - \frac{1}{e}\right)^2 + \frac{1}{2C}\left(1 - \frac{1}{e}\right)^2$$

As $L = 1\text{H}$ and $C = 1\text{F}$

$$\frac{1}{2}1.i_{\max}^2=\frac{1}{2}\frac{q_{\max}^2}{1}=\frac{1}{2}1.\left(1-\frac{1}{e}\right)^2+\frac{1}{2.1}\left(1-\frac{1}{e}\right)^2 \qquad \text{...(iv)}$$

From Eq. (iv) we get

$$i_{\max}^2=2.\left(1-\frac{1}{e}\right)^2 \quad \Rightarrow i_{\max}=\sqrt{2}\left(1-\frac{1}{e}\right)$$

$$q_{\max}^2=2.\left(1-\frac{1}{e}\right)^2 \quad \Rightarrow q_{\max}=\sqrt{2}\left(1-\frac{1}{e}\right)$$

(b) After closing the switch S_2 and opening the switches S_1 and S_3. Let us define this time as $t = 0$. The middle loop forms a LC circuit. In LC circuit we can express the charge and current in relation with time as,

$$q=q_{\max}\sin(\omega t+\phi)=\sqrt{2}\left(1-\frac{1}{e}\right)\sin(\omega t+\phi) \qquad \text{...(iv)}$$

$$i=(q_{\max})\omega\cos(\omega t+\phi)=\sqrt{2}\left(1-\frac{1}{e}\right)\omega\cos(\omega t+\phi) \qquad \text{...(v)}$$

$$\text{where, } \omega=\frac{1}{\sqrt{LC}}=\frac{1}{\sqrt{1\times 1}}=1$$

At $t = 0$, the charge in capacitor is $\left(1-\frac{1}{e}\right)$, from Eq. (iv) we have

$$\left(1-\frac{1}{e}\right)=\sqrt{2}\left(1-\frac{1}{e}\right)\sin\phi$$

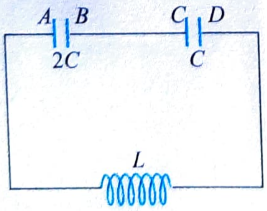
$$\Rightarrow \sin\phi=\frac{1}{\sqrt{2}} \Rightarrow \phi=\frac{\pi}{4}$$

Hence charge in capacitor as a function of time

$$q=\sqrt{2}\left(1-\frac{1}{e}\right)\sin\left(t+\frac{\pi}{4}\right)$$

EXAMPLE 5.9

Two capacitors of capacitance $2C$ and C, respectively, are connected in series with an inductor of inductance L. Initially the capacitors have charge such that $V_B - V_A = 4V_0$ and $V_C - V_D = V_0$. Initial current in the circuit is zero.



Find

(a) the maximum current that will flow in the circuit,

(b) the potential difference across each capacitor at that instant,

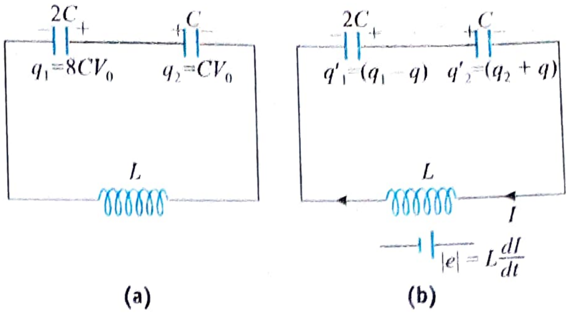
(c) the equation of current flowing toward left in the inductor.

Sol.

(a) Applying loop equation in Fig. (b)

$$\frac{q_1'}{2C}-\frac{q_2'}{C}-L\frac{dI}{dt}=0$$

$$\text{or} \quad L\frac{dI}{dt}=\left(\frac{q_1-2q_2}{2C}\right)-\frac{3q}{2C} \qquad \text{...(i)}$$



(a) (b)

Differentiating (i), we get

$$L\frac{d^2I}{dt^2}=-\frac{3}{2C}I \qquad \text{...(ii)}$$

Solution of (ii) is $I = I_0\sin(\omega t+\phi)$

$$\text{where } \omega^2=\frac{3}{2CL} \Rightarrow \omega=\sqrt{\frac{3}{2CL}}$$

but initial current is zero, i.e., at $t = 0$, $I = 0$, putting this above we get $I = I_0 \sin\omega t$ ...(iii)

$$\Rightarrow \quad \frac{dI}{dt}=I_0\omega\cos\omega t$$

Putting dI/dt in Eq. (i),

$$LI_0\omega\cos\omega t=\left(\frac{q_1-2q_2}{2C}\right)-\frac{3q}{2C} \qquad \text{...(iv)}$$

At $t = 0$, $q = 0$; putting this in the above equation, we get

$$LI_0\omega=\left(\frac{q_1-2q_2}{2C}\right)\Rightarrow LI_0\sqrt{\frac{3}{2CL}}=\left(\frac{8CV_0-2CV_0}{2C}\right)$$

$$\Rightarrow \quad I_0=V_0\sqrt{\frac{6C}{L}} \rightarrow \text{the maximum value of current}$$

(b) From (iii) maximum current occurs at

$$\omega t=\pi/2, 3\pi/2, \ldots etc,$$

and at these instants from (i) we get $q=\left(\frac{q_1-2q_2}{3}\right)$

Potential difference across each capacitor at this instant,

$$V_1=\frac{q_1'}{2C}=\frac{1}{2C}\left[q_1-\left(\frac{q_1-2q_2}{3}\right)\right]$$

$$V_1=\frac{(q_1+q_2)}{3C}=3V_0$$

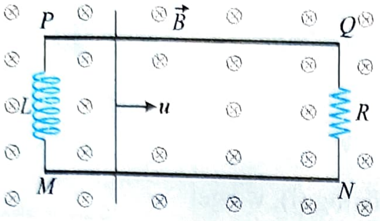
$$V_2=\frac{q_2'}{C}=\frac{1}{C}\left[q_2-\left(\frac{q_1-2q_2}{3}\right)\right]$$

$$V_2=\frac{(q_1+q_2)}{3C}=3V_0$$

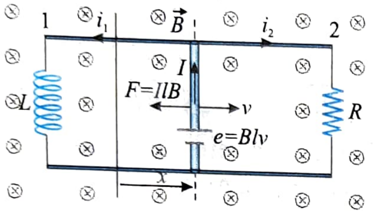
(c) Equation of current: $I=I_0\sin\omega t=V_0\sqrt{\frac{6C}{L}}\sin\sqrt{\frac{3}{2CL}}t$

EXAMPLE 5.10

In figure, a rod of length l and mass m moves with an initial velocity u on a fixed frame containing inductor L and resistance R. PQ and MN are smooth conducting wires. There is a uniform magnetic field of strength B. Initially, there is no current in the inductor. Find the total charge flown through the inductor by the time, velocity of rod becomes v_f and the rod has travelled a distance x.



Sol. $I = i_1 + i_2$



$$\frac{Blv}{R} = i_2 \Rightarrow Blv = L\frac{di_1}{dt}$$

$$m\frac{dv}{dt} = -IlB = -\left(i_1 + \frac{Blv}{R}\right)lB$$

$$\Rightarrow \quad m\int dv = -lB\int i_1 dt - \frac{B^2l^2}{R}\int v dt$$

$$\Rightarrow \quad m\left(v_f - u\right) = -lBQ - \frac{B^2l^2}{R}x$$

(v_f = velocity when it has moved a distance x)

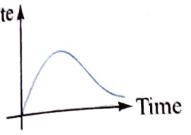
$$\Rightarrow \quad Q = \frac{-\frac{B^2l^2}{R}x - m\left(v_f - u\right)}{Bl}$$

Exercises

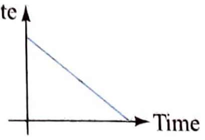
Single Correct Answer Type

1. In an LR circuit connected to a battery, the rate at which energy is stored in the inductor is plotted against time during the growth of current in the circuit. Which of the following best represents the resulting curve?

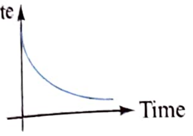
(1)



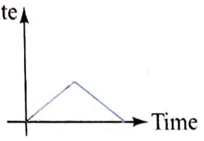
(2)



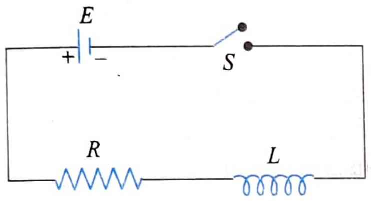
(3)



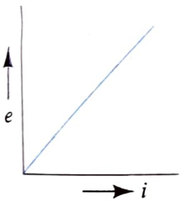
(4)



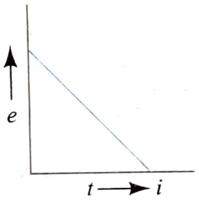
2. Switch S of the circuit shown in figure is closed at $t = 0$. If e denotes the induced emf in L and i the current flowing through the circuit at time t, then which of the following graphs correctly represents the variation of e with i?



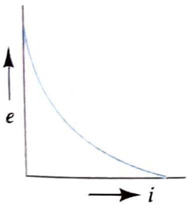
(1)



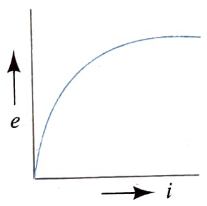
(2)



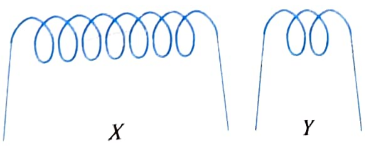
(3)



(4)

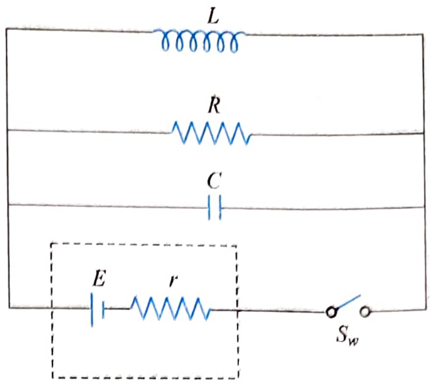


3. A mutual inductor consists of two coils X and Y as shown in figure in which one-quarter of the magnetic flux produced by X links with Y, giving a mutual inductance M. What will be the mutual inductance when Y is used as the primary?



(1) $M/4$
(2) $M/2$
(3) M
(4) $2M$

4. A pure inductor L, a capacitor C and a resistance R are connected across a battery of emf E and internal resistance r as shown in figure. Switch S_w is closed at $t = 0$, select the correct alternative(s).



(1) current through resistance R is zero all the time
(2) current through resistance R is zero at $t = 0$ and $t \to \infty$
(3) maximum charge stored in the capacitor is CE
(4) maximum energy stored in the inductor is equal to the maximum energy stored in the capacitor

5. A coil carrying a steady current is short-circuited. The current in it decreases α times in time t_0. The time constant of the circuit is

(1) $\tau = t_0 \ln \alpha$
(2) $\tau = \dfrac{t_0}{\ln \alpha}$
(3) $\tau = \dfrac{t_0}{\alpha}$
(4) $\tau = \dfrac{t_0}{\alpha - 1}$

6. A solenoid has 2000 turns wound over a length of 0.3 m. Its cross-sectional area is equal to 1.2×10^{-3} m^2. Around its central cross-section a coil of 300 turns is wound. If an initial current of 2 A flowing in the solenoid is reversed in 0.25 s, the emf induced in the coil is

(1) 0.6 mV
(2) 60 mV
(3) 48 mV
(4) 0.48 mV

7. The time constant of an inductance coil is 2×10^{-3} s. When a 90-Ω resistance is joined in series, the same constant becomes 0.5×10^{-3} s. The inductance and resistance of the coil are

(1) 30 mH; 30 Ω
(2) 60 mH; 30 Ω
(3) 30 mH; 60 Ω
(4) 60 mH; 60 Ω

8. The coefficient of mutual inductance of two circuits A and B is 3 mH and their respective resistances are 10 and 4 Ω. How much current should change in 0.02 s in circuit A, so that the induced current in B should be 0.0060 A?

(1) 0.24 A
(2) 1.6 A
(3) 0.18 A
(4) 0.16 A

9. An emf of 15 V is applied in a circuit containing 5 H inductance and 10 Ω resistance. The ratio of the currents at time $t = \infty$ and $t = 1$ s is

(1) $\dfrac{e^{1/2}}{e^{1/2} - 1}$
(2) $\dfrac{e^2}{e^2 - 1}$
(3) $1 - e^{-1}$
(4) e^{-1}

10. A coil of inductance 0.20 H is connected in series with a switch and a cell of emf 1.6 V. The total resistance of the

circuit is 4.0 Ω. What is the initial rate of growth of the current when the switch is closed?
(1) 0.050 As^{-1} (2) 0.40 As^{-1}
(3) 0.13 As^{-1} (4) 8.0 As^{-1}

11. A straight solenoid of length 1 m has 5000 turns in the primary and 200 turns in the secondary. If the area of cross section is 4 cm^2, the mutual inductance will be
(1) 503 H (2) 503 mH
(3) 503 μH (4) 5.03 H

12. The approximate formula expressing the formula of mutual inductance of two thin coaxial loops of the same radius a when their centers are separated by a distance l with $l \gg a$ is
(1) $\frac{1}{2}\frac{\mu_0 \pi a^4}{l^3}$ (2) $\frac{1}{2}\frac{\mu_0 a^4}{l^2}$
(3) $\frac{\mu_0}{4\pi}\frac{\pi a^4}{l^2}$ (4) $\frac{\mu_0}{\pi}\frac{a^4}{l^3}$

13. The length of a thin wire required to manufacture a solenoid of length l = 100 cm and inductance L = 1 mH, if the solenoid's cross-sectional diameter is considerably less than its length is
(1) 1 km (2) 0.10 km
(3) 0.010 km (4) 10 km

14. Current in a coil of self-inductance 2.0 H is increasing as $i = 2 \sin t^2$. The amount of energy spent during the period when the current changes from 0 to 2 A is
(1) 1 J (2) 2 J
(3) 3 J (4) 4 J

15. A long solenoid having 200 turns per centimeter carries a current of 1.5 A. At the center of the solenoid is placed a coil of 100 turns of cross-sectional area 3.14×10^{-4} m^2 having its axis parallel to the field produced by the solenoid. When the direction of current in the solenoid is reversed within 0.05 s, the induced emf in the coil is
(1) 0.48 V (2) 0.048 V
(3) 0.0048 V (4) 48 V

16. Two coils are at fixed locations. When coil 1 has no current and the current in coil 2 increases at the rate of 15.0 As^{-1}, the emf in coil 1 is 25 mV, when coil 2 has no current and coil 1 has a current of 3.6 A, the flux linkage in coil 2 is
(1) 16 mWb (2) 10 mWb
(3) 4.00 mWb (4) 6.00 mWb

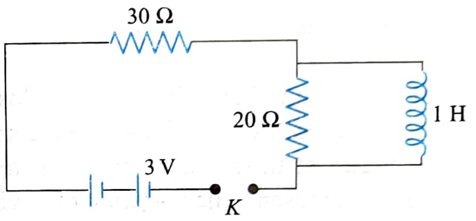
17. A current i_0 is flowing through an L-R circuit of time constant t_0. The source of the current is switched off at time $t = 0$. Let r be the value of $(-di/dt)$ at time $t = 0$. Assuming this rate to be constant, the current will reduce to zero in a time interval of
(1) t_0 (2) et_0
(3) $\frac{t_0}{e}$ (4) $\left(1-\frac{1}{e}\right)t_0$

18. A small coil of radius r is placed at the center of a large coil of radius R, where $R >> r$. The two coils are coplanar. The mutual inductance between the coils is proportional to
(1) r/R (2) r^2/R
(3) r^2/R^2 (4) r/R^2

19. The length of a wire required to manufacture a solenoid of length l and self-induction L is (cross-sectional area is negligible)
(1) $\sqrt{\frac{2\pi L l}{\mu_0}}$ (2) $\sqrt{\frac{\mu_0 L l}{4\pi}}$
(3) $\sqrt{\frac{4\pi L l}{\mu_0}}$ (4) $\sqrt{\frac{\mu_0 L l}{2\pi}}$

20. The total heat produced in resistor R in an RL circuit when the current in the inductor decreases from I_0 to 0 is
(1) LI_0^2 (2) $\frac{1}{2}LI_0^2$
(3) $\frac{3}{2}LI_0^2$ (4) $\frac{1}{3}LI_0^2$

21. In the circuit (figure), the final current through 30 Ω resistance when circuit is completed is

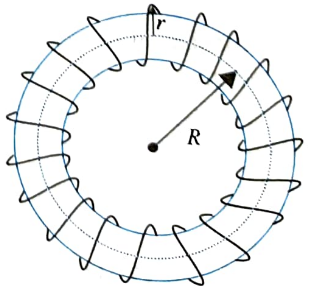


(1) 3 A (2) 0.1 A
(3) 5 A (4) 0.5 A

22. The inductance L of a solenoid of length l, whose windings are made of material of density D and resistivity ρ, is (the winding resistance is R)
(1) $\frac{\mu_0}{4\pi l}\frac{Rm}{\rho D}$ (2) $\frac{\mu_0}{4\pi R}\frac{lm}{\rho D}$
(3) $\frac{\mu_0}{4\pi l}\frac{R^2 m}{\rho D}$ (4) $\frac{\mu_0}{2\pi R}\frac{lm}{\rho D}$

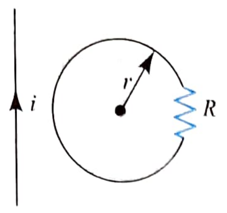
23. The capacitance in an oscillatory LC circuit is increased by 1%. The change in inductance required to restore its frequency of oscillation is to
(1) decrease it by 0.5% (2) increase it by 1%
(3) decrease it by 1% (4) decrease it by 2%

24. A toroid is wound over a circular core. Radius of each turn is r and radius of toroid is R (>> r). The coefficient of self-inductance of the toroid is given by



(1) $L = \frac{\mu_0 N r^2}{2R}$ (2) $L = \frac{\mu_0 N r}{2R}$
(3) $L = \frac{\mu_0 N r^2}{R}$ (4) $L = \frac{\mu_0 N^2 r^2}{2R}$

25. In figure, the mutual inductance of a coil and a very long straight wire is M, the coil has resistance R and self-inductance L. The current in the wire varies according to the law $i = at$, where a is a constant and t is the time, the time dependence of current in the coil is



(1) $\dfrac{M}{aR}$

(2) $MaR\, e^{-Rt/L}$

(3) $\dfrac{M}{R} e^{-tR/L}$

(4) $\dfrac{Ma}{R}(1 - e^{-tR/L})$

26. A closed circuit consists of a resistor R; inductor of inductance L and a source of emf E are connected in series. If the inductance of the coil is abruptly decreased to $L/4$ (by removing its magnetic core), the new current immediately after this moment is (before decreasing the inductance the circuit is in steady state)

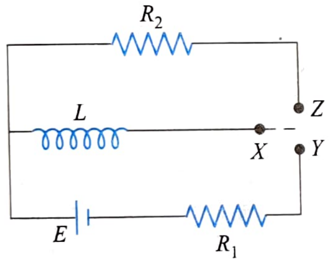
(1) zero

(2) $\dfrac{E}{R}$

(3) $4\dfrac{E}{R}$

(4) $\dfrac{E}{4R}$

27. In the circuit shown (figure), X is joined to Y for a long time and then X is joined to Z. The total heat produced in R_2 is



(1) $\dfrac{LE^2}{2R_1^2}$

(2) $\dfrac{LE^2}{2R_2^2}$

(3) $\dfrac{LE^2}{2R_1R_2}$

(4) $\dfrac{LE^2 R_2}{2R_1^3}$

28. A horizontal ring of radius $r = \dfrac{1}{2}$ m is kept in a vertical constant magnetic field 1 T. The ring is collapsed from maximum area to zero area in 1 s. Then the emf induced in the ring is

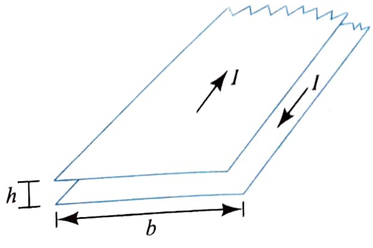
(1) 1 V

(2) ($\pi/4$) V

(3) ($\pi/2$) V

(4) π V

29. Calculate the inductance of a unit length of a double tape line as shown in figure if the tapes are separated by a distance h which is considerably less than their width b.



(1) $\dfrac{\mu_0 h}{b}$

(2) $\dfrac{\mu_0 h}{2b}$

(3) $\dfrac{2\mu_0 h}{b}$

(4) $\dfrac{\sqrt{2}\,\mu_0 h}{b}$

30. Find the inductance of a unit length of two parallel wires, each of radius a, whose centers are a distance d apart and carry equal currents in opposite directions. Neglect the flux within the wire.

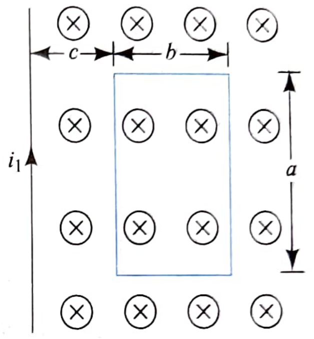
(1) $\dfrac{\mu_0}{2\pi}\ln\left(\dfrac{d-a}{a}\right)$

(2) $\dfrac{\mu_0}{\pi}\ln\left(\dfrac{d-a}{a}\right)$

(3) $\dfrac{3\mu_0}{\pi}\ln\left(\dfrac{d-a}{a}\right)$

(4) $\dfrac{\mu_0}{3\pi}\ln\left(\dfrac{d-a}{a}\right)$

31. Figure shows a rectangular coil near a long wire. The mutual inductance of the combination is



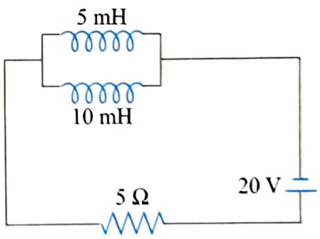
(1) $\dfrac{\mu_0 a}{2\pi}\ln\left(1-\dfrac{b}{c}\right)$

(2) $\dfrac{\mu_0 a}{2\pi}\ln\left(1+\dfrac{b}{c}\right)$

(3) $\dfrac{\mu_0 a}{\pi}\ln\left(1+\dfrac{b}{c}\right)$

(4) $\dfrac{\mu_0 a}{\sqrt{2\pi}}\ln\left(1+\dfrac{b}{c}\right)$

32. In the given circuit (figure), current through the 5 mH inductor in steady state is



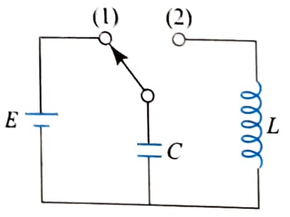
(1) $\dfrac{2}{3}$A

(2) $\dfrac{8}{3}$A

(3) $\dfrac{1}{3}$A

(4) $\dfrac{2}{3}$A

33. In the following electrical network at $t < 0$ (figure), key is placed on (1) till the capacitor got fully charged. Key is placed on (2) at $t = 0$. Time when the energy in both the capacitor and the inductor will be same for the first time is



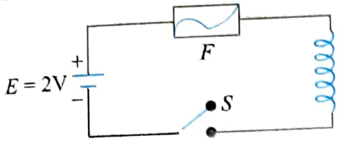
(1) $\dfrac{\pi}{4}\sqrt{LC}$

(2) $\dfrac{3\pi}{4}\sqrt{LC}$

(3) $\dfrac{\pi}{3}\sqrt{LC}$

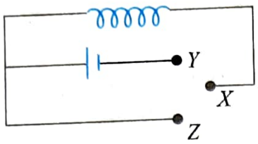
(4) $\dfrac{2\pi}{3}\sqrt{LC}$

34. In the circuit shown (figure), the cell is ideal. The coil has an inductance of 4 H and zero resistance. F is a fuse of zero resistance and will blow when the current through it reaches 5 A. The switch is closed at $t = 0$. The fuse will blow



(1) almost at once
(2) after 2 s
(3) after 5 s
(4) after 10 s

35. In the circuit shown (figure), the coil has inductance and resistance. When X is joined to Y, the time constant is τ during the growth of current.



When the steady state is reached, heat is produced in the coil at a rate P. X is now joined to Z. After joining X and Z:

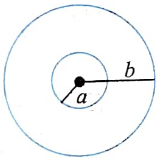
(1) The total heat produced in the coil is $P\tau$

(2) The total heat produced in the coil is $\frac{1}{2}P\tau$

(3) The total heat produced in the coil is $2P\tau$

(4) The data given are not sufficient to reach a conclusion

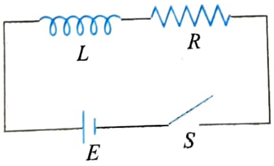
36. Two concentric and coplanar coils have radii a and b ($>> a$) as shown in figure. Resistance of the inner coil is R. Current in the outer coil is increased from 0 to i, then the total charge circulating the inner coil is



(1) $\frac{\mu_0 i a^2 \pi}{2Rb}$
(2) $\frac{\mu_0 iab}{2R}$

(3) $\frac{\mu_0 iab}{2a}\frac{\pi b^2}{R}$
(4) $\frac{\mu_0 ib}{2\pi R}$

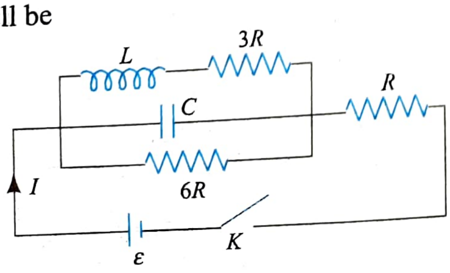
37. In the circuit shown in figure, switch S is closed at time $t = 0$. The charge that passes through the battery in one time constant is



(1) $\frac{eR^2E}{L}$
(2) $E\left(\frac{L}{R}\right)$

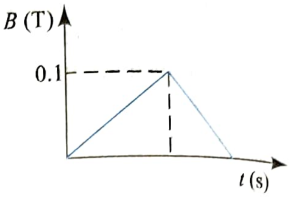
(3) $\frac{EL}{eR^2}$
(4) $\frac{eL}{ER}$

38. In the given circuit diagram (figure), key K is switched on at $t = 0$. The ratio of current i through the cell at $t = 0$ to that at $t = \infty$ will be



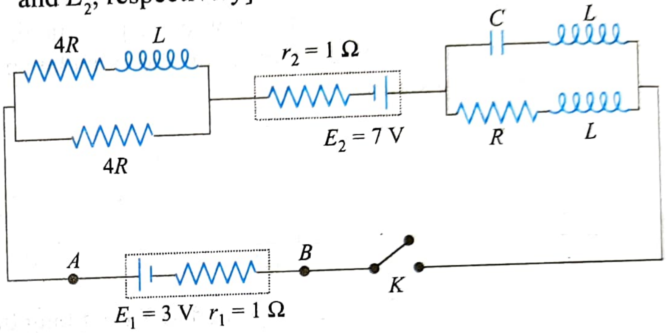
(1) 3 : 1
(2) 1 : 3
(4) 2 : 1

39. A closed loop of cross-sectional area 10^{-2} m^2 which has inductance $L = 10$ mH and negligible resistance is placed in a time-varying magnetic field. Figure shows the variation of B with time for the interval 4 s. The field is perpendicular to the plane of the loop (given at $t = 0$, $B = 0$, $I = 0$). The value of the maximum current induced in the loop is



(1) 0.1 mA
(2) 10 mA
(3) 100 mA
(4) Data insufficient

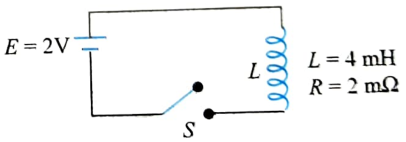
40. In figure, key K is closed at $t = 0$. After a long time, the potential difference between A and B is zero, the value of R will be [$r_1 = r_2 = 1\ \Omega$, $E_1 = 3$ V and $E_2 = 7$ V, $C = 2\ \mu$F, $L =$ 4 mH, where r_1 and r_2 are the internal resistances of cells E_1 and E_2, respectively].



(1) $\frac{4}{3}\ \Omega$
(2) $\frac{4}{9}\ \Omega$

(3) $\frac{2}{3}\ \Omega$
(4) 4 Ω

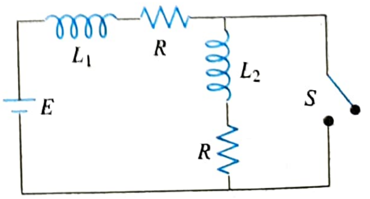
41. The cell in the circuit shown in figure is ideal. The coil has an inductance of 4 mH and a resistance of 2 mΩ. The switch is closed at $t = 0$. The amount of energy stored in the inductor at $t = 2$ s is (take $e = 3$)



(1) $\frac{4}{3}$ J
(2) $\frac{8}{9} \times 10^3$ J

(3) $\frac{8}{3} \times 10^{-3}$ J
(4) 2×10^3 J

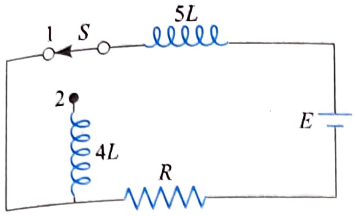
42. Switch S shown in figure is closed for $t < 0$ and is opened at $t = 0$. When currents through L_1 and L_2 are equal, their common value is



(1) $\frac{E}{R}$
(2) $\frac{E(L_2 + L_1)}{RL_1}$

(3) $\frac{EL_1}{R(L_1 + L_2)}$
(4) $\frac{E}{R}\frac{(L_1 + L_2)}{L_2}$

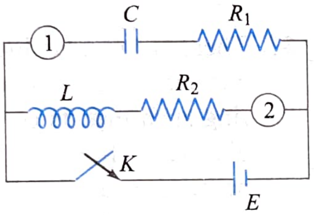
43. In the circuit shown in figure, switch S is shifted to position 2 from position 1 at $t = 0$, having been in position 1 for a long time. The current in the circuit just after shifting of switch will be (battery and both the inductors are ideal)



(1) $\frac{4}{5}\frac{E}{R}$ (2) $\frac{5}{4}\frac{E}{R}$

(3) $\frac{5}{9}\frac{E}{R}$ (4) $\frac{E}{R}$

44. In the circuit of figure, (1) and (2) are ammeters. Just after key K is pressed to complete the circuit, the reading is



(1) maximum in both 1 and 2

(2) zero in both 1 and 2

(3) zero in 1, minimum in 2

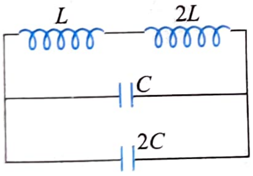
(4) maximum in 1, zero in 2

5. A wire of fixed length is wound on a solenoid of length l and radius r. Its self-inductance is found to be L. Now, if the same wire is wound on a solenoid of length $l/2$ and radius $r/2$, then the self-inductance will be

(1) $2L$ (2) L

(3) $4L$ (4) $8L$

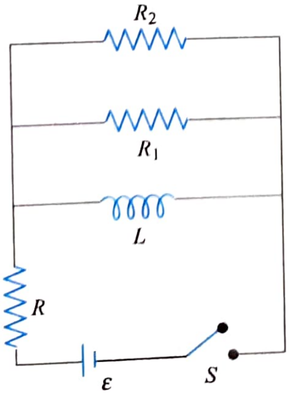
46. The frequency of oscillation of current in the inductance is



(1) $\frac{1}{3\sqrt{LC}}$ (2) $\frac{1}{6\pi\sqrt{LC}}$

(3) $\frac{1}{\sqrt{LC}}$ (4) $\frac{1}{2\pi\sqrt{LC}}$

47. In figure switch S is closed for a long time. At $t = 0$, if it is opened, then



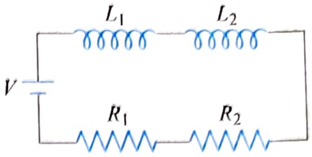
(1) total heat produced in resistor R after opening the switch is $\frac{1}{2}\frac{L\varepsilon}{R^2}$

(2) total heat produced in resistor R_1 after opening the switch is $\frac{1}{2}\frac{L\varepsilon^2}{R^2}\left(\frac{R_1}{R_1 + R_2}\right)$

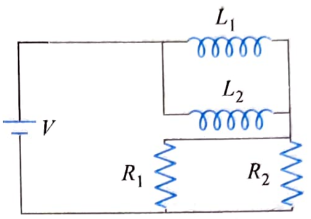
(3) heat produced in resistor R_1 after opening the switch is $\frac{1}{2}\frac{R_2 L\varepsilon^2}{(R_1 + R_2)R^2}$

(4) no heat will be produced in R_1

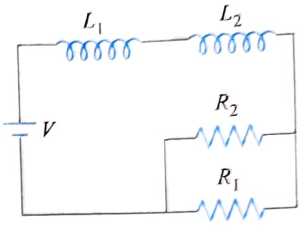
48. Given $L_1 = 1$ mH, $R_1 = 1\ \Omega$, $L_2 = 2$ mH, $R_2 = 2\ \Omega$



(i)



(ii)



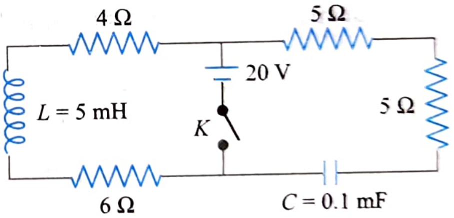
(iii)

Neglecting mutual inductance, the time constants (in ms) for circuits (i), (ii), and (iii) are

(1) $1, 1, \frac{9}{2}$ (2) $\frac{9}{4}, 1, 1$

(3) 1, 1, 1 (4) $1, \frac{9}{4}, 1$

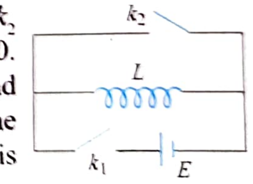
49. In the circuit shown in figure, key K is closed at $t = 0$, the current through the key at the instant $t = 10^{-3}\ln 2$ s is

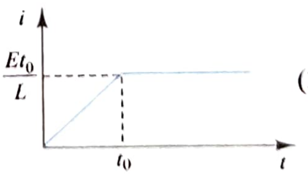


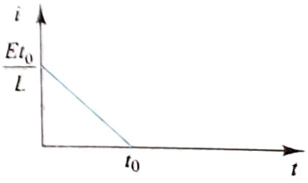
(1) 2 A (2) 3.5 A

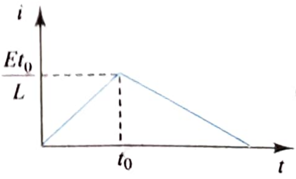
(3) 2.5 A (4) 0

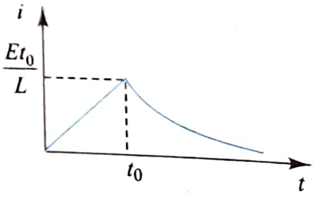
50. In the circuit shown in figure, switch k_2 is open and switch k_1 is closed at $t = 0$. At time $t = t_0$, switch k_1 is opened and switch k_2 is simultaneously closed. The variation of inductor current with time is



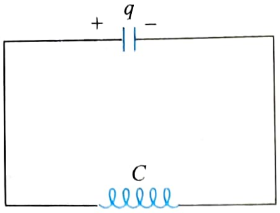
(1)


(2)


(3)


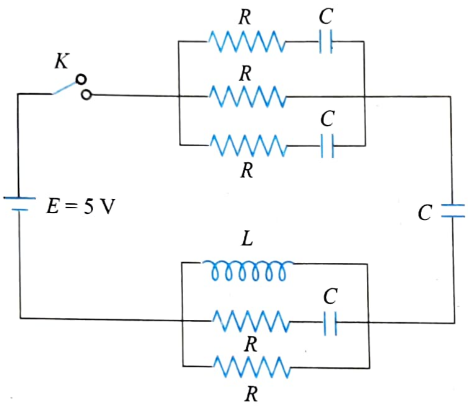
(4)


51. In an LC circuit shown in figure, $C = 1$ F, $L = 4$ H. At time $t = 0$, charge in the capacitor is 4 C and it is decreasing at the rate of $\sqrt{5}$ C s^{-1}. Choose the correct statement.



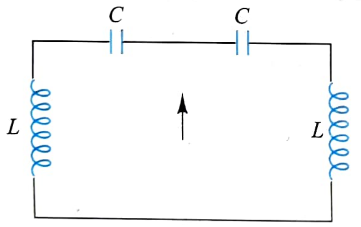
(1) Maximum charge in the capacitor can be 6 C
(2) Maximum charge in the capacitor can be 8 C
(3) Charge in the capacitor will be maximum after time 3 $\sin^{-1}(2/3)$ s
(4) None of these

52. The current passing through the battery immediately after key K is closed [it is given that initially all the capacitors are uncharged (given that $R = 6\ \Omega$ and $C = 4\ \mu$F)] is



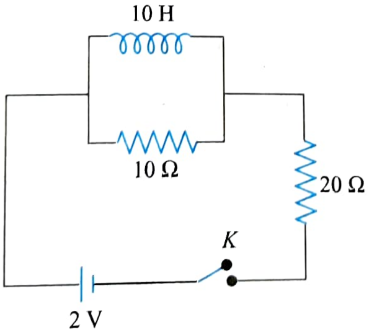
(1) 1 A
(2) 5 A
(3) 3 A
(4) 2 A

53. The natural frequency of the circuit shown in figure is



(1) $\dfrac{1}{\sqrt{LC}}$
(2) $\dfrac{1}{\sqrt{2LC}}$
(3) $\dfrac{2}{\sqrt{LC}}$
(4) none of these

54. Two resistors of 10 Ω and 20 Ω and an ideal inductor of 10 H are connected to a 2 V battery as shown in figure. Key K is inserted at time $t = 0$. The initial ($t = 0$) and final ($t \to \infty$) currents through the battery are



(1) $\dfrac{1}{15}$ A, $\dfrac{1}{10}$ A
(2) $\dfrac{1}{10}$ A, $\dfrac{1}{15}$ A
(3) $\dfrac{2}{15}$ A, $\dfrac{1}{10}$ A
(4) $\dfrac{1}{15}$ A, $\dfrac{2}{25}$ A

55. A simple LR circuit is connected to a battery at time $t = 0$. The energy stored in the inductor reaches half its maximum value at time

(1) $\dfrac{R}{L}\ln\left[\dfrac{\sqrt{2}}{\sqrt{2}-1}\right]$
(2) $\dfrac{L}{R}\ln\left[\dfrac{\sqrt{2}-1}{\sqrt{2}}\right]$
(3) $\dfrac{L}{R}\ln\left[\dfrac{\sqrt{2}}{\sqrt{2}-1}\right]$
(4) $\dfrac{R}{L}\ln\left[\dfrac{\sqrt{2}-1}{\sqrt{2}}\right]$

56. A square conducting loop of side L is situated in gravity-free space. A small conducting circular loop of radius r ($r \ll L$) is placed at the center of the square loop, with its plane perpendicular to the plane of the square loop. The mutual inductance of the two coils is

(1) $\dfrac{2\sqrt{2}\mu_0 I}{L}r^2$
(2) $\dfrac{\sqrt{2}\mu_0 I_0}{L}r^2$
(3) 0
(4) none of these

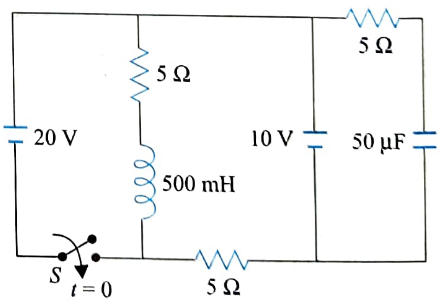
57. There is a conducting ring of radius R. Another ring having current i and radius r ($r << R$) is kept on the axis of bigger ring such that its center lies on the axis of bigger ring at a distance x from the center of bigger ring and its plane is perpendicular to that axis. The mutual inductance of the bigger ring due to the smaller ring is

(1) $\dfrac{\mu_0 \pi R^2 r^2}{(R^2+x^2)^{3/2}}$
(2) $\dfrac{\mu_0 \pi R^2 r^2}{4(R^2+x^2)^{3/2}}$
(3) $\dfrac{\mu_0 \pi R^2 r^2}{16(R^2+x^2)^{3/2}}$
(4) $\dfrac{\mu_0 \pi R^2 r^2}{2(R^2+x^2)^{3/2}}$

58. Two coils of self inductance 100 mH and 400 mH are placed very close to each other. Find the maximum mutual inductance between the two when 4 A current passes through them.

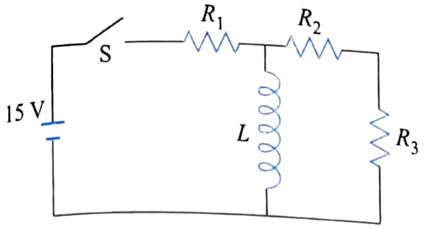
(1) 200 mH
(2) 300 mH
(3) 100 $\sqrt{2}$ mH
(4) none of these

59. Switch S is closed at $t = 0$, in the circuit shown in figure. The change in flux in the inductor ($L = 500$ mH) from $t = 0$ to an instant when it reaches steady state is

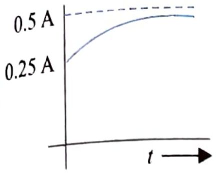


(1) 2 Wb
(2) 1.5 Wb
(3) 0 Wb
(4) None

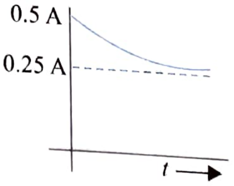
60. The figure below shows a battery with emf 15 V in a circuit with $R_1 = 30\ \Omega$, $R_2 = 10\ \Omega$, $R_3 = 20\ \Omega$ and $L = 3.0$ H. The switch S is initially in the open position and is then closed at time $t = 0$. Then the graph which shows the correct variation of current through battery after switch S is closed



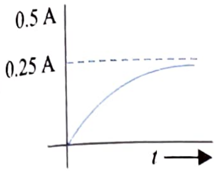
(1)



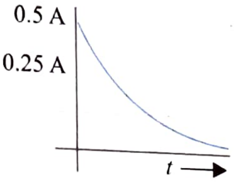
(2)



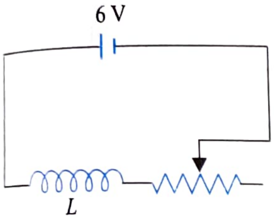
(3)

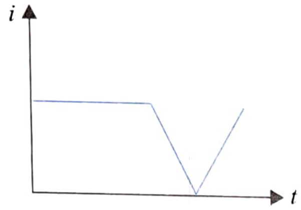


(4)



61. In the circuit shown in figure, sliding contact is moving with uniform velocity towards right. Its value at some instance is 12 Ω. The current in the circuit at this instant of time will be

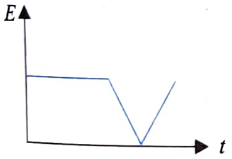




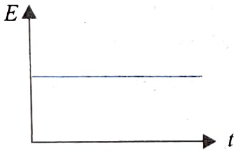
(1) 0.5 A
(2) More than 0.5 A
(3) Less than 0.5 A
(4) May be less or more than 0.5 A depending on the value of L.

62. The current i in an induction coil varies with time according to the graph shown in figure. Which of the following graphs shows induced emf in the coil with time:

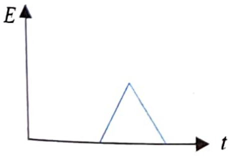
(1)



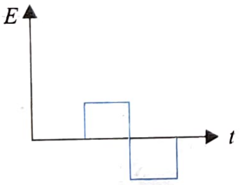
(2)



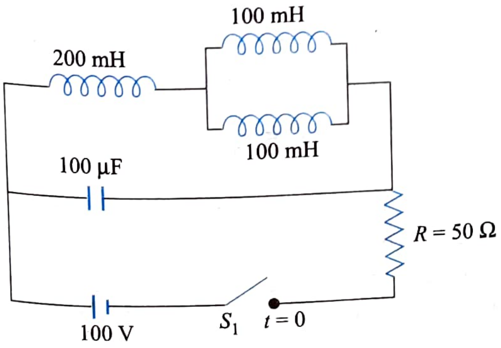
(3)



(4)

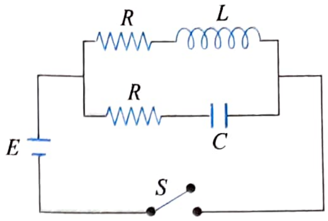


63. In the circuit shown in figure, switch S was closed for a long time. At time $t = 0$, the switch is opened again. The maximum potential difference across the plates of the capacitor after the switch is opened is



(1) 10 V
(2) 100 V
(3) 20 V
(4) 200 V

64. In the circuit shown in figure, the time constants of the two branches are equal (= T). Then if the key S is closed at the instant $t = 0$, the time in which the current in the circuit through the battery will rise to its final value of (E/R) will be, (assume that the internal resistance of the battery and of the connecting wires are negligible)

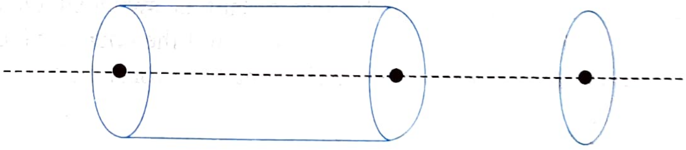


(1) Instantly
(2) $t = \frac{T}{2}$
(3) $t = 2T$
(4) t = infinity

65. A bulb of 100 W is connected in parallel to an ideal inductance of 1 H. This arrangement is connected to a 90 V battery through a switch. On pressing the switch the
(1) bulb does not glow
(2) bulb glows
(3) bulb glows after a short time and then continues to glow
(4) bulb glows for a short time and then stops glowing.

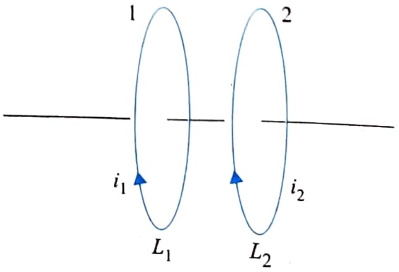
66. A uniformly wound solenoid coil of self inductance 4 mH and resistance 12 Ω is broken into two parts. These two coils are connected in parallel across a 12 V battery. The steady current through battery is
(1) 1 A
(2) 2 A
(3) 4 A
(4) 8 A

67. The diagram given below shows a solenoid carrying time varying current $I = I_0 t$. On the axis of the solenoid, a ring has been placed. The mutual inductance of the ring and the solenoid is M and the self inductance of the ring is L. If the resistance of the ring is R then maximum current which can flow through the ring is



(1) $\frac{(2M+L)I_0}{R}$
(2) $\frac{MI_0}{R}$
(3) $\frac{(2M-L)I_0}{R}$
(4) $\frac{(M+L)I_0}{R}$

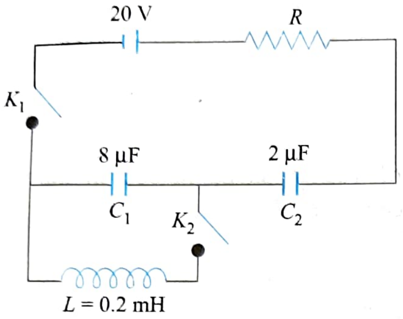
68. A system consists of two coaxial current carrying circular loops as shown in figure. The self inductance of loop 1 is L_1 and self inductance of loop 2 is L_2. Magnitude of mutual inductance is M. If at any instant current flowing through loop 1 and loop 2 are i_1 and i_2 respectively, then total magnetic energy of the system is



(1) $\frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 - |M| i_1 i_2$ (2) $\frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 + |M| i_1 i_2$

(3) $\frac{1}{2}L_1 i_2^2 + \frac{1}{2}L_2 i_1^2 - |M| i_1 i_2$ (4) $\frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2$

69. A circuit containing capacitors C_1 and C_2, shown in figure is in the steady state with key K_1 closed. At the instant $t = 0$, K_1 is opened and K_2 is closed. The angular frequency of oscillation of the circuit and charge across C_1 at $t = 125$ μs are respectively



(1) 25×10^3 rad s^{-1}, 32 μC (2) 5×10^4 rad s^{-1}, 16 μC

(3) 5×10^4 rad s^{-1}, −16 μC (4) 25×10^3 rad s^{-1}, −32 μC

70. An inductor coil stores 32 J of magnetic field energy and dissipates energy as heat at the rate of 320 W when a current of 4 A is passed through it. Find the time constant of the circuit when it is formed across an ideal battery.

(1) $t = 0.2$ s (2) $t = 0.32$ s

(3) $t = 0.5$ s (4) $t = 1$ s

71. A solenoid of inductance 50 mH and resistance 10 Ω is connected to a battery of 6 V. Find the time elapsed before the current acquires half of its steady state value.

(1) 3.5 ms (2) 2.5 ms

(3) 0.693 ms (4) 2 ms

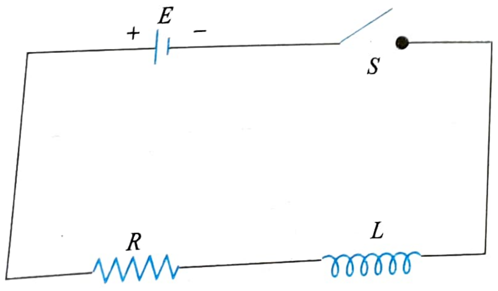
72. An inductor resistance battery circuit is switched on at $t = 0$. If the emf of the battery is E, find the charge which passes through the battery during one time constant τ.

(1) $\frac{E}{R} \cdot \frac{\tau}{e}$ (2) $\frac{E}{R} \cdot \frac{\tau}{(e-1)}$

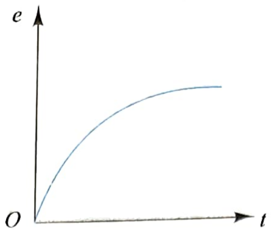
(3) $\frac{E}{R} \cdot \frac{\tau}{(e+1)}$ (4) $\frac{E}{R} \cdot \frac{(e-1)}{\tau}$

Multiple Correct Answers Type

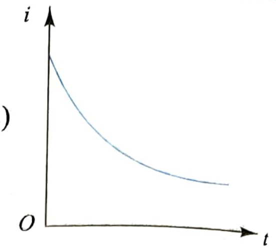
1. Switch S of the circuit shown in figure is closed at $t = 0$. If e denotes the induced emf in L and I is the current flowing through the circuit at time t, which of the following graphs is/are correct?



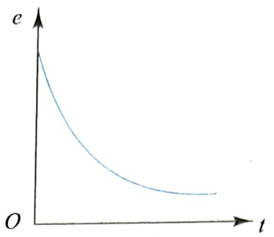
(1)



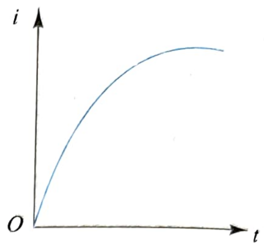
(2)



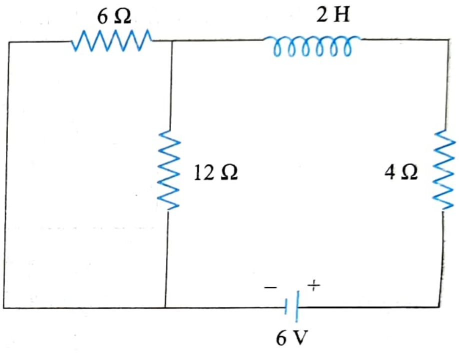
(3)



(4)



2. For the circuit shown in figure, which of the following statements is/are correct?



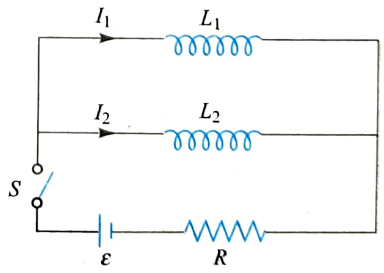
(1) Its time constant is 0.25 s

(2) In steady state, current through the inductance will be equal to zero

(3) In steady state, current through the battery will be equal to 0.75 A

(4) None of these

3. In the circuit shown in figure, the switch is closed at $t = 0$.



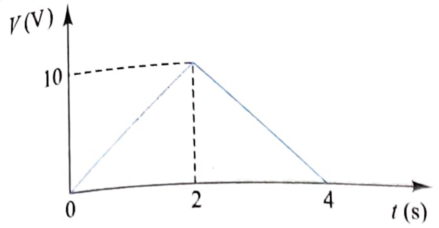
(1) At $t = 0$, $I_1 = I_2 = 0$

(2) At any time t, $\frac{I_1}{I_2} = \frac{L_2}{L_1}$

(3) At any time t, $I_1 + I_2 = \frac{\varepsilon}{R}$

(4) At $t = \infty$, I_1 and I_2 are independent of L_1 and L_2

4. The potential difference across a 2-H inductor as a function of time is shown in figure. At $t = 0$, current is zero. Choose the correct statement.

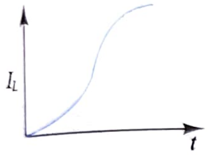


(1) Current at $t = 2$ s is 5 A

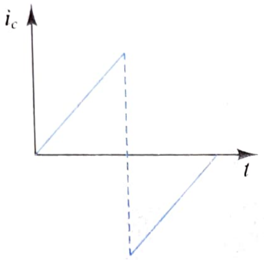
(2) Current at $t = 2$ s is 10 A

(3) Current versus time graph across the inductor will be [Figure (a)]

(4) Current versus time graph across inductor will be [Figure (b)]

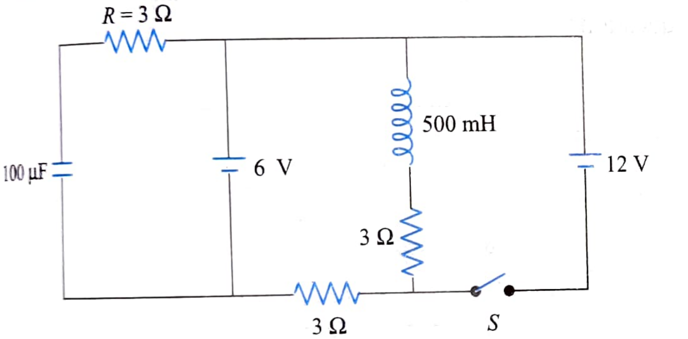


(a)



(b)

5. In the given circuit (figure), the switch is closed at $t = 0$. Choose the correct answers.



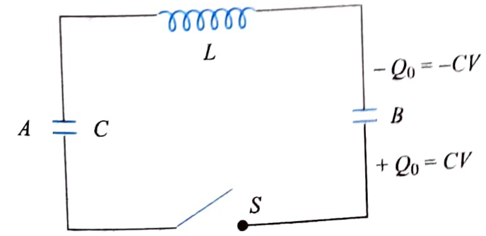
(1) Current in the inductor when the circuit reaches the steady state is 4 A.

(2) The net change in flux in the inductor is 1.5 Wb.

(3) The time constant of the circuit after closing S is 555.55 s.

(4) The charge stored in the capacitor in steady state is 1.2 mC.

6. An inductor and two capacitors are connected in the circuit as shown in figure. Initially capacitor A has no charge and capacitor B has CV charge. Assume that the circuit has no resistance at all. At $t = 0$, switch S is closed, then [given $LC = \dfrac{2}{\pi^2 \times 10^4}$ s^2 and $CV = 100$ mC]



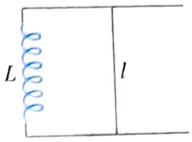
(1) when current in the circuit is maximum, charge on each capacitor is same

(2) when current in the circuit is maximum, charge on capacitor A is twice the charge on capacitor B

(3) $q = 50(1 + \cos 100\ \pi t)$ mC, where q is the charge on capacitor B at time t

(4) $q = 50(1 - \cos 100\pi t)$ mC, where q is the charge on capacitor B at time t

7. Two parallel resistanceless rails are connected by an inductor of inductance L at one end as shown in figure. A magnetic field B exists in the space which is perpendicular to the plane of the rails. Now a conductor of length l and mass m is placed transverse on the rail and given an impulse J toward the rightward direction. Then choose the correct option(s).



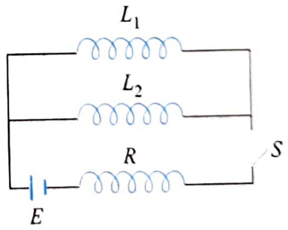
(1) Velocity of the conductor is half of the initial velocity after a displacement of the conductor $d = \sqrt{\dfrac{3J^2L}{4B^2l^2m}}$

(2) Current flowing through the inductor at the instant when velocity of the conductor is half of the initial velocity is $i = \sqrt{\dfrac{3J^2}{4Lm}}$

(3) Velocity of the conductor is half of the initial velocity after a displacement of the conductor $d = \sqrt{\dfrac{3J^2L}{B^2l^2m}}$

(4) Current flowing through the inductor at the instant when velocity of the conductor is half of the initial velocity is $i = \sqrt{\dfrac{3J^2}{mL}}$

8. In the circuit, a battery of emf E, a resistance R and inductance coil L_1 and L_2 and switch S are connected as shown. in figure. Initially the switch is open



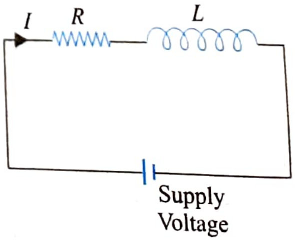
(1) The time constant of the circuit is $\dfrac{1}{R}\left(\dfrac{L_1L_2}{L_1 + L_2}\right)$

(2) Steady state current in the inductor L_1 is $\dfrac{EL_2}{R(L_1 + L_2)}$

(3) Steady state, current in the inductor L_2 is $\dfrac{EL_1}{R(L_1 + L_2)}$

(4) In steady state the total energy stored in the inductor coils is $\dfrac{1}{2}\left(\dfrac{L_1L_2}{L_1 + L_2}\right)\dfrac{E^2}{R^2}$

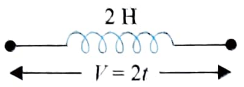
9. When current (I) in R–L series circuit becomes constant, where L is a pure inductor, which of the following given statements is/are correct?



(1) voltage across R is RI.

(2) some part (not 100 %) of the energy supplied by the battery will be dissipated in R and remaining will continue to store in L.
(3) voltage across L is equal to zero
(4) magnetic energy stored is $\frac{1}{2}LI^2$

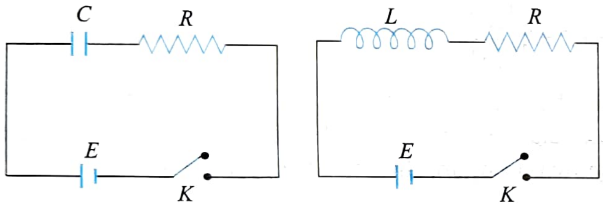
10. A time varying voltage $V = 2t$ (Volt) is applied across an ideal inductor of inductance $L = 2\text{H}$ as shown in figure. Then (assume current to be zero at $t = 0$)



(1) current versus time graph is a parabola
(2) energy stored in magnetic field at $t = 2$ s is 4 J
(3) potential energy at time $t = 1$ s in magnetic field is increasing at a rate of 1 J/s
(4) energy stored in magnetic field is zero all the time

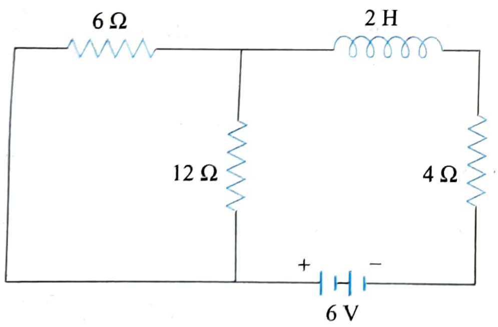
11. An LR circuit with a battery connected at $t = 0$. Which of the following quantities will be zero just after the connection?
(1) Current in the circuit
(2) Magnetic field energy in the inductor
(3) Power delivered by battery
(4) emf induced in the inductor

12. The switches in following figures are closed at $t = 0$ and respond after a long time at $t = t_0$. Then



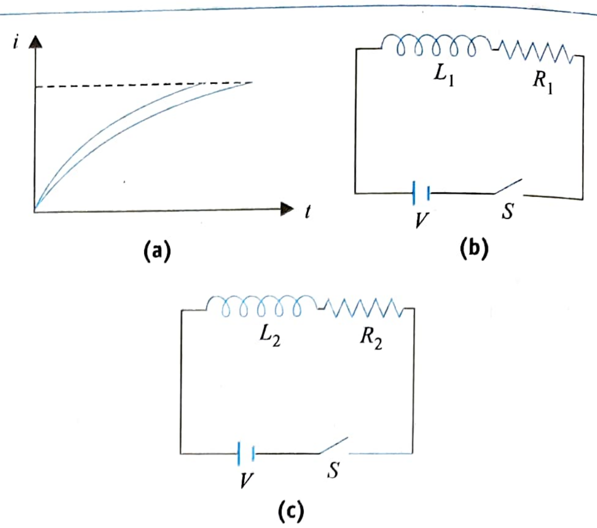
(1) the charge on C just after $t = 0$ is EC
(2) the charge on C long after $t = 0$ is EC
(3) the current in L just before $t = t_0$ is E/R
(4) the current in L long after $t = t_0$ is E/R

13. For the circuit shown in figure, which of the following statements are correct?



(1) Its time constant is 0.25 s.
(2) In steady state, current through inductance will be equal to zero.
(3) In steady state, current through the battery will be equal to 0.75 A.
(4) None of the above.

4. The current growth in two L–R circuits (b) and (c) is as shown in Fig. (a). Let L_1, L_2, R_1, and R_2 be the corresponding values in two circuits. Then



(a) (b)

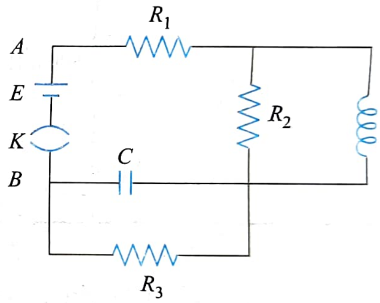
(c)

(1) $R_1 > R_2$ (2) $R_1 = R_2$
(3) $L_1 > L_2$ (4) $L_1 < L_2$

Linked Comprehension Type

For Problems 1–2

In the given circuit (figure), all the symbols have their usual meanings. At $t = 0$, key K is closed. Now answer the following questions.

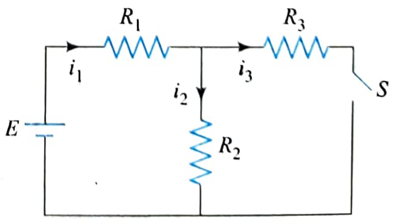


1. At $t = 0$, the equivalent resistance between A and B is
(1) $R_1 + R_2 + R_3$ (2) $R_1 + R_2$
(3) $R_1 + R_3$ (4) indeterminate

2. At $t \to \infty$, the equivalent resistance between A and B is
(1) $R_1 + R_2 + R_3$ (2) $R_1 + R_2$
(3) $R_1 + R_3$ (4) none of these

For Problems 3–5

In the circuit shown in figure, $E = 15$ V, $R_1 = 1\Omega$, $R_2 = 1\ \Omega$, $R_3 = 2\ \Omega$, and $L = 1.5$ H. The currents flowing through R_1, R_2, and R_3 are i_1, i_2, and i_3, respectively.



3. Immediately after turning switch S on,
(1) $i_1 = i_2 = 7.5$ A, $i_3 = 0$ A (2) $i_1 = i_3 = 5$ A, $i_2 = 0$ A
(3) $i_1 = i_2 = 9$ A, $i_3 = 0$ A (4) $i_1 = i_2 = i_3 = 0$ A

4. After the circuit reaches the steady state,
(1) $i_1 = 9$ A, $i_2 = 6$ A, $i_3 = 3$ A
(2) $i_1 = 9$ A, $i_2 = 3$ A, $i_3 = 6$ A

(3) $i_1 = 6$ A, $i_2 = 6$ A, $i_3 = 0$ A

(4) $i_1 = 0$ A, $i_2 = 0$ A, $i_3 = 0$ A

5. Immediately after connecting switch S,

(1) $i_3 = 0$ A and $\frac{di_3}{dt} = 0\ \text{A s}^{-1}$

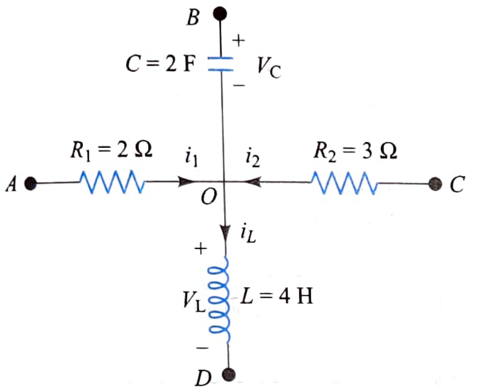
(2) $i_3 = 0$ A and $\frac{di_3}{dt} \neq 0\ \text{A s}^{-1}$

(3) $i_3 = 0$ A and the rate at which magnetic energy stored is not zero

(4) none of these

For Problems 6–11

In figure, $i_1 = 10\,e^{-2t}$ A, $i_2 = 4$ A, and $V_C = 3e^{-2t}$ V.



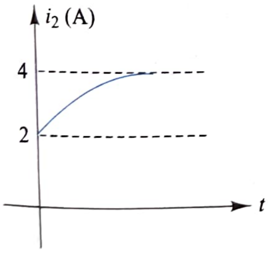
6. The current i_L is

(1) $[2 - 2(1 - e^{-2t})]$ A　(2) $[2 + 2(1 - e^{-2t})]$ A

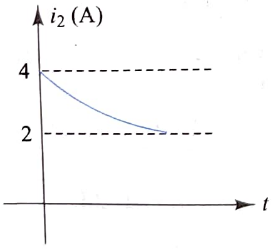
(3) $[3 - 2(1 - e^{-2t})]$ A　(4) $[2 + 3(1 - e^{-2t})]$ A

7. The variation of current in the inductor with time can be represented as

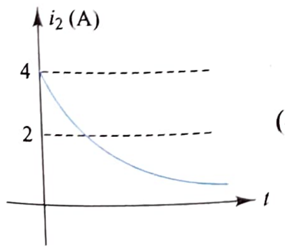
(1)



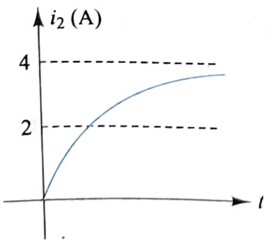
(2)



(3)



(4)



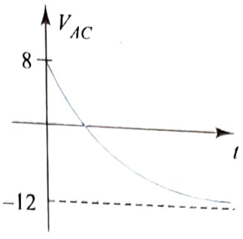
8. The potential difference across inductor V_L is

(1) $8e^{-2t}$ V　(2) $9e^{-2t}$ V

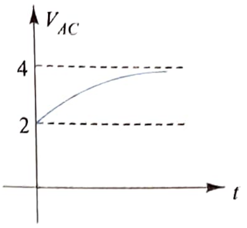
(3) $16e^{-2t}$ V　(4) $18e^{-2t}$ V

9. The variation of potential difference across A and C (V_{AC}) with time can be represented as

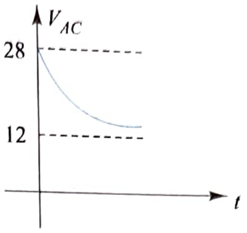
(1)



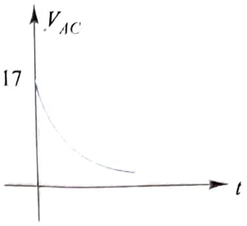
(2)



(3)



(4)



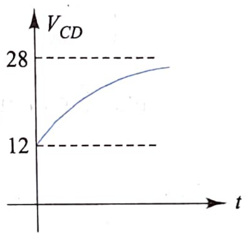
10. The potential difference across AB (V_{AB}) is

(1) $8e^{-2t}$ V　(2) $\frac{1}{2}e^{-3t}$ V

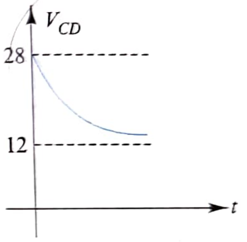
(3) $17e^{-2t}$ V　(4) $16e^{-2t}$ V

11. The variation of potential difference across C and D (V_{CD}) with time can be expressed as

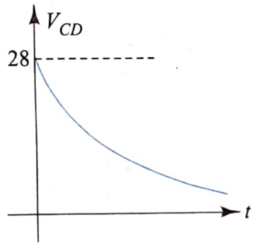
(1)



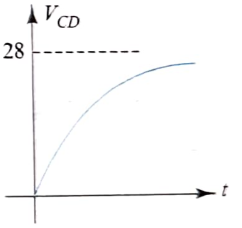
(2)



(3)

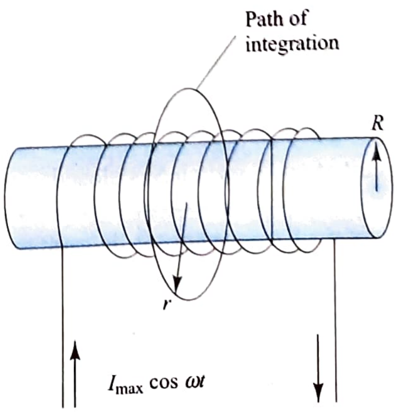


(4)



For Problems 12–13

A long solenoid of radius R has n turns of wire per unit length and carries a time-varying current that varies sinusoidally as $I = I_{max} \cos \omega t$, where I_{max} is the maximum current and ω is the angular frequency of the alternating current source (shown in figure).



12. The magnitude of the induced electric field inside the solenoid, a distance $r < R$ from its long central axis is

(1) $\frac{3\mu_0 n I_{\max}\omega}{2} r \sin \omega t$ (2) $\frac{\mu_0 n I_{\max}\omega}{2} r \cos \omega t$

(3) $\mu_0 n I_{\max}\omega r \sin \omega t$ (4) $\frac{\mu_0 n I_{\max}\omega}{2} r \sin \omega t$

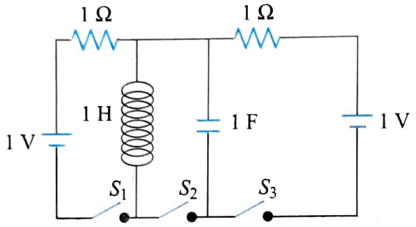
13. The magnitude of electric field outside the solenoid at a distance $r > R$ from its long central axis is

(1) $\frac{\mu_0 n I_{\max}\omega R^2}{2r}\sin\omega t$ (2) $\frac{2\mu_0 n I_{\max}\omega R^2}{r}\sin\omega t$

(3) $\frac{\mu_0 n I_{\max}\omega R^2}{3r}\sin\omega t$ (4) $\frac{3\mu_0 n I_{\max}\omega R^2}{2r}\sin\omega t$

For Problems 14–16

In the circuit shown (figure), switches S_1 and S_3 have been closed for 1 s and S_2 remained open. Just after 1 s, switch S_2 is closed and S_1 and S_3 are opened. Find after that instant ($t = 0$):



14. The maximum current in the circuit containing inductor and capacitor only (only S_2 is closed)

(1) $\sqrt{3}\left(1-\frac{1}{e}\right)$ (2) $\sqrt{2}\left(1-\frac{1}{e}\right)$

(3) $\sqrt{3}\left(1+\frac{1}{e}\right)$ (4) $\sqrt{2}\left(1+\frac{1}{e}\right)$

15. The maximum charge on the capacitor

(1) $\sqrt{3}\left(1+\frac{1}{e}\right)$ (2) $\sqrt{3}\left(1-\frac{1}{e}\right)$

(3) $\sqrt{2}\left(1+\frac{1}{e}\right)$ (4) $\sqrt{2}\left(1-\frac{1}{e}\right)$

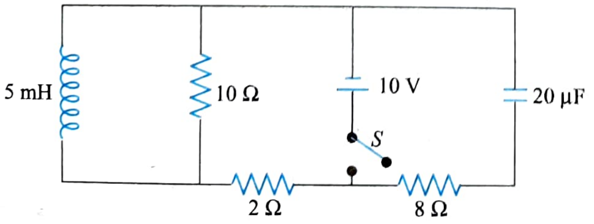
16. The charge on the upper plate of the capacitor as a function of time

(1) $\sqrt{2}\left(1-\frac{1}{e}\right)\sin\left(t+\frac{3\pi}{4}\right)$ (2) $\sqrt{2}\left(1-\frac{1}{e}\right)\sin\left(t+\frac{\pi}{4}\right)$

(3) $\sqrt{3}\left(1-\frac{1}{e}\right)\sin\left(t+\frac{\pi}{4}\right)$ (4) $\sqrt{3}\left(1+\frac{1}{e}\right)\sin\left(t+\frac{\pi}{4}\right)$

For Problems 17–19

In the given circuit at $t = 0$, switch S is closed.



17. The current through the 10 Ω resistor at any instant t ($0 < t < \infty$) will be

(1) $\frac{1}{6}e^{-(1000/3)t}$ (2) $\frac{5}{6}e^{-(1000/3)t}$

(3) $\frac{1}{6}e^{(1000/3)t}$ (4) $\frac{6}{5}e^{(1000/3)t}$

18. The energy stored in the inductor at any instant t ($0 < t < \infty$) will be

(1) $\frac{1}{2}\left[5-5e^{-(1000/3)t}\right]^2$ mJ (2) $\frac{125}{2}\left[1-e^{-(1000/3)t}\right]^2$ mJ

(3) $\frac{25}{2}\left[1-e^{-(1000/3)t}\right]^2$ mJ (4) $\frac{5}{2}\left[1-e^{-(1000/3)t}\right]^2$ mJ

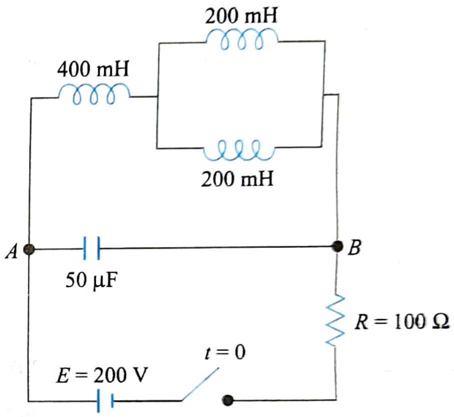
19. The energy stored in the capacitor and inductor, respectively, at $t \to \infty$ will be

(1) 1 mJ and 62.5 mJ (2) 62.5 mJ and 1 mJ

(3) 2 mJ and 62.5 mJ (4) 1 mJ and 60 mJ

For Problem 20–21

In the circuit shown in figure, switch S_1 was closed for a long time. At time $t = 0$ the switch is opened.



20. Find the maximum potential difference across the plates of the capacitor after the switch is opened.

(1) 100 V (2) 200 V

(3) 300 V (4) 400 V

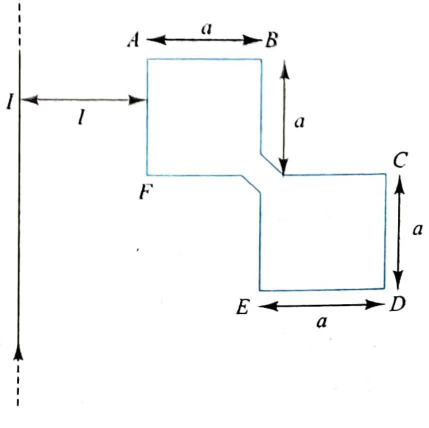
21. Find the angular frequency of oscillation of the charge on the capacitor.

(1) 100 rad s^{-1} (2) 200 rad s^{-1}

(3) 300 rad s^{-1} (4) 400 rad s^{-1}

For Problems 22–24

In figure, there is a conducting loop $ABCDEF$ of resistance λ per unit length placed near a long straight current-carrying wire. The dimensions are shown in the figure. The long wire lies in the plane of the loop. The current in the long wire varies as $I = I_0(t)$.



2. The mutual inductance of the pair is

(1) $\frac{\mu_0 a}{2\pi}\ln\left(\frac{2a+l}{l}\right)$ (2) $\frac{\mu_0 a}{2\pi}\ln\left(\frac{2a-l}{l}\right)$

(3) $\frac{2\mu_0 a}{\pi}\ln\left(\frac{a+l}{l}\right)$ (4) $\frac{\mu_0 a}{\pi}\ln\left(\frac{a+l}{l}\right)$

3. The emf induced in the closed loop is

(1) $\frac{\mu_0 I_0 a}{2\pi}\ln\left(\frac{2a+l}{l}\right)$ (2) $\frac{\mu_0 I_0 a}{2\pi}\ln\left(\frac{2a-l}{l}\right)$

(3) $\frac{2\mu_0 I_0 a}{\pi}\ln\left(\frac{a+l}{l}\right)$ (4) $\frac{\mu_0 I_0 a}{\pi}\ln\left(\frac{a+l}{l}\right)$

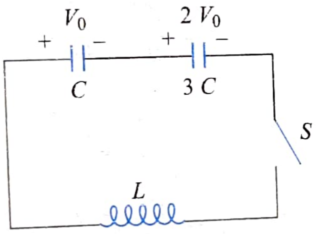
4. The heat produced in the loop in time t is

(1) $\frac{\left[\frac{\mu_0 I_0}{2\pi}\ln\left(\frac{a+l}{l}\right)\right]^2 at}{4\lambda}$ (2) $\frac{\left[\frac{\mu_0 I_0}{2\pi}\ln\left(\frac{2a+l}{l}\right)\right]^2 at}{8\lambda}$

(3) $\frac{\left[\frac{2\mu_0 I_0}{\pi}\ln\left(\frac{a+l}{l}\right)\right]^2 at}{3\lambda}$ (4) $\frac{\left[\frac{\mu_0 I_0}{2\pi}\ln\left(\frac{3a+l}{l}\right)\right]^2 at}{6\lambda}$

For Problems 25–27

Two capacitors of capacitance C and $3C$ are charged to potential difference V_0 and $2V_0$, respectively, and connected to an inductor of inductance L as shown in figure. Initially, the current in the inductor is zero. Now, switch S is closed.



25. The maximum current in the inductor is

(1) $\frac{3V_0}{2}\sqrt{\frac{3C}{L}}$ (2) $V_0\sqrt{\frac{3C}{L}}$

(3) $2V_0\sqrt{\frac{3C}{L}}$ (4) $V_0\sqrt{\frac{C}{L}}$

26. Potential difference across capacitor of capacitance C when the current in the circuit is maximum is

(1) $\frac{V_0}{4}$ (2) $\frac{3V_0}{4}$

(3) $\frac{5V_0}{4}$ (4) none of these

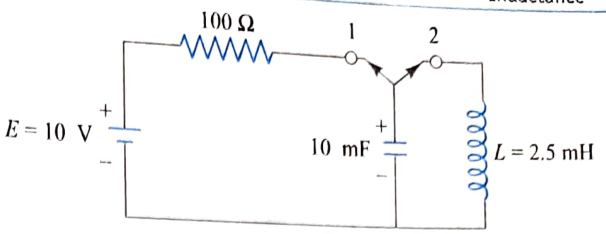
27. Potential difference across capacitor of capacitance $3C$ when the current in the circuit is maximum is

(1) $\frac{V_0}{4}$ (2) $\frac{V_0}{4}$

(3) $\frac{5V_0}{4}$ (4) none of these

For Problems 28–30

In figure, initially the capacitor is charged to a potential of 5 V and then connected to position 1 with the shown polarity for 1 s. After 1 s, it is connected across the inductor at position 2.



28. The potential across the capacitor after 1 s of its connection to position 1is

(1) $5\times10^3\left(2+\frac{1}{e}\right)$V (2) $5\times10^3\left(2-\frac{1}{e}\right)$V

(3) $5\times10^3\left(1+\frac{2}{e}\right)$V (4) none of these

29. The maximum current flowing in the LC circuit when the capacitor is connected across the inductor is

(1) $\left(2-\frac{1}{e}\right)\times10^4$ A (2) $\left(1+\frac{2}{e}\right)\times10^4$ A

(3) $\left(1-\frac{2}{e}\right)\times10^4$ A (4) none of these

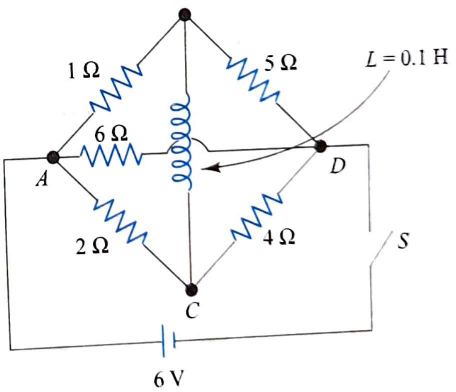
30. The frequency of LC oscillations is

(1) $(20/\pi)$ Hz (2) $(2/\pi)$ Hz

(3) $(40/\pi)$ Hz (4) $100/\pi$ Hz

For Problems 31–33

There is no current in any part of this circuit for time $t < 0$. Switch S is closed at $t = 0$.



31. The rate at which the current through the inductor increases initially is

(1) zero (2) 10 A s^{-1}

(3) 1 A s^{-1} (4) 5 A s^{-1}

32. Current through the 6 Ω resistor

(1) increases linearly with time

(2) increases non-linearly with time

(3) decreases non-linearly with time

(4) remains constant

33. The current through the inductor after a long time will be

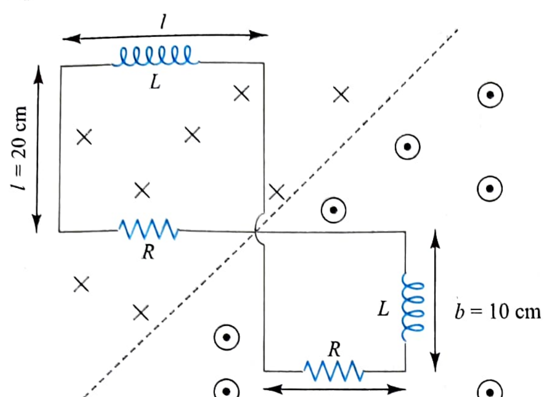
(1) zero (2) infinite

(3) $\frac{6}{13}$A (4) none of these

For Problems 34–36

In figure, there is a frame consisting of two square loops having resistors and inductors as shown. This frame is placed in a uniform but time-varying magnetic field in such a way that one of the loops is placed in crossed magnetic field and the other is placed in dot magnetic field. Both magnetic fields are perpendicular to the planes of the loops.

If the magnetic field is given by $B = (20 + 10\,t)$ Wb m^{-2} in both regions [$l = 20$ cm, $b = 10$ cm, and $R = 10\ \Omega$, $L = 10$ H),



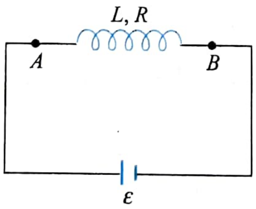
34. The direction of induced current in the bigger loop will be
   (1) clockwise
   (2) anticlockwise
   (3) first clockwise for some time, then anticlockwise, and so on
   (4) first anticlockwise for some time, then clockwise, and so on

35. The induced emf in the frame only due to the variation of magnetic field will be
   (1) 0.3 V (2) 0.1 V
   (3) 0.5 V (4) 0.4 V

36. The current in the frame as a function of time will be
   (1) $\frac{1}{20}\,(1 - e^{-t})$ (2) $\frac{1}{40}\,(1 - e^{-t})$
   (3) $\frac{1}{20}\,e^{-t}$ (4) $\frac{1}{10}\,e^{-t}$

For Problems 37–39

An inductor having self inductance L with its coil resistance R is connected across a battery of emf ε. When the circuit is in steady state at $t = 0$, an iron rod is inserted into the inductor due to which its inductance becomes nL ($n > 1$).



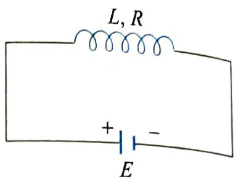
37. After insertion of rod which of the following quantities will change with time?
   (a) Potential difference across terminals A and B
   (b) Inductance
   (c) Rate of heat produced in coil
   (1) only (a) (2) (a) and (c)
   (3) only (c) (4) (a), (b) and (c)

38. After insertion of rod, current in the circuit:
   (1) Increases with time
   (2) Decreases with time
   (3) Remains constant with time
   (4) First decreases with time then becomes constant

39. When again circuit is in steady state, the current in it is
   (1) $I < \varepsilon/R$ (2) $I > \varepsilon/R$
   (3) $I = \varepsilon/R$ (4) None of these

For Problems 40–42

A solenoid of resistance R and inductance L has a piece of soft iron inside it. A battery of emf E and of negligible internal resistance is connected across the solenoid as shown in figure. At any instant, the piece of soft iron is pulled out suddenly so that inductance of the solenoid decreases to ηL ($\eta < 1$) with battery remaining connected.



40. The work done to pull out the soft iron piece is
   (1) $\frac{\eta LE^2}{2R^2}$ (2) $\frac{(1-\eta)LE^2}{2R^2}$
   (3) $\frac{(1-\eta)LE^2}{\eta R^2}$ (4) $\frac{(1-\eta)LE^2}{2\eta R^2}$

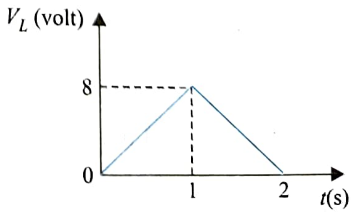
41. Assume $t = 0$ is the instant when iron piece has been pulled out, the current as a function of time after this is
   (1) $i = \frac{E}{R}\left[1-\left(1-\frac{1}{\eta}\right)e^{-t/\lambda}\right]$ (2) $i = \frac{E}{R}\left[1+\left(1+\frac{1}{\eta}\right)e^{-t/\lambda}\right]$
   (3) $i = \frac{E}{R}\left[1-\left(1+\frac{1}{\eta}\right)e^{-t/\lambda}\right]$ (4) $i = \frac{E}{R}\left[1+\left(1-\frac{1}{\eta}\right)e^{-t/\lambda}\right]$

$\left(\text{where } \lambda = \frac{\eta L}{R}\right)$

42. Power supplied by the battery as a function of time
   (1) $P = \frac{E^2}{R}\left[1+\left(1-\frac{1}{\eta}\right)e^{-t/\lambda}\right]$
   (2) $P = \frac{E^2}{R}\left[1+\left(1+\frac{1}{\eta}\right)e^{-t/\lambda}\right]$
   (3) $P = \frac{E^2}{R}\left[1-\left(1+\frac{1}{\eta}\right)e^{-t/\lambda}\right]$
   (4) $P = \frac{E^2}{R}\left[1-\left(1-\frac{1}{\eta}\right)e^{-t/\lambda}\right]$

For Problems 43–45

The potential difference across a 2-H inductor as a function of time is shown in figure. At time $t = 0$, current is zero.



43. Area under V_L–t graph gives
   (1) current
   (2) change in current

(3) product of inductance and current
(4) product of inductance and change in current

44. Current at $t = 1$s is

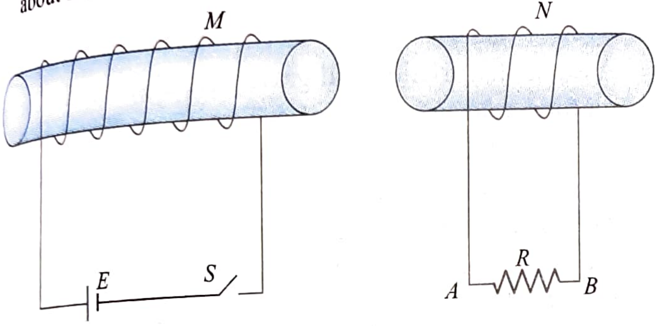
(1) 1 A (2) 2 A
(3) 4 A (4) 8 A

45. The variation of current versus time is

(1) a straight line (2) a parabola
(3) a hyperbola (4) exponential

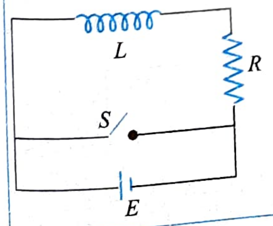
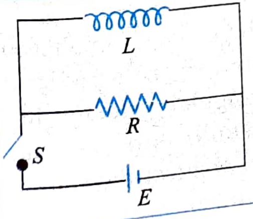
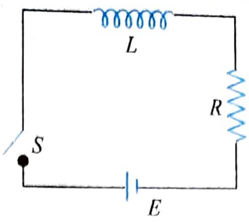
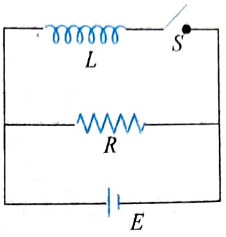
Matrix Match Type

1. Figure shows two coaxial coils M and N. Column I is regarding some operations done with coil M and column II about induced current in coil N.

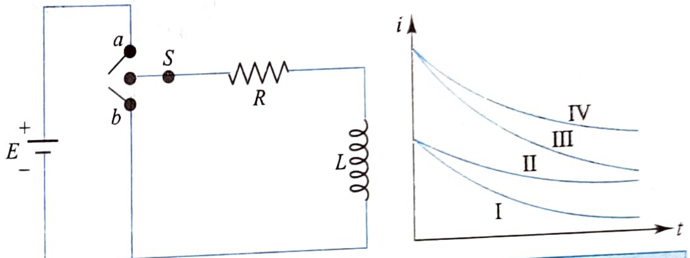


Column I	Column II
i. Just after switch S is closed	**a.** Current is induced from A to B
ii. Switch S is opened after keeping it closed for a long time.	**b.** Current is induced from B to A
iii. After switch S is closed for a long time	**c.** No current is induced
iv. Just after switch S is closed while moving M away from N	**d.** the wire will attract or repel the loop.

2. In column I, some circuits are given. In all the circuits except in (i), switch S remains closed for a long time and then it is opened at $t = 0$; while for (i), the situation is reversed. Column II tells something about the circuit quantities. Match the entries of column I with the entries of coumn II.

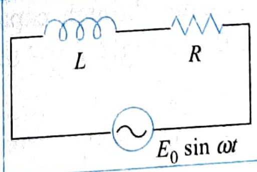
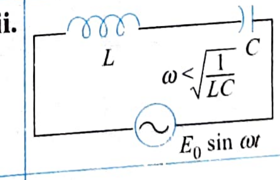
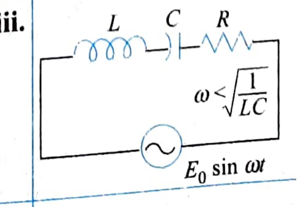
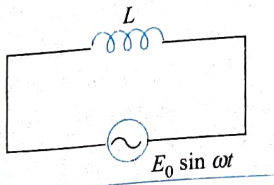
Column I	Column II
i. 	**a.** Induced emf can be greater than E.
ii. 	**b.** Induced emf would be less than E.
iii. 	**c.** Finally, energy stored in inductor is zero.
iv. 	**d.** Finally, energy stored in inductor is non-zero.

3. The switch S in the circuit is connected with point a for a very long time, then it is shifted to position b. The resulting current through the inductor is shown by curves in the graph for four sets of values for the resistance R and inductance L (given in column 1). Which set corresponds with which curve?



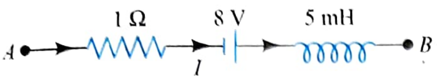
Column I	Column II
i. R_0 and L_0	**a.** I
ii. $2R_0$ and L_0	**b.** II
iii. R_0 and $2L_0$	**c.** III
iv. $2R_0$ and $2L_0$	**d.** IV

4. Column I shows some ac circuits. Column II gives some possibilities in the circuit. Match appropriately.

Column I	Column II
i. 	**a.** $\lvert\cos \phi\rvert \neq 1$ (ϕ is the phase difference between current and emf)
ii.  L C $\omega < \sqrt{\frac{1}{LC}}$ $E_0 \sin \omega t$	**b.** Current leads the emf
iii.  L C R $\omega < \sqrt{\frac{1}{LC}}$ $E_0 \sin \omega t$	**c.** Current lags the emf
iv. 	**d.** Average power dissipated is zero

Numerical Value Type

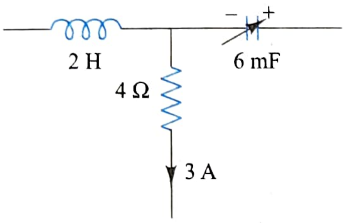
1. In the circuit (figure) what is potential difference $V_B - V_A$ (in V) when current I is 5 A and is decreasing at the rate of 10^3 A s^{-1}?



2. A current of 2 A is increasing at a rate of 4 A s^{-1} through a coil of inductance 1 H. Find the energy stored in the inductor per unit time in the units of J s^{-1}.

3. Two coils, 1 and 2 have a mutual inductance $M = 5$ H and resistance $R = 10\ \Omega$ each. A current flows in coil 1, which varies with time as: $I_1 = 2t^2$, where t is time. Find the total charge (in C) that has flown through coil 2 between $t = 0$ and $t = 2$ s.

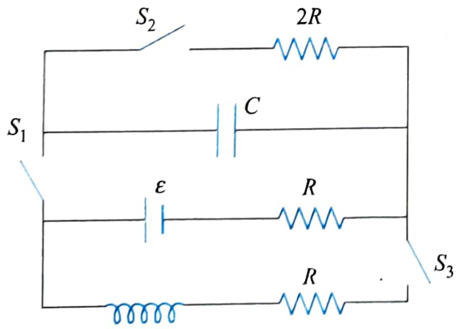
4. Figure shows a part of a bigger circuit. The capacity of the capacitor is 6 mF and is decreasing at the constant rate 0.5 mF s^{-1}. The potential difference across the capacitor at the shown moment is changing as follows:



$$\frac{dV}{dt} = 2\ \mathrm{V\,s^{-1}},\ \frac{d^2V}{dt^2} = \frac{1}{2}\ \mathrm{V\,s^{-2}}$$

The current in the 4 Ω resistor is decreasing at the rate of 1 mA s^{-1}. What is the potential difference (in mV) across the inductor at this moment?

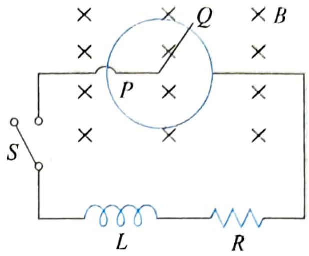
5. In the given circuit, initially switch S_1 is closed, and S_2 and S_3 are open. After charging of capacitor, at $t = 0$, S_1 is opened and S_2 and S_3 are closed. If the relation between inductance, capacitance and resistance is $L = 4CR^2$, then find the time (in s) after which current passing through capacitor and inductor will be same (given $R = \ln 2$ mΩ, $L = 2$ mH)



6. A long solenoid of diameter 0.1 m has 2×10^4 turns per metre. At the center of the solenoid, a 100-turn coil of radius 0.01 m is placed with its axis coinciding with the solenoid axis. The current in the solenoid is decreased at a constant rate from + 2 A to − 2 A in 0.05 s. Find the total charge (in μC) flowing through the coil during this time when the resistance of the coil is $40\ \pi^2\ \Omega$.

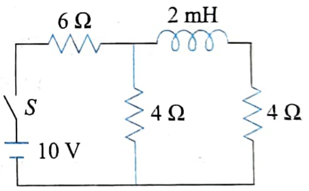
7. A capacitor of capacity 2 μF is charged to a potential difference of 12 V. It is then connected across an inductor of inductance 6 μH. What is the current (in A) in the circuit at a time when the potential difference across the capacitor is 6.0 V?

8. Figure shows a circuit having a coil of resistance $R = 2.5\ \Omega$ and inductance L connected to a conducting rod PQ which can slide on a perfectly conducting circular ring of radius 10 cm with its center at P.



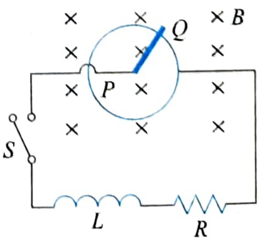
Assume that friction and gravity are absent and a constant uniform magnetic field of 5 T exists as shown in figure. At $t = 0$, the circuit is switched on and simultaneously a time-varying external torque is applied on the rod so that it rotates about P with a constant angular velocity 40 rad s^{-1}. Find the magnitude of this torque (in mNm) when current reaches half of its maximum value. Neglect the self inductance of the loop formed by the circuit.

9. In the given circuit, what is the current I (in A) drawn from battery at time $t = 0$.



10. At any instant a current of 2 A is increasing at a rate of 1 A/s through a coil of inductance 2 H. Find the energy (in SI units) being stored in the inductor per unit time at that instant.

11. The diagram shows a circuit having a coil of resistance $R = 2.5\ \Omega$ and inductance L connected to a conducting rod PQ which can slide on a perfectly conducting circular ring of radius 10 cm with its centre at 'P'. Assume that friction and gravity are absent and a constant uniform magnetic field of 5 T exists as shown in figure.



At $t = 0$, the circuit is switched on and simultaneously a time varying external torque is applied on the rod so that it rotates about P with a constant angular velocity 40 rad/s. Find magnitude of this torque (in milli Nm) when current reaches half of its maximum value. Neglect the self-inductance of the loop formed by the circuit.

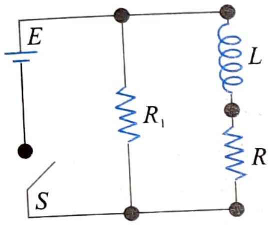
12. A certain volume of copper is drawn into a wire of radius a and is wrapped in shape of a helix having radius r ($>> a$). The windings are as close as possible without overlapping. Self-inductance of the inductor so obtained is L_1. Another wire of radius $2a$ is drawn using same volume of copper and wound in the fashion as described above. This time the inductance is L_2. Find $\frac{L_1}{L_2}$.

Archives

JEE MAIN

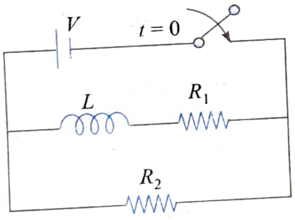
Single Correct Answer Type

1. An inductor of inductance L = 400 mH and resistors of resistances R_1 = 2 ohm and R_2 = 2 Ω are connected to a battery of emf 12 V as shown in the figure. The internal resistance of the battery is negligible. The switch S is closed at $t = 0$. The potential drop across L as a function of time is



(1) $6e^{-5t}$ V
(2) $\frac{12}{t}e^{-3t}$ V
(3) $6(1 - e^{-t/0.2})$ V
(4) $12e^{-5t}$ V **(AIEEE 2009)**

2. In the circuit shown below, the key K is closed at $t = 0$. The current through the battery is

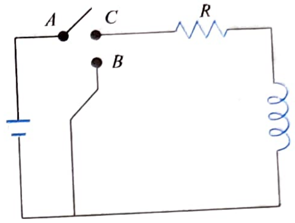


(1) $\frac{VR_1R_2}{\sqrt{R_1^2 + R_2^2}}$ at $t = 0$ and $\frac{V}{R_2}$ at $t = \infty$
(2) $\frac{V}{R_2}$ at $t = 0$ and $\frac{V(R_1 + R_2)}{R_1 R_2}$ at $t = \infty$
(3) $\frac{V}{R_2}$ at $t = 0$ and $\frac{VR_1R_2}{\sqrt{R_1^2 + R_2^2}}$ at $t = \infty$
(4) $\frac{V(R_1 + R_2)}{R_1R_2}$ at $t = 0$ and $\frac{V}{R_2}$ at $t = \infty$ **(AIEEE 2010)**

3. A fully charged capacitor C with initial charge q_0 is connected to a coil of self inductance L at $t = 0$. The time at which the energy is stored equally between the electric and the magnetic fields is

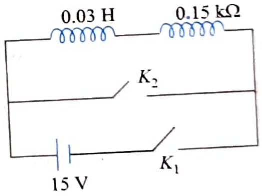
(1) $\pi\sqrt{LC}$
(2) $\frac{\pi}{4}\sqrt{LC}$
(3) $2\pi\sqrt{LC}$
(4) $\sqrt{LC}$ **(AIEEE 2011)**

4. In the circuit shown here, the point 'C' is kept connected to point 'A' till the current flowing through the circuit becomes constant. Afterward, suddenly, point 'C' is disconnected from point 'A' and connected to point 'B' at time $t = 0$. Ratio of the voltage across resistance and the inductor at $t = L/R$ will be equal to:



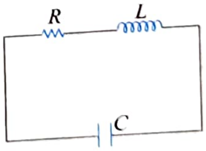
(1) −1
(2) $\frac{1-e}{e}$
(3) $\frac{e}{1-e}$
(4) 1 **(JEE Main 2014)**

5. An inductor (L = 0.03 H) and a resistor (R = 0.15 kΩ) are connected in series to a battery of 15 V EMF in a circuit shown below. The key K_1 has been kept closed for a long time. Then at t = 0, K_1 is opened and key K_2 is closed simultaneously. At t = 1 ms, the current in the circuit will be ($e^5 \cong 150$)

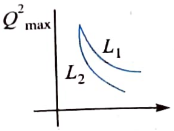


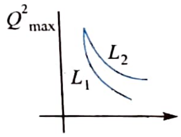
(1) 100 mA
(2) 67 mA
(3) 6.7 mA
(4) 0.67 mA **(JEE Main 2015)**

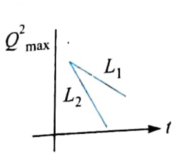
6. An LCR circuit is equivalent to a damped pendulum. In an LCR circuit the capacitor is charged to Q_0 and then connected to the L and R as shown below:

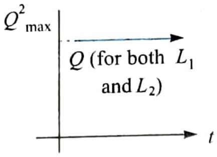


If a student plots graphs of the square of maximum charge (Q^2_{Max}) on the capacitor with time (t) for two different values L_1 and L_2 ($L_1 > L_2$) of L then which of the following represents this graph correctly?
(Plots are schematic and not drawn to scale)

(1)


(2)


(3)


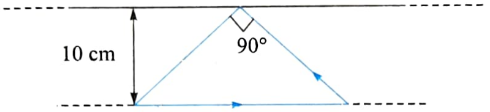
(4)


(JEE Main 2015)

JEE ADVANCED

Multiple Correct Answers Type

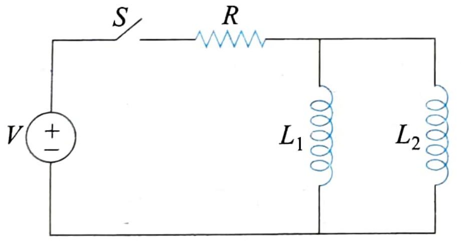
1. A conducting loop in the shape of right angled isosceles triangle of height 10 cm is kept such that the 90° vertex is very close to an infinitely long conducting wire (see the figure). The wire is electrically insulated from the loop. The hypotenuse of the triangle is parallel to the wire. The current in the triangular loop is in counterclockwise direction and increased at constant rate of 10 As^{-1}. Which of the following statement(s) is (are) true?



(1) The induced current in the wire is in opposite direction to the current along the hypotenuse.
(2) There is a repulsive force between the wire and the loop.
(3) If the loop is rotated at a constant angular speed about the wire, an additional emf of $\left(\frac{\mu_0}{\pi}\right)$ volt is induced in the wire.
(4) The magnitude of induced emf in the wire is $\left(\frac{\mu_0}{\pi}\right)$ volt.

(JEE Advanced 2016)

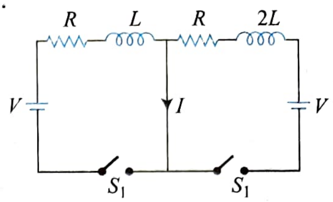
2. A source of constant voltage V is connected to a resistance R and two ideal inductors L_1 and L_2 through a switch S as shown. There is no mutual inductance between the two inductors. The switch S is initially open. At $t = 0$, the switch is closed and current begins to flow. Which of the following options is/are correct?



(1) The ratio of the currents through L_1 and L_2 is fixed at all times ($t > 0$)
(2) After a long time, the current through L_1 will be $\frac{V}{R}\frac{L_2}{L_1 + L_2}$
(3) After a long time, the current through L_2 will be $\frac{V}{R}\frac{L_1}{L_1 + L_2}$
(4) At $t = 0$, the current through the resistance R is $\frac{V}{R}$

(JEE Advanced 2017)

3. In the figure below, the switches S_1 and S_2 are closed simultaneously at $t = 0$ and a current starts to flow in the circuit. Both the batteries have the same magnitude of the electromotive force (emf) and the polarities are as indicated in the figure.



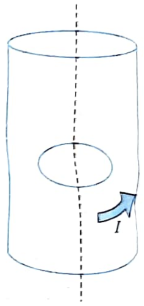
Ignore mutual inductance between the inductors. The current I in the middle wire reaches its maximum magnitude I_{max} at time $t = \tau$. Which of the following statement(s) is (are) true?

(1) $I_{max} = \frac{V}{2R}$
(2) $I_{max} = \frac{V}{4R}$
(3) $\tau = \frac{L}{R}\ln 2$
(4) $\tau = \frac{2L}{R}\ln 2$

(JEE Advanced 2018)

Numerical Value Type

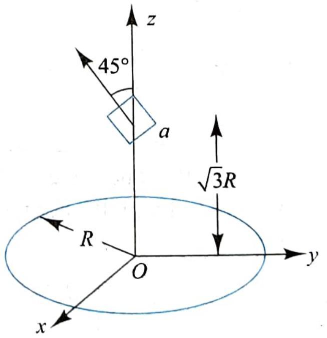
1. A long circular tube of length 10 m and radius 0.3 m carries a current I along its curved surface as shown in figure. A wire-loop of resistance 0.005 Ω and of radius 0.1 m is placed inside the tube with its axis coinciding with the axis of the tube. The current varies as $= I = I_0 \cos(300\ t)$ where I_0 is constant. If the magnetic moment of the loop is $N\mu_0 I_0 \sin(300t)$, then N is



(IIT JEE, 2011)

2. A circular wire loop of radius R is placed in the x–y plane centered at the origin O. A square loop of side a ($a << R$) having two turns is placed with its center at along the axis of the circular wire loop as shown in figure. The plane of the square loop makes an angle of 45° with respect to the z-axis. If the mutual inductance between the loops is given by $\frac{\mu_0 a^2}{2^{p/2}R}$, then the value of p is

(IIT JEE 2012)



3. Two inductors L_1 (inductance 1 mH, internal resistance 3 Ω) and L_2 (inductance 2 mH, internal resistance 4 Ω), and a resistor R (resistance 12 Ω) are all connected in parallel across a 5 V battery. The circuit is switched on at time $t = 0$. The ratio of the maximum to the minimum current (I_{max}/I_{min}) drawn from the battery is

(JEE Advanced 2016)

Answers Key

EXERCISES

Single Correct Answer Type

1. (1) 2. (2) 3. (3) 4. (2) 5. (2)
6. (3) 7. (2) 8. (4) 9. (2) 10. (4)
11. (3) 12. (1) 13. (2) 14. (4) 15. (2)
16. (4) 17. (1) 18. (2) 19. (3) 20. (2)
21. (2) 22. (1) 23. (3) 24. (4) 25. (4)
26. (3) 27. (1) 28. (2) 29. (1) 30. (2)
31. (2) 32. (2) 33. (1) 34. (4) 35. (2)
36. (1) 37. (3) 38. (1) 39. (3) 40. (2)
41. (2) 42. (3) 43. (3) 44. (4) 45. (1)
46. (2) 47. (3) 48. (1) 49. (3) 50. (1)
51. (1) 52. (1) 53. (1) 54. (1) 55. (3)
56. (3) 57. (4) 58. (1) 59. (2) 60. (1)
61. (2) 62. (4) 63. (2) 64. (1) 65. (4)
66. (3) 67. (2) 68. (1) 69. (4) 70. (1)
71. (1) 72. (1)

Multiple Correct Answers Type

1. (3),(4) 2. (1),(3) 3. (1),(2)
4. (1),(3) 5. (1),(2) 6. (1),(3)
7. (1),(2) 8. (1),(2),(3),(4) 9. (1),(3),(4)
10. (1),(2),(3) 11. (1),(2),(3) 12. (2),(3)
13. (1),(3) 14. (2),(4)

Linked Comprehension Type

1. (2) 2. (3) 3. (1) 4. (1) 5. (2)
6. (2) 7. (1) 8. (3) 9. (1) 10. (3)
11. (2) 12. (4) 13. (1) 14. (2) 15. (4)
16. (1) 17. (2) 18. (2) 19. (1) 20. (2)
21. (2) 22. (1) 23. (1) 24. (2) 25. (1)
26. (3) 27. (3) 28. (2) 29. (1) 30. (4)
31. (2) 32. (4) 33. (3) 34. (2) 35. (3)
36. (2) 37. (3) 38. (1) 39. (3) 40. (4)
41. (1) 42. (4) 43. (4) 44. (2) 45. (2)

Matrix Match Type

1. i. → a.; ii. → b.; iii. → c.; iv. → a.
2. i. → c.; ii. → c.; iii. → a., c.; iv. → a., c.
3. i. → c.; ii. → a.; iii. → d.; iv. → b.
4. i. → a., c.; ii. → a., c., d.; iii. → a., b.; iv. → a., c., d.

Numerical Value Type

1. (8) 2. (8) 3. (4) 4. (4) 5. (1)
6. (8) 7. (6) 8. (5) 9. (1) 10. (4)
11. (5) 12. (8)

ARCHIVES

JEE Main

Single Correct Answer Type

1. (4) 2. (2) 3. (2) 4. (4) 5. (4)
6. (1)

JEE Advanced

Multiple Correct Answers Type

1. (2),(4) 2. (1),(2),(3) 3. (2),(4)

Numerical Value Type

1. (6) 2. (7) 3. (8)

6

Alternating Current

INTRODUCTION

We have studied about the dc circuits. But most of the power generated and used in the world is in the form of alternating current (ac). An alternating current is the current whose value changes with time in both magntiude and direction. We will study about a special kind of alternating current which varies sinusoidally. It is represented by $I = I_0 \sin(\omega t + \phi)$

where $I \rightarrow$ instantaneous current

$I_0 \rightarrow$ peak value or maximum value of ac

$\omega t + \phi \rightarrow$ phase at any time t

$\phi \rightarrow$ initial phase or phase constant

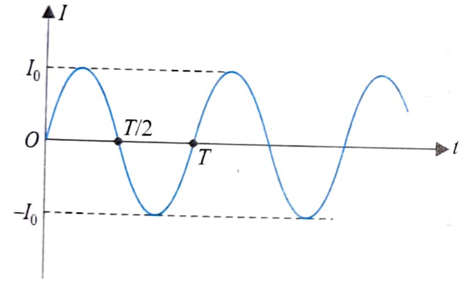
$\omega \rightarrow$ angular frequency

$T \rightarrow$ time period

(where $T = 2\pi/\omega$)

$f \rightarrow$ frequency (where $f = 1/T$)

The graph of I vs. t is shown for $\phi = 0$



Time period, T: It is the time taken to complete one complete cycle or one oscillation (one cycle consists of variation of current from 0 to maximum, maximum to 0, 0 to negative maximum, and negative maximum to 0).

Frequency, f: It is the number of cycles or oscillations completed per unit time.

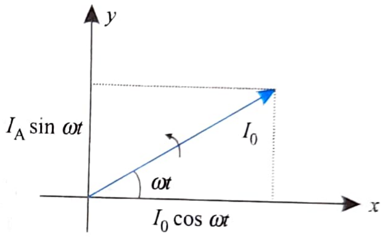
Note:

1. The equation of current can also be written in cosine form as $I = I_0 \cos(\omega t + \phi')$
2. To produce alternating current, emf should be alternating. An alternating emf can be represented by $E = E_0 = \sin(\omega t + \phi_1)$
3. An alternating emf is produced by a dynamo or an electronic oscillator.
4. The frequency of ac in India is 50 Hz, i.e., $f = 50$ Hz, so $\omega = 2\pi f = 100\pi$ rad s^{-1}.
5. The ac can be converted into dc with the help of a rectifier while dc into ac with the help of an inverter.
6. It cannot produce chemical effects of current such as electroplating or electrolysis as due to large ions, they cannot follow the frequency of ac.
7. It can be stepped up or stepped down with the help of transformer (while dc cannot be).

PHASOR DIAGRAMS

Generally, currents and voltages in ac circuits are represented in the form of phasors or anticlockwise rotating vectors. The length of arrow represents the peak value of the quantity and its projection on x- or y-axis gives its instantaneous value.

For example, let $I = I_0 \sin \omega t$, then it will be represented as shown in figure. Length of arrow is I_0 which represents the peak value of I. Its projection on y-axis is $I_0 \sin \omega t$ which represents the instantaneous value. ωt is the phase angle which increases with time.



Note: If the equation of current were in cosine form as $I = I_0 \cos \omega t$, then projection on x-axis will represent the instantaneous value.

MEAN OR AVERAGE VALUE OF CURRENT

Mean or Average value of any current in a given time is defined as that constant value of current which will send the same amount of charge in a given circuit as sent by actual current in the same time interval.

$$I_{av}(t_2 - t_1) = \int_{t_1}^{t_2} I(t)\,dt \Rightarrow I_{av} = \frac{\int_{t_1}^{t_2} I(t)\,dt}{t_2 - t_1}$$

Average value of $I = I_0 \sin \omega t$

1. Over first half cycle: for this $t_1 = 0$, $t_2 = T/2 = \pi/\omega$

$$I_{av}\left(\frac{T}{2} - 0\right) = \int_0^{T/2} I_0 \sin \omega t\,dt = \left[\frac{-I_0 \cos \omega t}{\omega}\right]_0^{\pi/\omega}$$

$$\Rightarrow I_{av}\left(\frac{T}{2}\right) = \frac{2I_0}{\omega} = \frac{2I_0 T}{2\pi}$$

$$\Rightarrow I_{av} = \frac{2I_0}{\pi} = 0.637\,I_0$$

2. Over full cycle: for this $t_1 = 0$, $t_2 = T = 2\pi/\omega$

$$I_{av}(T - 0) = \int_0^{T} I_0 \sin \omega t\,dt = \left[\frac{-I_0 \cos \omega t}{\omega}\right]_0^{2\pi/\omega}$$

$\Rightarrow I_{av} = 0$

So average value of current for the full cycle is zero. We can also think in this way:

Average value during positive half cycle: $0.637\, I_0$

Average value during negative half cycle: $-0.637\, I_0$

So over full cycle, $I_{av} = 0.637\, I_0 - 0.637\, I_0 = 0$

Note:

1. Average value of emf can also be found in the same way as we find for current. For this we can write the formula:

$$E_{av} = \frac{\int_{t_1}^{t_2} E(t)\, dt}{t_2 - t_1}$$

2. Average value of alternating current or emf over a long period of time will be zero, because for this time interval $t_2 - t_1$ will be large whereas quantity in the numerator will be finite.

ROOT MEAN SQUARE (RMS) VALUE OF CURRENT

The root mean square value of any current is defined as that value of steady current (constant), which would generate the same amount of heat in a given resistance in a given time as generated by actual current passing through the same resistance for the same given time.

The rms value is also known as *effective value* or *virtual value* of current. It is denoted by I_v.

$$I_v^2 R(t_2 - t_1) = \int_{t_1}^{t_2} [I(t)]^2 R dt \quad \Rightarrow \quad I_v = \sqrt{\frac{\int_{t_1}^{t_2} [I(t)]^2 dt}{t_2 - t_1}}$$

We square the instantaneous current I, take the average (mean) value of I^2, and finally take the square root of that average. This procedure defines the root mean square current, denoted as I_{rms} or I_v. Even when I is negative, I^2 is always positive, so I_v is never zero (unless I is zero at every instant).

rms value of $I = I_0 \sin \omega t$:

1. Over first half cycle: for this $t_1 = 0$, $t_2 = T/2 = \pi/\omega$

$$I_v^2\left(\frac{T}{2} - 0\right) = \int_0^{T/2} I_0^2 \sin^2 \omega t\, dt = I_0^2 \int_0^{T/2} \left(\frac{1 - \cos 2\omega t}{2}\right) dt$$

$$\Rightarrow I_v = \frac{I_0}{\sqrt{2}} = 0.707\, I_0$$

2. Over full cycle: for this $t_1 = 0$, $t_2 = T = 2\pi/\omega$

$$I_v^2 (T - 0) = \int_0^T I_0^2 \sin^2 \omega t\, dt = I_0^2 \int_0^T \left(\frac{1 - \cos 2\omega t}{2}\right) dt$$

$$\Rightarrow I_v = \frac{I_0}{\sqrt{2}} = 0.707\, I_0$$

The rms value of ac over half cycle or full cycle is same. This is also valid over a long period of time.

Note:

1. Similarly, root mean square value of an emf.

$$E_v = \frac{E_0}{\sqrt{2}} = 0.707\, E_0$$

2. Ordinary dc ammeters and voltmeters cannot measure alternating current or alternating emf. They will show zero reading. For this purpose, we use ac ammeters and voltmeters. They are known as hot wire instruments. They measure virtual values of current or voltage. For example, if we measure $I = 10 \sin \omega t$ A from ac ammeter, it will show $I_v = \frac{10}{\sqrt{2}}$ A.

3. The symbol for an ac source is shown below.

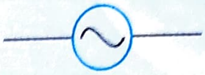


ILLUSTRATION 6.1

The electric mains in a house are marked 220 V-50 Hz. Write down the equation for instantaneous voltage.

Sol. Here 220 V is rms voltage. So peak voltage is $220\sqrt{2}$ V. Instantaneous voltage is given by

$$E = E_0 \sin(\omega t + \phi) = 220\sqrt{2} \sin(2\pi\, 50\, t + \phi)$$
$$= 220\sqrt{2} \sin(100\pi t + \phi)$$

ILLUSTRATION 6.2

The plate on the back of a personal computer says that it draws 2.7 A from a 120-V, 60-Hz line. For this computer, what is **(a)** the average of the square of the current, **(b)** the current amplitude, **(c)** the average current for a positive half cycle, and **(d)** the average current for a full cycle?

Sol.

(a) The current given is the rms value: $I_{rms} = 2.7$ A. The average of the square of the current is nothing but $(I_{rms})^2$.

So, $(I_{rms})^2 = (2.7\text{ A})^2 = 7.3\text{ A}^2$

(b) The current amplitude I_0 is

$$I_0 = \sqrt{2}\, I_{rms} = \sqrt{2}\,(2.7\text{ A}) = 3.8\text{ A}$$

(c) For a positive half cycle, $I_{av} = \frac{2I_0}{\pi} = \frac{2 \times 3.8}{\pi} = 2.42$ A

(d) For a full cycle, $I_{av} = 0$

ILLUSTRATION 6.3

An alternating current is given by the following equation: $I = 3\sqrt{2} \sin(100\pi t + \pi/4)$. Give the frequency and rms value of the current.

Sol. $I = 3\sqrt{2} \sin[100\pi t + (\pi/4)]$

We have $2\pi f = 100\pi \Rightarrow f = 50$ Hz

and $I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{3\sqrt{2}}{\sqrt{2}}\text{ A} = 3\text{ A}$

ILLUSTRATION 6.4

A voltage, $E = 60 \sin(314t)$, is applied across a resistor of 20 Ω. What will be the reading of I_{rms}

(a) in an ac ammeter?

(b) in an ordinary moving coil ammeter in series with the resistor?

Sol. Given that $E = 60 \sin(314t)$

(a) An ac ammeter will read the rms value

$$E_{\text{rms}} = \frac{E_0}{\sqrt{2}} = \frac{60}{\sqrt{2}} = 42.4 \text{ V}$$

$$I_{\text{rms}} = \frac{E_{\text{rms}}}{R} = \frac{42.4}{20} = 2.12 \text{ A}$$

Therefore, ac ammeter will read 2.12 A.

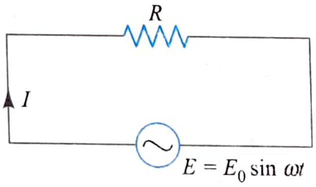
(b) An ordinary moving coil ammeter will read the average value of alternating current. Since the average value of ac is zero, this meter will give zero reading.

AC CIRCUITS

Basic ac circuit elements are resistors, inductors, and capacitors. We will discuss the behaviour of each of them when connected in ac circuits.

AC CIRCUITS CONTAINING RESISTOR ONLY

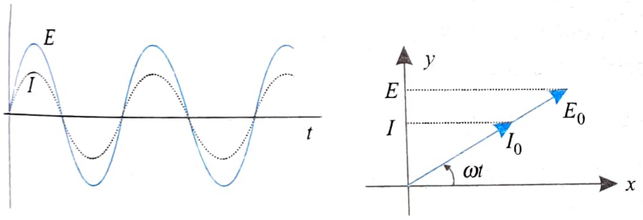
Consider resistor R is connected across emf $E = E_0 \sin \omega t$ as shown in figure. Due to this emf, current will flow in the resistance which will vary in both magnitude and direction. At any time, current is given by



$$I = \frac{E}{R} = \frac{E_0}{R} \sin \omega t$$

$$= I_0 \sin \omega t \qquad \left[\text{where } I_0 = \frac{E_0}{R} \rightarrow \text{Current amplitude}\right]$$

Variation of E and I is shown in Fig. (a) and their phasor diagram representation is shown in Fig. (b).



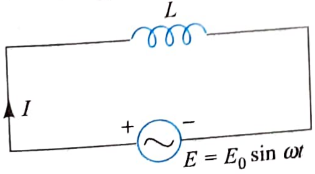
(a) (b)

From both the diagrams we see that phase of current and emf are same.

Same phase means both are varying in the same manner. Both are becoming zero at the same time, attaining their positive maximum, and negative maximum values at the same time, etc. The current and voltage phasors rotate together; they are parallel at each instant. Their projections on the vertical axis represent their instantaneous values.

AC CIRCUITS CONTAINING INDUCTOR ONLY

Consider a pure inductor (having no resistance) of inductance L is connected as shown in figure.



Induced emf in the inductor (with the polarities as shown in the figure, assuming current increases with time): $e = L\dfrac{dI}{dt}$

As there is only inductor in the circuit, induced emf should be equal to applied emf.

$$\Rightarrow \quad \frac{L dI}{dt} = E_0 \sin \omega t$$

$$\Rightarrow \quad \int dI = \int \frac{E_0}{L} \sin \omega t \, dt$$

$$\Rightarrow \quad I = -\frac{E_0}{\omega L} \cos \omega t + C$$

where C is constant of integration. To determine the value of C, let us take the average of current for one time period.

$$I_{\text{av}} = \frac{\int_0^T I\,dt}{T} = \frac{1}{T}\int_0^T \left(-\frac{E_0}{\omega L}\cos \omega t + C\right) dt$$

$$= \frac{1}{T}\left[\frac{-E_0}{\omega^2 L}\sin \omega t + Ct\right]_0^T = \frac{1}{T}[Ct] = C$$

But for any kind of alternating current $I_{\text{av}} = 0$ over one cycle. Hence, $C = 0$. So, finally, we get

$$I = -\frac{E_0}{\omega L}\cos \omega t$$

$$= \frac{E_0}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

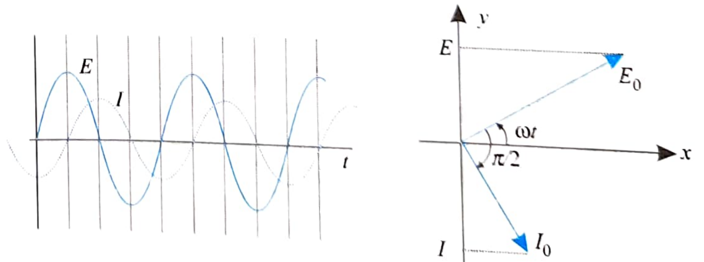
$$= \frac{E_0}{X_L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

$$= I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$$

where $X_L = \omega L$ is the resistance offered by inductor. It is also known as inductive reactance. Its unit is Ω.

$$I_0 = \frac{E_0}{X_L} \rightarrow \text{Current amplitude}$$

Variation of E and I is shown in Fig. (a) and their phasor diagram respresentation is shown in Fig. (b).



(a) (b)

From Fig. (a), voltage and current are "out of step" or out of phase by a quarter cycle. Voltage peak occurs a quarter cycle earlier than the current peak. Thus, we say as that the voltage leads the current by 90°.

Also from Fig. (b), we see that the phase of emf is ωt and that of current is $\omega t - \pi/2$. So, phase of current is $\pi/2$ less than emf. Hence, current lags behind emf by phase $\pi/2$, or emf leads current by phase $\pi/2$.

A phase difference of $\pi/2$ is equivalent to time difference of $T/4$. We see that current attains its zero or maximum (positive or negative) values after time $T/4$ of emf has attained the same type of value.

Note:

1. For dc circuit $\omega = 0 \Rightarrow X_L = 0$. So inductor offers zero resistance to a dc circuit.
2. X_L depends upon the frequency of source.

INDUCTIVE REACTANCE

Inductive reactance X_L is a description of the self-induced emf that opposes any change in the current through the inductor. Inductive reactance and self-induced emf increase with more rapid variation in current (that is, increasing angular frequency, ω) and increasing inductance L.

If an oscillating voltage of a given amplitude E_0 is applied across the inductor terminals, the resulting current will have a smaller amplitude I for larger values of X_L, since X_L is proportional to frequency, a high-frequency voltage applied to the inductor gives only a small current, while a lower-frequency voltage of the same amplitude gives rise to a larger current. Inductors are used in some circuit applications, such as power supplies and radio-interference filters, to block high frequencies while permitting lower frequencies or dc to pass through. A circuit device that uses an inductor for this purpose is called a low-pass filter.

ILLUSTRATION 6.5

Suppose you want the current amplitude in a pure inductor in a radio receiver to be 250 μA when the voltage amplitude is 3.60 V at a frequency of 1.60 MHz (corresponding to the upper is 3.60 V AM broadcast band). What inductive reactance is needed? What inductance is required?

Sol. We are given the current amplitude, I_0, and the voltage amplitude, E_0. The inductive reactance

$$X_L = \frac{E_0}{I_0} = \frac{3.60}{250 \times 10^{-6}} = 1.44 \times 10^4\ \Omega = 14.4\ \text{k}\Omega$$

$$L = \frac{X_L}{2\pi f} = \frac{1.44 \times 10^4\ \Omega}{2\pi(1.60 \times 10^6\ \text{Hz})} = 1.43 \times 10^{-3}\ \text{H} = 1.43\ \text{mH}$$

ILLUSTRATION 6.6

A pure inductance of 1.0 H is connected across a 110 V, 70 Hz source. Find the **(a)** reactance, **(b)** current, and **(c)** peak value of current.

Sol. $L = 1.0$ H, $V = 110$ V, $f = 70$ Hz

(a) Reactance $= \omega L = 2\pi \times 70 \times 1 = 440\ \Omega$

(b) Current $= 110/440 = 0.25$ A

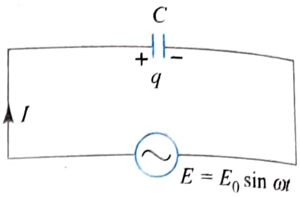
(c) Peak value of current $= 0.25\sqrt{2} = 0.353$ A

AC CIRCUITS CONTAINING CAPACITOR ONLY

Consider a capacitor of capacitance C is connected as shown in figure. Charge on the capacitor at any time is

$$q = CE = CE_0 \sin \omega t$$

Current in the circuit at any time is



$$I = \frac{dq}{dt} = CE_0\omega \cos \omega t$$

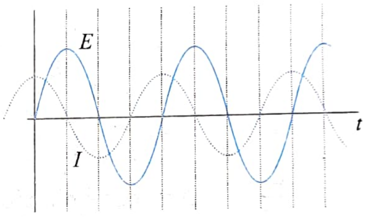
$$\Rightarrow \quad I = \frac{E_0}{(1/\omega C)} \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$\Rightarrow \quad I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

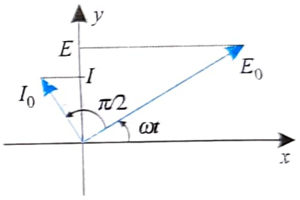
where $I_0 = \dfrac{E_0}{X_C}$ and $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi f C}$

X_C represents effective resistance offered by the capacitor. It is known as capacitive reactance. More the frequency, lesser is the reactance. Further, if $f = 0$, $X_C = \infty$. So for dc circuit, capacitor offers infinite resistance or it blocks dc. Unit of X_C is Ω.

Variation of E and I is shown in Fig. (a) and their phasor diagram respresentation is shown in Fig. (b).



(a)



(b)

The capacitor voltage and the current are out of phase by a quarter cycle. The peak of voltage occurs quarter cycle after the corresponding current peak. And we say that the voltage phasor is behind the current phasor by a quarter cycle or 90°.

CAPACITIVE REACTANCE

The capacitive reactance of a capacitor is inversely proportional to both the capacitance C and the angular frequency ω. Greater the capacitance and higher the frequency, smaller is the capacitive reactance, X_C. Capacitors tend to pass high-frequency current and block low-frequency currents and dc, just the opposite of inductors. A device that preferentially passes signals of high frequency is called a high-pass filter.

Note: In case of resistor, resistance is offered due to the obstruction to the passage of current. In inductor, it is induced emf. In capacitor, it is potential difference developed across the capacitor.

The whole of the above discussion can be generalized as:

Let $E = E_0 \sin \omega t$ and $I = I_0 \sin(\omega t + \phi)$. Here we have assumed that current leads emf by an angle, ϕ. ϕ is also known as phase factor. Here $I_0 = E_0/Z$, where Z is known as impedance of the circuit. Its unit is Ω. We can also write $Z = \dfrac{E_0}{I_0} = \dfrac{E_v}{I_v}$

Values of ϕ and Z for different types of circuits is given in the table below:

Type of circuit	Phase factor	Impedance
Purely resistive circuit	$\phi = 0°$	$Z = R$
Purely inductive circuit	$\phi = -\pi/2$	$Z = X_L = \omega L$
Purely capacitive circuit	$\phi = \pi/2$	$Z = X_C = \dfrac{1}{\omega C}$

The reciprocal of reactance is called susceptance and that of impedance is called admittance.

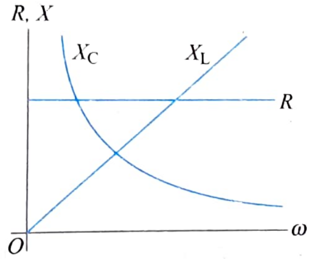


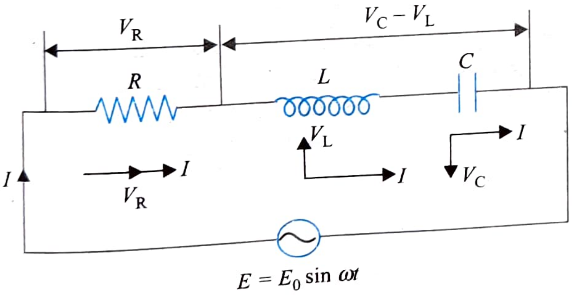
Figure shows how the resistance of a resistor and the reactances of an inductor and a capacitor vary with angular frequency, ω. Resistance R is independent of frequency, while the reactances X_L and X_C are not. If $\omega = 0$, corresponding to a dc circuit, there is no current through a capacitor because $X_C \to \infty$, and there is no inductive effect because $X_L = 0$. In the limit $\omega \to \infty$, X_L also approaches infinity, and the current through an inductor becomes vanishingly small; recall that the self-induced emf opposes rapid changes in the current. In this same limit, X_C and the voltage across a capacitor both approach zero; the current changes direction so rapidly that no charge can build up on either plate.

AC CIRCUIT CONTAINING RESISTOR, INDUCTOR, AND CAPACITOR IN SERIES (SERIES LCR CIRCUIT)

Figure shows a circuit in which a resistor of resistance R, an inductor of inductance L, and a capacitor of capacitance C are connected in series across an alternating source of emf

$$E = E_0 \sin \omega t \qquad \text{...(i)}$$

We want to determine the instantaneous current I in the circuit and its phase relationship to the applied voltage.



Let us assume that current I is flowing in the circuit as

$$I = I_0 \sin(\omega t + \phi) \qquad \text{...(ii)}$$

where I_0 is peak current and ϕ is the phase by which current leads emf.

V_R, V_L, and V_C are the instantaneous voltages across R, L and C, respectively. We can write the following relations:

$$V_R = V_{R0} \sin(\omega t + \phi) \qquad \text{...(iii)}$$

$$V_L = V_{L0} \sin\left(\omega t + \phi + \frac{\pi}{2}\right) \qquad \text{...(iv)}$$

$$= V_{L0} \cos(\omega t + \phi)$$

$$V_C = V_{C0} \sin\left(\omega t + \phi - \frac{\pi}{2}\right) \qquad \text{...(v)}$$

$$= -V_{C0} \cos(\omega t + \phi)$$

The above equations are written keeping in mind the phase relationship between current and voltage across each of the element resistor, inductor, and capacitor.

Equation (iii) shows that current and voltage across R are in phase (having same phase $\omega t + \phi$).

Equation (iv) shows that voltage across inductor leads the current by $\pi/2$.

Equation (v) shows that voltage across capacitor lags current by $\pi/2$.

V_{R0}, V_{L0}, and V_{C0} are the peak values of voltages across R, L, and C, respectively. We can write

$$\begin{bmatrix} V_{R0} = I_0 R \\ V_{L0} = I_0 X_L \\ V_{C0} = I_0 X_C \end{bmatrix} \qquad \text{...(vi)}$$

At any instant:

$$E = V_R + V_L + V_C$$

$$\Rightarrow \quad E_0 \sin \omega t = V_{R0} \sin(\omega t + \phi) + (V_{L0} - V_{C0}) \cos(\omega t + \phi) \qquad \text{...(vii)}$$

Equation (vii) is a general equation and it is true at any instant of time. Replacing ωt by $\omega t + \pi/2$ in Eq. (vii), we get

$$E_0 \cos \omega t = V_{R0} \cos(\omega t + \phi) - (V_{L0} - V_{C0}) \sin(\omega t + \phi) \qquad \text{...(viii)}$$

Squaring and adding (vii) and (viii),

$$E_0^2 = V_{R0}^2 + (V_{L0} - V_{C0})^2 \qquad \text{...(ix)}$$

$$\Rightarrow \quad = I_0^2\left[R^2 + (X_L - X_C)^2\right] \qquad \text{[using (vi)]}$$

$$\Rightarrow \quad \frac{E_0}{I_0} = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\Rightarrow \quad Z = \sqrt{R^2 + (X_L - X_C)^2} \qquad \text{...(x)}$$

Equation (x) gives impedance of the circuit.

Dividing Eq. (vii) by Eq. (viii) and simplifying, we get

$$\tan \phi = \frac{V_{C0} - V_{L0}}{V_{R0}} \qquad \text{...(xi)}$$

$$\Rightarrow \quad = \frac{I_0 X_C - I_0 X_L}{I_0 R}$$

$$\Rightarrow \quad = \frac{X_C - X_L}{R} \qquad \text{...(xii)}$$

Using Eq. (xii), we can find ϕ, the phase difference between current and voltage. It is to be noted that $-\pi/2 \le \phi, \le \pi/2$.

If $X_C < X_L$, then ϕ is negative

If $X_C > X_L$, then ϕ is positive

If $X_C = X_L$, then $\phi = 0$

If $R = 0$, then $\phi = \dfrac{\pi}{2}$ (if $X_C > X_L$)

or $\quad \phi = -\dfrac{\pi}{2}$ (if $X_L > X_C$)

Note:

Equation (ix) can also be written in terms of virtual voltages.

$$E_v^2 = V_{Rv}^2 + (V_{Lv} - V_{Cv})^2$$

because $E_0 = \sqrt{2}\,E_v, V_{R0} = \sqrt{2}\,V_{Rv}, V_{L0} = \sqrt{2}\,V_{Lv}$

and $V_{C0} = \sqrt{2}\,V_{Cv}$

Similarly, Eq. (xi) can also be written in terms of virtual voltages:

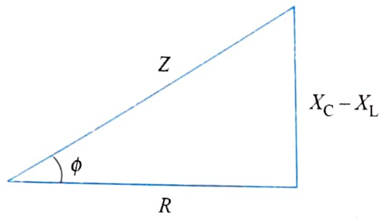
$$\tan\phi = \frac{V_{Cv} - V_{Lv}}{V_{Rv}}$$

We can also write $V_{RV} = I_{VR}$

$$V_{LV} = I_V X_L$$

$$V_{CV} = I_V X_C$$

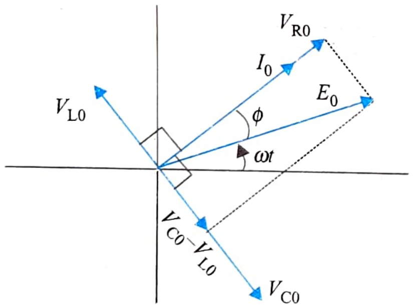
Impedance triangle: Equations (x) and (xii) are shown graphically in figure. It is known as impedance diagram or impedance triangle.



Hypotenuse of this triangle is the impedance of the circuit:

$$\cos\phi = \frac{R}{Z},\ \sin\phi = \frac{X_C - X_L}{Z}$$

Figure below shows the phasor diagram of series *LCR* circuit. Equations (ix) and (xi) can be obtained from this diagram

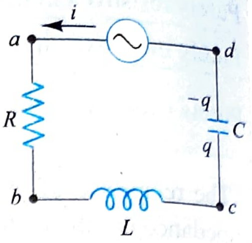


All of the expressions that we have developed for an *L-R-C* series circuit are still valid if one of the circuit elements is missing. If the resistor is missing, we set $R = 0$; if the inductor is missing, we set $L = 0$, but if the capacitor is missing, we set $C = \infty$, corresponding to the absence of any potential difference ($V_C = q/C = 0$) or any capacitive reactance ($X_C = 1/\omega C = 0$).

Finally, we remark that in this section we have been describing the steady state condition of a circuit, the state that exists after the circuit has been connected to the source for a long time. When the source is first connected, there may be additional voltages and currents, called transients, whose nature depends on the time in the cycle when the circuit is initially completed. A detailed analysis of transients is beyond our scope. They always die out after a sufficiently long time and they do not affect the steady state behaviour of the circuit. But they can cause dangerous and damaging surges in power lines, which is why delicate electronic systems such as computers are often provided with power-line surge protectors.

ILLUSTRATION 6.7

In the series circuit of figure, suppose $R = 300\ \Omega$, $L = 60$ mH, $C = 0.50\ \mu$F, source amplitude is $E_0 = 50$V and $\omega = 10{,}000$ rad s^{-1}. Find the reactances X_L and X_C, the impedance Z, the current amplitude I_0, the phase angle ϕ, and the voltage amplitude across each circuit element.



Sol. The inductive and capacitive reactances are

$$X_L = \omega L = (10{,}000\text{ rad/s})(60\text{ mH}) = 600\ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{(10{,}000\text{ rad s}^{-1})(0.50\times10^{-6}\text{ F})} = 200\ \Omega$$

The impedance Z of the circuit is

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{(300\,\Omega)^2 + (600\,\Omega - 200\,\Omega)^2} = 500\ \Omega$$

With source voltage amplitude $E_0 = 50$ V, the current amplitude is $I_0 = \dfrac{E_0}{Z} = \dfrac{50\text{ V}}{500\,\Omega} = 0.10$ A

The phase angle ϕ is

$$\phi = \tan^{-1}\frac{X_C - X_L}{R} = \tan^{-1}\left(-\frac{400\,\Omega}{300\,\Omega}\right) = -53°$$

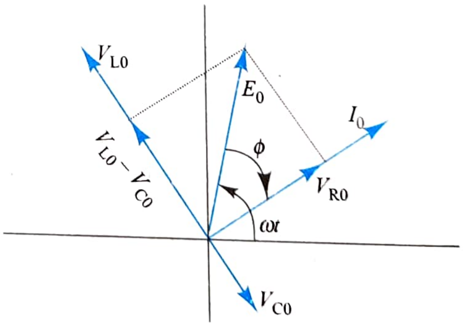
The voltage amplitudes V_{RO}, V_{LO}, and V_{CO} across the resistor, inductor, and capacitor, respectively, are

$$V_{RO} = I_0 R = (0.10\text{ A})(300\ \Omega) = 30\text{ V}$$

$$V_{LO} = I_0 X_L = (0.10\text{ A})(600\ \Omega) = 60\text{ V}$$

$$V_{CO} = I_0 X_C = (0.10\text{ A})(200\ \Omega) = 20\text{ V}$$

Note that $X_L > X_C$ and hence the voltage amplitude across the inductor is greater than that across the capacitor and ϕ is negative. The value $\phi = -53°$ means that the voltage leads the current by 53°; this is like the situation shown in figure.



Note that the source voltage amplitude $V = 50$ V is not equal to the sum of the voltage amplitude across the separate circuit elements (that is, 50 V π 30 V + 60 V + 0 V).

ILLUSTRATION 6.8

For the *L-R-C* series circuit described in the above Illustration 6.7, describe the time dependence of the instantaneous current and each instantaneous voltage.

Sol. In Illustration 6.7, we found the amplitude of the current and voltages. Now we have to find the expressions for the instantaneous values of the current and voltages. As we learned, the voltage across a resistor is in phase with the current but the voltages across an inductor or capacitor are not. We also learned in this section that ϕ is the phase angle between the source voltage and the current.

The current and all the voltages oscillate with the same angular frequency.

Let us choose

$$E = E_0 \sin \omega t \quad \Rightarrow \quad E = 50\sin(10{,}000t)$$

then $I = I_0 \sin(10{,}000t + \phi)$,

where $\phi = -53° = -\dfrac{53 \times \pi}{180}$ rad $= -0.93$ rad

$$\Rightarrow \quad I = 0.10 \sin(10{,}000t - 0.93)$$

Current and voltage are in phase in resistor so

$$V_R = V_{R0} \sin(10{,}000t - 0.93) = 30 \sin(10{,}000t - 0.93)$$

In inductor voltage leads current by $\pi/2$ so

$$V_L = V_{L0} \sin[10{,}000t - 0.93 + (\pi/2)] = 60 \cos(10{,}000t - 0.93)$$

In capacitor, voltage lags current by $\pi/2$ so

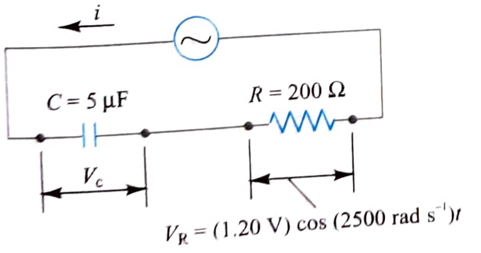
$$V_C = V_{C0} \sin[10{,}000t - 0.93 - (\pi/2)]$$
$$= -20 \cos(10{,}000t - 0.93)$$

LLUSTRATION 6.9

A 200 Ω resistor is connected in series with a 5 μF capacitor. The voltage across the resistor is $V_R = (1.20 \text{ V}) \cos (2500 \text{ rad s}^{-1})t$.

(a) Derive an expression for the circuit current.
(b) Determine the capacitive reactance of the capacitor.
(c) Derive an expression for the voltage across the capacitor.

Sol. This is a series circuit. The current is the same through the capacitor as through the resistor. Our target variables are current i, capacitive reactance X_C, and capacitor voltage V_C.



(a) Using $V_R = iR$, we find that current i in the resistor and through the circuit as a whole is

$$i = \frac{V_R}{R} = \frac{(1.20 \text{ V}) \cos(2500 \text{ rad s}^{-1})t}{200\ \Omega}$$
$$= (6.0 \times 10^{-3}\text{A}) \cos (2500 \text{ rad s}^{-1})t$$

(b) The capacitive reactance at $\omega = 2500$ rad s^{-1} is

$$X_C = \frac{1}{\omega C} = \frac{1}{(2500 \text{ rad s}^{-1})(5.0 \times 10^{-6} \text{ F})} = 80\,\Omega$$

(c) The amplitude V_C of the voltage across the capacitor is

$$V_{C0} = I_0 X_C = (6.0 \times 10^{-3}\text{A})\ (80\ \Omega) = 0.48 \text{ V}$$

The instantaneous capacitor voltage V_C is given by

$$V_C = V_{C0} \cos(\omega t - 90°]$$
$$= (0.48 \text{ V}) \cos [(2500 \text{ rad s}^{-1})t - \pi/2 \text{ rad}]$$

ILLUSTRATION 6.10

A 200 Ω resistor and 1 H inductor are joined in series with an ac source of emf $10\sqrt{2}\ \sin(200\ t)$ V. Calculate the phase difference between emf and current.

Sol. $L = 1$ H, $R = 200\ \Omega$, $E = 10\sqrt{2}\sin(200t)$ V

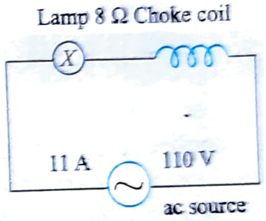
$$\tan\phi = \frac{X_C - X_L}{R} = \frac{-\omega L}{R} = -\frac{200 \times 1}{200} = -1$$

$$\Rightarrow \quad \phi = \tan^{-1}(-1) = -\frac{\pi}{4}$$

ϕ is negative, hence current lags behind voltage by $\pi/4$.

ILLUSTRATION 6.11

A lamp with a resistance of 8 Ω is connected to a choke coil as shown in figure. This arrangement is connected to an alternating source of 110 V. The current in the circuit is 11 A. The frequency of the ac is 60 Hz. Find



(a) the impedence of the circuit, and
(b) the value of inductive reactance of the choke coil.

Sol. $R = 8\ \Omega$, $\omega = 2\pi \times 60 = 120\pi$ Hz, $I_{rms} = 1$ A, $E_{rms} = 110$ V

(a) Impedence $(Z) = \sqrt{R^2 + X_L^2} = \dfrac{E_{rms}}{I_{rms}} = \dfrac{110}{11} = 10\ \Omega$

(b) $Z^2 = R^2 + X_L^2$

$$10^2 = 8^2 + X_L^2 \quad \text{or} \quad X_L = \sqrt{(100 - 64)} = \sqrt{36} = 6\,\Omega$$

ILLUSTRATION 6.12

A resistor of 200 Ω and a capacitor of 15.0 μF are connected in series to a 220 V, 50 Hz source.

(a) Calculate the current in the circuit.
(b) Calculate the voltage (rms) across the resistor and the inductor. Is the algebraic sum of these voltages more than the source voltage? If yes, resolve the paradox.

Sol. Given: $R = 200\ \Omega$, $C = 15.0\ \mu\text{F} = 15.0 \times 10^{-6}$ F
$E_{rms} = 220$ V, $f = 50$ Hz

(a) In order to calculate the current, we need the impedance of the circuit.

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + (2\pi f C)^{-2}}$$
$$= \sqrt{(200\ \Omega)^2 + (2 \times 3.14 \times 50 \times 15 \times 10^{-6} \text{ F})^{-2}}$$
$$= \sqrt{(200\ \Omega)^2 + (212\ \Omega)^2} = 291.5\,\Omega$$

Therefore, the current in the circuit is

$$I_{\text{rms}} = \frac{E_{\text{rms}}}{Z} = \frac{220\text{ V}}{291.5\,\Omega} = 0.755\text{ A}$$

(b) Since the current is the same throughout the circuit,

$$V_R = I_{\text{rms}} R = (0.755\text{ A})(200\,\Omega) = 151\text{ V}$$

$$V_C = I_{\text{rms}} X_C = (0.755\text{ A})(212\,\Omega) = 160\text{ V}$$

The algebraic sum of the two voltages V_R and V_C is 311 V, which is more than the source voltage of 220 V. How to resolve this paradox? You may recall that the two voltages are not in the same phase. Therefore, they cannot be added like the ordinary numbers. The two voltages are out of phase by 90°. Therefore, the total of these voltages must be obtained using the Pythagoras theorem:

$$V_{RC} = \sqrt{V_R^2 + V_C^2} = \sqrt{(151\text{V})^2 + (160\text{ V})^2} = 220\text{ V}$$

Thus, if the phase difference between two voltages is properly taken into account, the total voltage across the resistor and the inductor is equal to the voltage of the source.

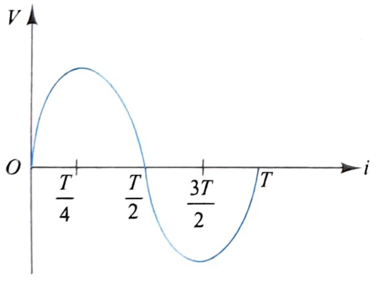
ILLUSTRATION 6.13

In an LR series circuit, a sinusoidal $V = V_0 \sin \omega t$ is applied. It is given that $L = 35$ mH, $R = 11\,\Omega$, $V_{\text{rms}} = 220$ V, $\omega/2\pi = 50$ Hz and $\pi = 22/7$. Find the amplitude of the current in steady state and obtain the phase difference between the current and the voltage. Also plot the variation of the current for one cycle on the given graph.

Sol. Inductive reactance,

$$X_L = \omega L = 2\pi \times 50 \times 35 \times 10^{-3}$$

$$= 2 \times \frac{22}{7} \times 50 \times 35 \times 10^{-3} = 11\,\Omega$$



Therefore, impedance of circuit

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{11^2 + 11^2} = 11\sqrt{2}\,\Omega$$

Given $V_{\text{rms}} = 220$ V

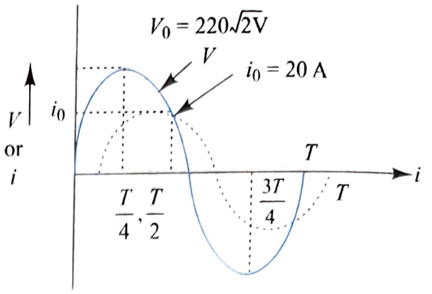
Amplitude of voltage $V_0 = \sqrt{2}\,V_{\text{rms}} = 220\sqrt{2}$ V

$$\text{Amplitude of current } i_0 = \frac{V_0}{Z} = \frac{220\sqrt{2}}{11\sqrt{2}} = 20\text{A}$$

Phase lag of current over voltage

$$\phi = \tan^{-1}\frac{\omega L}{R} = \tan^{-1}\frac{11}{11} = \tan^{-1}(1) = \frac{\pi}{4}$$

Voltage and current variations are shown in figure.



The voltage and the current are expressed as $V = 220\sqrt{2}\sin \omega t$

$$I = 20 \sin\left(\omega t - \frac{\pi}{4}\right)$$

POWER IN SERIES LCR CIRCUITS

Alternating currents play a central role in the system for distributing, converting, and using electrical energy, so it is important to look at power relationships in ac circuits.

The power in an electric circuit is the rate at which electrical enegy is consumed in the circuit. Let an emf $E = E_0 \sin \omega t$ be applied to a series LCR circuit. The curent in the circuit is $I = I_0 \sin(\omega t + \phi)$, where ϕ is the phase difference between current and voltage. The instantaneous power is given by

$$P = EI = E_0 I_0 \sin \omega t \sin(\omega t + \phi)$$

If the average power consumed in the cycle from time t_1 to t_2 is P_{av}, then

$$P_{\text{av}}(t_2 - t_1) = \int_{t_1}^{t_2} P dt$$

Average power over 1 cycle, i.e., $t_1 = 0$ to $t_2 = T$ is given by

$$P_{\text{av}} T = \int_0^T E_0 I_0 \sin \omega t \sin(\omega t + \phi)\, dt$$

$$\Rightarrow \quad P_{\text{av}} T = E_0 I_0 \int_0^T \left(\sin^2 \omega t \cos\phi + \frac{\sin 2\omega t}{2}\sin\phi\right) dt$$

$$\left[\begin{array}{ll} \text{remember} & \int_0^T \sin^2 \omega t\, dt = \frac{T}{2} \\ \text{and} & \int_0^T \sin 2\omega t\, dt = 0 \end{array}\right]$$

$$\Rightarrow \quad P_{\text{av}} T = \frac{E_0 I_0}{2}\cos\phi\, T$$

$$\Rightarrow \quad P_{\text{av}} = E_v I_v \cos\phi$$

$$= P_{\text{ap}} \cos\phi,$$

where $P_{\text{ap}} = E_v I_v \rightarrow$ apparent power or virtual power

and $\quad \cos\phi = \dfrac{P_{\text{av}}}{P_{\text{ap}}} \rightarrow$ power factor

also $\quad P_{\text{av}} = E_v I_v \cos\phi = (I_v Z) I_v \dfrac{R}{Z} = I_v^2 R$

For pure resistive circuit: $\phi = 0$ and $Z = R$

$$P_{\text{av}} = E_v I_v = (I_v Z) I_v = I_v^2 Z = I_v^2 R$$

For pure inductive circuit: $\phi = -\dfrac{\pi}{2}$

So $P_{av} = 0$

Therefore, the average power over a complete cycle of ac through an ideal inductor is zero. Actually, whatever energy is needed in building up current in inductance is returned back during the decay of current.

For pure capacitive circuit: $\phi = \frac{\pi}{2}$

$P_{av} = 0$

In this case too, the average power is zero. Actually, whatever energy is needed in building up the voltage across capacitor is returned to the source during discharging of capacitor.

So average power consumed in pure inductive or pure capacitive circuit is zero. So there is no loss of energy in the inductor or capacitor. But in a resistance, loss of energy occurs. It cannot store the energy like inductor or capacitor.

The current through pure L or C, which dissipates no power is called idle current or wattless current. L and C are most suitable for controlling the current in ac circuits.

WATTLESS COMPONENT OF CURRENT

$I_v \sin\phi$ is known as wattless component of current.

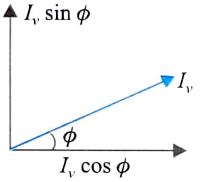


ILLUSTRATION 6.14

The potential difference E and current I flowing through the ac circuit is given by $E = 5\cos(\omega t - \pi/6)$ V and $I = 10\sin\omega t$ A. Find the average power dissipated in the circuit.

Sol.

$E = 5\cos[\omega t - (\pi/6)]$, $I = 10\sin\omega t = 10\cos[\omega t - (\pi/2)]$

Phase difference between E and I is given by:

$$\phi = \frac{\pi}{2} - \frac{\pi}{6} = \frac{2\pi}{6} = \frac{\pi}{3}$$

Average power dissipated:

$$P = \frac{E_0 I_0}{2}\cos\phi = \frac{5\times 10}{2}\times\frac{1}{2} = 12.5\text{ W}$$

CHOKE COIL

Choke coil is required to reduce the current in a given circuit with a minimum wastage of energy when the current is derived at constant voltage from a given supply. In dc circuits, a resistance is used for this purpose but there is a wastage of energy i^2R. Similarly, if in ac circuit, resistance is used, the wastage of energy will also be i^2R. However, with an alternating supply, a more economical method to reduce the current without any wastage is by introducting an inductance. The inductance sets up a back emf which opposes the applied emf. So, the current is reduced. When the resistance of the inductance is negligible, no power is dissipated. Such an inductance coil used in ac circuits to limit the current without dissipation of power is called a choke coil.

RESONANCE IN SERIES LCR CIRCUIT

When the current in the series LCR circuit is maximum possible, then the circuit is said to be in resonance. We know that in a series LCR circuit, amplitude of current is given by

$$I_0 = \frac{E_0}{Z} = \frac{E_0}{\sqrt{R^2 + (X_L - X_C)^2}}$$

where $X_L = \omega L$ and $X_C = 1/\omega C$

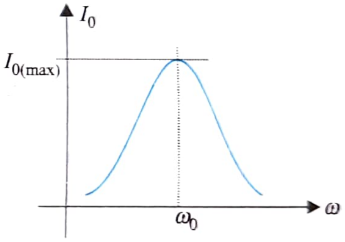
If we vary the angular frequency ω of the source, X_L and X_C both vary and I_0 also varies. I_0 is maximum when Z is minimum. Z is minimum when $X_L = X_C$. Let at $\omega = \omega_0$ when $X_L = X_C$

$$\Rightarrow \quad \omega_0 L = \frac{1}{\omega_0 C} \quad \Rightarrow \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

$$\Rightarrow \quad 2\pi f_0 = \frac{1}{\sqrt{LC}} \quad \Rightarrow \quad f_0 = \frac{1}{2\pi\sqrt{LC}}$$

So at $\omega = \omega_0$, maximum possible current flows in circuit and the circuit is said to be in resonance. ω_0 is known as resonance frequency. At resonance frequency, impedance is minimum which is given by $Z_{min} = R$ and at resonance, the value of I_0 is maximum which is given by

$$I_{0(\max)} = \frac{E_0}{R}$$

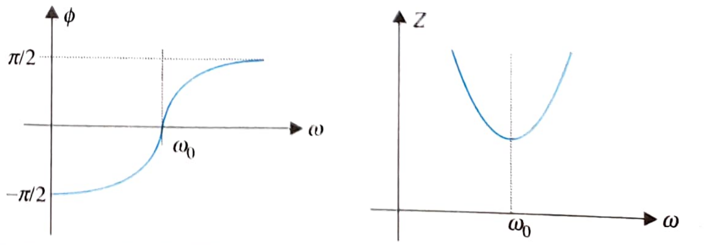


The variation of I_0 with ω is as shown in figure.

A series resonance circuit is used in tuning circuits. The tuning circuits of radio and TV sets, etc. are series LCR circuits.

The instantaneous voltages across L and C always differ in phase by 180°; they have opposite sign at each instant. At the resonance frequency, $X_L = X_C$ and the voltages $V_L = V_C$. Then the instantaneous voltages across L and C add to zero at each instant, and the total voltage across the L–C combination is exactly zero. The voltage across the resistor is then equal to the source voltage. So at the resonance frequency, the circuit behaves as if the inductor and the capacitor were not there at all.

This variation of ϕ and Z with angular frequency is shown in figure.



Note:

At resonance power, factor is maximum. It is given by

$$\cos\phi = \frac{Z}{R} = \frac{R}{R} = 1 \qquad (\because \; Z = R \text{ at resonance})$$

Thus, power factor is unity at resonance. Maximum power is dissipated during resonance.

Q-FACTOR (OR QUALITY FACTOR) OF RESONANCE

The quality factor Q is a parameter which is used to describe the sharpness of resonance curve. The quality factor Q of a series resonance circuit is defined as the ratio of voltage drop across inductor (or capacitor) at resonance to the applied voltage. Thus,

$$Q = \frac{\text{voltage across } L \text{ (or } C\text{) at resonance}}{\text{applied voltage}} \qquad \text{(i)}$$

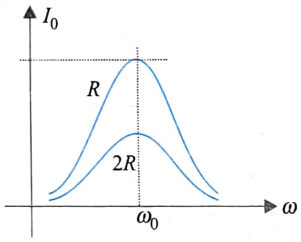
$$= \frac{I_v \omega_0 L}{I_v R} = \omega_0 \left(\frac{L}{R}\right)$$

$$= \frac{1}{\sqrt{LC}} \times \frac{L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}} \qquad \text{(ii)}$$

The rapidity with which the current falls from its resonant value $I_{0(\max)} = E_0/R$ with change in applied frequency is known as sharpness of resonance. More quickly the current falls from its maximum value with change in frequency, more is the sharpness of resonance.

If Q-factor is high, the current will respond to very narrow range of frequencies and the resonance will be sharper. Usual value of Q is 10 to 100 but it can go upto 200. Q-factor is a dimensionless quantity. Equation (ii) shows that when R increases, Q-factor of the circuit decreases.

In figure, two curves are plotted, one when the circuit resistance is low, i.e., R, and the other when the circuit resistance is high, i.e., $2R$. We observe the following points:



- The peak of the curve depends on the resistance of the circuit. When R is low, the peak is high and vice-versa. The peak is related to sharpness of resonance. More is height of peak, more sharp is the resonance.
- It is clear from figure that the width of resonance curve increases on increasing the value of R. Therefore, the resonance curve becomes flatter.
- The series resonant circuit is sometimes called the acceptor circuit. The reason is that impedance of the circuit is minimum at resonance and, due to this fact, it readily accepts that current out of the many currents whose frequency is equal to its resonant frequency.

Alternate method: The quality factor Q is also defined as 2π times the ratio of the energy stored in L (or C) to the average energy loss per period. Therefore,

$$Q = 2\pi\left[\frac{\text{maximum energy stored in circuit}}{\text{energy loss per period}}\right]$$

The maximum energy stored in inductor: $U = \dfrac{1}{2}L(I_0)^2$

The energy dissipated per second: $P_R = I_{\text{rms}}^2 R = \dfrac{I_0^2 R}{2}$

Energy dissipated per time period: $U_R = \dfrac{I_0^2 R}{2} \times T$

Substituting these values, we get

$$Q = 2\pi\left[\frac{\frac{1}{2}LI_0^2}{\left(\frac{I_0^2 R}{2}\right)T}\right] = \frac{2\pi}{T} \times \left(\frac{L}{R}\right)$$

$$= \omega_0\left(\frac{L}{R}\right) = \frac{1}{\sqrt{LC}} \times \frac{L}{R} \qquad \left(\because\ \omega_0 = \frac{1}{\sqrt{LC}}\right)$$

$$= \frac{1}{R}\sqrt{\frac{L}{C}}$$

So, the Q-factor of the circuit varies inversely as R. Thus, at resonance, the voltage drop across inductance or capacitance is Q-times the applied voltage.

ANOTHER DEFINITION OF Q-FACTOR

We know that current in series LCR is given by

$$I_v = \frac{E_v}{\sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2}} \qquad \text{...(i)}$$

Now consider the value of I_v for which power dissipated in the circuit is half of maximum power. For this we have

$$P = \frac{P_{\max}}{2} = I_v^2 R, \quad \text{where} \quad P_{\max} = I_{v(\max)}^2 R$$

From above: $I_v = \dfrac{I_{v(\max)}}{\sqrt{2}} = \dfrac{E_v}{R\sqrt{2}}$

Putting the values of I_v in Eq. (i):

$$\frac{E_v}{R\sqrt{2}} = \frac{E_v}{\sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2}}$$

$$\Rightarrow \quad \sqrt{2}\,R = \sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2}$$

$$\Rightarrow \quad 2R^2 = R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2$$

$$\Rightarrow \quad R^2 = \left(\frac{1}{\omega C} - \omega L\right)^2$$

$$\Rightarrow \quad \frac{1}{\omega C} - \omega L = \pm R$$

$$\Rightarrow \quad 1 - \omega^2 LC = \pm R\omega C$$

$$\Rightarrow \quad LC\omega^2 \pm RC\omega - 1 = 0$$

From here, we get

$$\omega = \frac{\mp RC \pm \sqrt{R^2C^2 + 4LC}}{2LC}$$

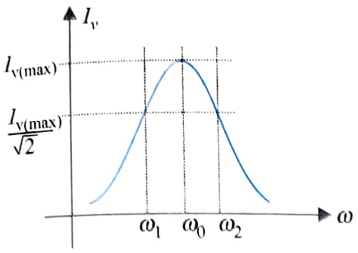
As ω cannot be negative, so

$$\omega = \frac{\mp RC + \sqrt{R^2C^2 + 4LC}}{2LC}$$

We get the following two values of ω

$$\omega_1 = \frac{-RC+\sqrt{R^2C^2+4LC}}{2LC} \text{ and } \omega_2 = \frac{RC+\sqrt{R^2C^2+4LC}}{2LC}$$

These values are indicated in figure also.



These two frequencies ω_1 and ω_2 are known as half power frequencies. From these, we can get

$$\omega_1\omega_2 = \frac{1}{LC} \text{ and } \omega_2 - \omega_1 = \frac{R}{L}$$

$\omega_2 - \omega_1 = 2\Delta\omega$ is known as band width.

Now the ratio $\frac{\omega_0}{2\Delta\omega}$ is also known as Q-factor. Thus,

$$Q = \frac{\omega_0}{2\Delta\omega} = \frac{\omega_0}{\omega_2-\omega_1} = \frac{1}{\sqrt{LC}} \times \frac{L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Lesser the band width, sharper is the resonance.

LLUSTRATION 6.15

A series LCR circuit with $L = 0.12$ H, $C = 480$ nF, and $R = 23$ Ω is connected to a 230-V variable frequency supply.

(a) What is the source frequency for which current amplitude is maximum? Find this maximum value.

(b) What is the source frequency for which average power absorbed by the circuit is maximum? Obtain the value of maximum power.

(c) For which frequencies of the source is the power transferred to the circuit half the power at resonant frequency?

(d) What is the Q-factor of the circuit?

Sol.

(a) Current amplitude is maximum at resonance and source frequency at resonance is given by

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.12\times480\times10^{-9}}} = 663.14 \text{ Hz}$$

and $\omega_0 = 2\pi f_0 = 4166.66$ rad s^{-1}

The maximum current amplitude is given by

$$I_{0(max)} = \frac{E_0}{R} = \frac{230\sqrt{2}}{23} = 14.14 \text{ A}$$

(b) Average power absorbed by the circuit is maximum at resonanceforwhichfrequencyiscalcultedaboveas663.14Hz. Maximum power absorbed is given by

$$P_{max} = \frac{E_v^2}{R} = \frac{(230)^2}{23} = 2300 \text{ W}$$

(c) $P = E_v I_v \cos\phi = E_v I_v \frac{R}{Z} = \frac{E_v^2}{Z^2}R$ and $P_{max} = \frac{E_v^2}{R}$

Given $P = P_{max}/2$

$$\Rightarrow \frac{E_v^2 R}{Z^2} = \frac{1}{2}\frac{E_v^2}{R} \Rightarrow Z^2 = 2R^2$$

$$\Rightarrow R^2 + (X_C - X_L)^2 = 2R^2$$

$$\Rightarrow X_C - X_L = \pm R$$

Solve to get

$$\omega_1 = \frac{-RC+\sqrt{R^2C^2+4LC}}{2LC} = 4071.9 \text{ rad s}^{-1}$$

$$\text{and } \omega_2 = \frac{RC+\sqrt{R^2C^2+4LC}}{2LC} = 4263.6 \text{ rad s}^{-1}$$

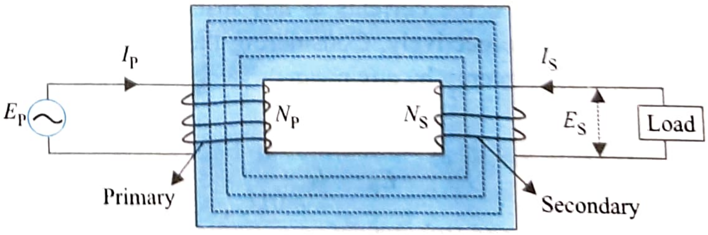
(d) Q-factor is given by

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{23}\sqrt{\frac{0.12}{480\times10^{-9}}} = 21.74$$

TRANSFORMERS

One of the great advantages of ac over dc for electric-power distribution is that it is much easier to step voltage levels up and down with ac than with dc. For a long distance power transmission, it is desirable to use as high a voltage and as small a current as possible; this reduces i^2R losses in the transmission lines, and as smaller wires can be used, it saves material costs. Present day transmission lines routinely operate at rms voltages of the order of 500 kV. On the other hand, safety considerations and insulation requirements dictate relatively low voltages in generating equipment and in household and industrial power distribution. The standard voltage for household wiring is 120 V in the United States and Canada and 240 V in many other countries. The necessary voltage conservation is accomplished by the use of transformers.

A transformer is an ac device which transfers electric power from one circuit to another. It can raise or lower the voltage in a circuit but with a corresponding decrease or increase in current. Here it is worth mentioning that while transferring the power, frequency is not altered. Figure shows a basic transformer.



As evident, any transformer has two coils electrically insulated but magnetically linked. By magnetic link we mean that the two coils are wound on a common core. The energy from one coil is transferred to other coil by means of magnetic coupling. The coil which receives energy from an ac source is called primary and the coil which delivers the energy to the load is called secondary.

A transformer operates on the principle of mutual induction. When an alternating voltage is applied to the primary coil, an alternating voltage is set up in it. As the winding is linked with a magnetic emf, it produces an alternating flux in the core. This alternating flux links with the turns of the secondary coil. Since the flux is alternating in magnitude, it induces a mutually induced emf in secondary coil of the same frequency as the flux. This follows from Faraday's law of electromagnetic induction, i.e., $e = -M(di/dt)$. Because of this induced emf, the secondary coil is capable of supplying current and, hence, energy.

Theory and working: Let N_P and N_S be the number of turns in the primary and secondary coils, respectively. Here we assume that the entire magnetic flux of the primary coil is linked to the secondary coil. This results in the same magnetic flux per turn linked with each of the two coils. Let ϕ be the flux linked with each turn of either coil at any instant. Then

$$\text{emf induced in primary coil, } E_P = -\frac{N_P d\phi}{dt}$$

$$\text{emf induced in secondary coil, } E_S = -\frac{N_S d\phi}{dt}$$

If an alternating emf E_P is applied across that primary coil, then Kirchhoff's loop law applied to primary circuit gives

$$E_P - N_P\frac{d\phi}{dt} = 0$$

or
$$E_P = N_P\frac{d\phi}{dt} \qquad \text{...(i)}$$

Here, we neglect the resistance of primary circuit.

If E_S represents the alternating emf in secondary coil, then

$$E_S = -N_S\frac{d\phi}{dt} \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{E_S}{E_P} = -\frac{N_S}{N_P} \quad \text{or} \quad E_S = -\frac{N_S}{N_P}E_P$$

The negative sign shows that E_S is 180° out of phase with E_P. Ignoring negative sign,

$$\frac{E_S}{E_P} = \frac{N_S}{N_P} \qquad \text{...(iii)}$$

Let I_P and I_S be the currents at any instant in primary and seconary coils, respectively. Considering the transformer to be ideal (no loss of energy by any means),

$$E_P I_P = E_S I_S \Rightarrow \frac{E_S}{E_P} = \frac{I_P}{I_S} \qquad \text{...(iv)}$$

From Eqs. (iii) and (iv), we get

$$\frac{E_S}{E_P} = \frac{N_S}{N_P} = \frac{I_P}{I_S}$$

or
$$I_S = I_P\left(\frac{N_P}{N_S}\right) = \frac{I_P}{K} \qquad \text{...(v)}$$

where $K = (N_S/N_P)$ = transformer ratio.

When $K > 1$, then $N_S > N_P$. The transformer is known as step up transformer. So, in a step up transformer, the number of turns in secondary coil is more than the number of turns in primary coil. This transformer converts a low voltage at high current to a high voltage at low current.

When $K > 1$, then $N_S < N_P$. The transformer is known as step-down transformer. So, in a step down transformer, the number of turns in primary coil is more than the number of turns in secondary coil. This transformer converts high voltage at low current to a low voltage at high current.

When a resistance R is introduced in the secondary circuit, then a current I_S flows in it. The current is in phase with the voltage. The current is given by $I_S = E_S/R$

$$\text{where } I_S = I_P\left(\frac{N_P}{N_S}\right) \quad \text{and} \quad E_S = E_P\left(\frac{N_S}{N_P}\right)$$

Putting these values, we get

$$I_P\left(\frac{N_P}{N_S}\right) = E_P \times \left(\frac{N_S}{N_P}\right) \times \frac{1}{R}$$

or
$$I_P = \frac{E_P}{(N_P/N_S)^2 R} = \frac{E_P}{R_{eq.}}$$

$$\text{where } R_{eq.} = \left(\frac{N_P}{N_S}\right)^2 R$$

This current I_P is the same as if we connect a resistance $R_{eq} = (N_P/N_S)^2 R$ across the primary. This effect is called impedance transformation. Impedance matching is an important function of the transformer.

Efficiency of a transformer: There are some losses of energy due to primary resistance, hysteresis in the core, eddy currents in the core, etc., in an ordinary tranformer. The efficiency of a transformer is defined as the ratio of output power to the input power

$$\eta = \frac{\text{output power}}{\text{input power}} = \frac{E_S I_S}{E_P I_P}$$

For an indeal transformer, $\eta = 1$ (100 %). Due to transformer. losses, the efficiency is less than one (less than 100 %).

It is important to mention that a transformer is an ac device and it cannot work on dc.

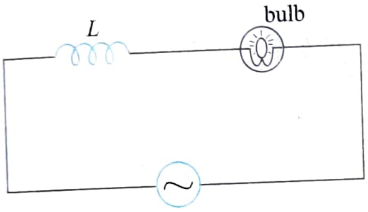
Applications: We know that power is generated at around 11 kV in a power-house. For transmission, we utilize higher ratings, say, around 110 kV or 200 kV. We need step-up transformers for this purpose. Again to supply voltage at load centres, it has to be reduced to, say, 3.6 kV and then to supply it to customers to 220 V. For this we need step-down transformers. This is the major application of a transformer. Here it would not be wrong to say that the wide application of ac, instead of dc, is due to the transformers alone. Since ac can be easily converted from high voltage to low voltage and vice-versa by using a transformer, it has led to widespread development of the ac systems.

SKIN EFFECT

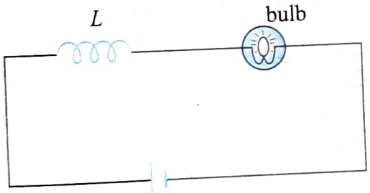
When a steady current flows through a cylindrical conductor, it is distriburted uniformly over the whole cross-section of the conductor. But when an alternating current of high frequency flows through the conductor, the curent density is not uniform throughout the cross-sectional area. The current density is more near the surface than inside the conductor. So the high frequency alternating current is confined to the surface layer. The phenomenon is called skin effect. Since the high frequency alternating current does not pass through the entire cross-section, hence the effective resistance of the conductor for ac is much higher, than dc. So in the transmission of ac, a bundle of wire is preferred than a thick wire to reduce the resistance caused by skin effect.

CONCEPT APPLICATION EXERCISE 6.1

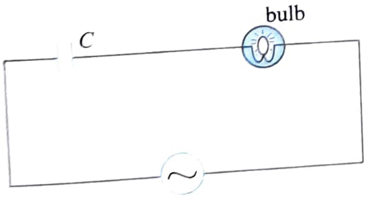
1. Can we use 15 Hz ac for lighting purposes?
2. A soft iron rod is inserted in the solenoid. After this what will be the effect on the brightness of the bulb?
3. In the above question, what will be the effect on the brightness of bulb if frequency of the source ω is increased?



4. An iron rod is inserted in the inductor, what will be the effect on the brightness of the bulb?



5. In the circuit shown in figure, what will be the effect on the brightness of bulb if frequency of the source ω is increased?



6. In the above question, what will be the effect on the brightness of bulb if capacitance C is reduced?
7. An inductor wire has some resistance R when connected to dc. It is connected to an ac source. Now the impedance of the wire will be more than R. (True/False)
8. The element of an electric heater is in the form of a coil. Once it is heated by dc voltage and then by ac voltage of equal potential difference. Will the production of heat in both cases be same or different?
9. When L and C are in series, voltage across them is in phase (True /False). The current in them is in phase (True/False).
10. In a series LCR circuit, the frequency of the source is more than resonance frequency. The current in the circuit leads the voltage in phase (True/False).
11. In a series LCR circuit, voltage drop across L can be more than applied voltage (True/False).
12. For circuits used for transporting electric power, a low power factor implies larger power loss in transmission. Explain?
13. At very high frequency of ac, capacitor behaves like a conductor. (True/False]
14. In a series LCR circuit, at resonance, power factor is __________.
15. Can we have resonance in LR or CR circuits?

ANSWERS

1. Yes **2.** Brightness will decrease
3. Brightness will decrease
4. First decrease, then becomes same
5. Brightness will increase **6.** Brightness will decrease
7. True **8.** Different **9.** False, True **10.** False
11. True **13.** True **14.** 1 **15.** No

Solved Examples

EXAMPLE 6.1

A 25 μF capacitor, a 0.1 H inductor and a 25 Ω resistor are connected in series with an ac source of emf $E = 310\sin 314t$. Find:

(a) the frequency of the emf
(b) the reactance of the circuit
(c) the impedance of the circuit
(d) the current in the circuit
(e) the phase angle
(f) the effective voltages across the capacitor, inductor, and resistor.

Sol.

(a) Given $\omega = 314 \Rightarrow 2\pi f = 314 \Rightarrow f = 50$ Hz

(b) Total reactance of the circuit is combination of capacitive and inductive reactance which is given by

$$|X_C - X_L| = \left|\frac{1}{\omega C} - \omega L\right| = \left|\frac{1}{314\times 25\times 10^{-6}} - 314\times 0.1\right|$$

$$= 96\ \Omega$$

(c) Impedance of the circuit is

$$Z = \sqrt{R^2 + (X_C - X_L)^2} = \sqrt{25^2 + 96^2} = 99.2\,\Omega$$

(d) We have to find rms current in the circuit. It is given by

$$I_v = \frac{E_v}{Z} = \frac{310/\sqrt{2}}{99.2} = 2.21\text{ A}$$

(e) Let ϕ be the angle by which current leads emf, then

$$\tan\phi = \frac{X_C - X_L}{R} = \frac{96}{25} \Rightarrow \phi = 75.4^\circ$$

(f) Effective volatges are given by

$$V_{Rv} = I_v R = 25 \times 2.21 = 55.26\text{ V}$$

$$V_{\text{Lv}} = I_{\text{v}}R_{\text{L}} = 2.21\times 314\times 0.1 = 69.4\text{ V}$$

$$V_{\text{Cv}} = I_{\text{v}}X_{\text{C}} = \frac{2.21\times 1}{314\times 25\times 10^{-6}} = 281.6\text{ V}$$

EXAMPLE 6.2

A circuit containing an 80 mH inductor and a 60 μF capacitor in series is connected to a 230 V–50 Hz supply. The resistance in the circuit is negligible.

(a) Obtain the current amplitude and rms current.

(b) Obtain the rms values of voltage across inductor and capacitor.

(c) What is the average power transferred to the inductor and to the capacitor?

(d) What is the total power absorbed by the circuit?

Sol.

(a) Given $L = 80$ mH, $C = 60$ μF, $E_{\text{v}} = 230$ V, $f = 50$ Hz

Since the circuit has no resistance, so impedance of the circuit is equal to its reactance.

$$Z = X = X_{\text{C}} - X_{\text{L}} = \frac{1}{100\,\pi C} - 100\pi L = 28\ \Omega$$

Current amplitude is: $I_0 = \dfrac{E_0}{Z} = \dfrac{230\sqrt{2}}{28} = 11.6$ A

(b) The rms volatge across capacitor is

$$V_{\text{Cv}} = I_{\text{v}}X_{\text{C}} = \frac{11.6/\sqrt{2}}{100\,\pi C} = 436\text{ V}$$

The rms voltage across inductor is

$$V_{\text{Lv}} = I_{\text{v}}X_{\text{L}} = (11.6/\sqrt{2})100\,\pi L = 206\ V$$

(c) Average power transferred to inductor and capacitor will be zero because whatever energy is used in building up of charge on capacitor (current in case of inductor) is returned in discharging the capacitor (decay of current in case of inductor).

(d) As there is no resistance in the circuit, power factor of the circuit is $\cos\phi = R/Z = 0$. Hence, no power is absorbed by the circuit.

EXAMPLE 6.3

An emf $E = 100\sin 314t$ V is applied across a pure capacitor of 637 μF. Find

(a) the instantaneous current i

(b) the instantaneous power p

(c) the frequency of power

(d) the maximum energy stored in the capacitor

Sol. Given $E = 100\sin(314t)$

We have $\omega = 314 \Rightarrow 2\pi f = 100\pi \Rightarrow f = 50$ Hz

Capacitive reactance:

$$X_{\text{C}} = \frac{1}{\omega C} = \frac{1}{314\times 637\times 10^{-6}} = 5\ \Omega$$

Since there is pure capacitor in the circuit, so current leads emf by $\phi = \pi/2$.

(a) Instantaneous current is given as

$$i = i_0\sin(\omega t + \phi) = i_0\sin[314t + (\pi/2)]$$

$$= \frac{E_0}{X_{\text{C}}}\cos(314t) = \frac{100}{5}\cos(314t) = 20\cos(314t)$$

(b) Instantaneous power is given by

$$p = EI = 100\sin(314t)\times 20\cos(314\,t) = 1000\sin(628\,t)$$

(c) From the expression of power above, we can calculate the frequency of power. We have

$$\omega_{\text{p}} = 628 \Rightarrow 2\pi f_{\text{p}} = 200\pi \Rightarrow f_{\text{p}} = 100\text{ Hz}$$

We see that the frequency of power is double that of the frequency of emf or current. This is obvious because during one cycle of oscillation, power becomes maximum and minimum two times.

(d) We know that energy in the capacitor is $U = \dfrac{1}{2}CE^2$. Energy stored in the capacitor will be maximum when $E = E_0$.

$$U_0 = \frac{1}{2}CE_0^2 = \frac{1}{2}(637\times 10^{-6})(100)^2 = 3.185\text{ J}$$

EXAMPLE 6.4

A radio can be tuned over a frequency range from 500 kHz to 1.5 MHz. If its L–C circuit has an effective inductance of 400 μH, what is the range of its variable capacitor?

Sol. We know that resonant frequency of a tuning circuit is given by $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$. From here we can find C, which is given as

$$C = \frac{1}{4\pi^2 f_0^2 L}$$

We are given the range of frequency and we have to find the range of variable capacitor. By putting the extreme values of frequency, we can find the range of capacitor.

By putting $f_0 = 500$ kHz, we get

$$C_1 = \frac{1}{4\pi^2(500\times 10^3)^2\times 400\times 10^{-6}} = 253\text{ pF}$$

By putting $f_0 = 1.5$ MHz, we get

$$C_2 = \frac{1}{4\pi^2(1.5\times 10^6)^2\times 400\times 10^{-6}} = 28\text{ pF}$$

So the range of variable capacitor comes as: 28 pF–253 pF.

EXAMPLE 6.5

Obtain the resonant frequency and Q-factor of a series LCR circuit with $L = 3.0$ H, $C = 27$ μF, and $R = 7.4\ \Omega$. How will you improve the sharpness of resonance of the circuit by a factor of 2 by reducing its full width at half maximum?

Sol. Resonant frequency is given by

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{3\times 27\times 10^{-6}}} = 17.68\text{ Hz}$$

Q-factor is given by: $Q = \dfrac{1}{R}\sqrt{\dfrac{L}{C}} = \dfrac{1}{7.4}\sqrt{\dfrac{3}{27\times 10^{-6}}} = 45$

We know that Q-factor is also given by $Q = \frac{\omega_0}{2\Delta\omega} = \frac{\omega_0}{\omega_2 - \omega_1}$

where bandwidth $= \omega_2 - \omega_1 = \frac{R}{L}$

Now Q-factor can be improved by decreasing bandwidth which in turn can be decreased by decreasing R. So sharpness or Q-factor can be made twice by making the value of R half, i.e., 3.7 Ω.

EXAMPLE 6.6

An inductive circuit draws a power 550 W from a 220 V–50 Hz source. The power factor of the circuit is 0.8. The current in the circuit lags behind the voltage. Show that a capacitor of about $\frac{1}{42\pi}\times10^{-2}$ F will have to be connected in the circuit to bring its power factor to unity.

Sol. Circuit is inductive, i.e., there is no capacitor in the circuit.

Given power factor:

$$\cos\phi = \frac{R}{\sqrt{R^2 + X_L^2}} = 0.8 \Rightarrow X_L = \frac{3}{4}R$$

Now power: $P = \frac{E_v^2}{Z^2}R$

$$550 = \frac{(220)^2 R}{R^2 + \left(\frac{3}{4}R\right)^2} \Rightarrow R = 56.32\ \Omega$$

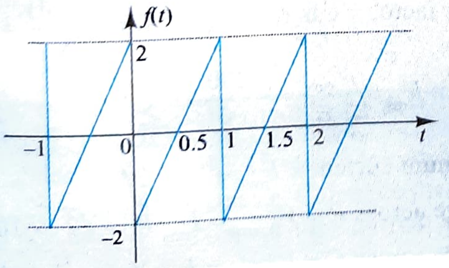
Now we have to bring the power factor to unity. For this

$$X_C = X_L \Rightarrow \frac{1}{\omega C} = \frac{3}{4}R \Rightarrow C = \frac{4}{3\omega R}$$

$$\Rightarrow C = \frac{4}{3\times 2\pi 50\times 56.32} = \frac{1}{4224\pi}\text{ F}$$

EXAMPLE 6.7

Find the rms and the average values of the saw tooth waveform shown in figure.



Sol. Let us solve the question for time interval $t = 0$ to $t = 1$ s. The rms and average value calculated for this time interval will be same for overall graph for a long time because the graph is repeating again and again.

Analytical Method: For $t = 0$ to $t = 1$ s:

$f(t) = 4t - 2$

Average value:

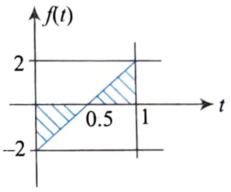
$$f(t)_{\text{av}} = \frac{\int_0^1 f(t)\,dt}{1-0} = \frac{\int_0^1 (4t-2)\,dt}{1} = \left[\frac{4t^2}{2} - 2t\right]_0^1 = 0$$

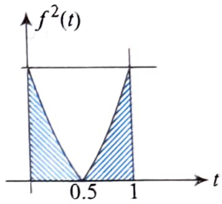
Hence, average value of waveform is zero.

$$\text{rms value} = f(t)_{\text{rms}} = \sqrt{\frac{\int_0^1 f^2(t)\,dt}{1-0}} = \sqrt{\frac{\int_0^1 (4t-2)^2\,dt}{1}}$$

$$= \sqrt{\left[\frac{(4t-2)^3}{4\times3}\right]_0^1} = \sqrt{\frac{4}{3}}$$

Alternate method (Graphically):





$$f(t)_{\text{av}} = \frac{\int_0^1 f(t)\,dt}{1-0}$$

$$= \frac{\text{area of } f(t)-t \text{ graph from } t=0 \text{ to } t=1\text{ s}}{1}$$

$$= \frac{\frac{1}{2}(0.5)(-2) + \frac{1}{2}(0.5)2}{1} = 0$$

$$f(t)_{\text{rms}} = \sqrt{\frac{\int_0^1 f^2(t)\,dt}{1-0}}$$

$$= \sqrt{\frac{\text{area under } f^2(t)-t \text{ graph from } t=0 \text{ to } t=1\text{s}}{1}}$$

$$= \sqrt{\frac{\frac{1}{3}\times0.5\times4 + \frac{1}{3}\times0.5\times4}{1}} = \sqrt{\frac{4}{3}}$$

EXAMPLE 6.8

An inductor 20×10^{-3} H, a capacitor 100 μF, and a resistor 50 Ω are connected in series across a source of emf $V = 10\sin 314t$. Find the energy dissipated in the circuit in 20 min. If the resistance is removed from the circuit and the value of inductance is doubled, then find the variation of current with time in the new circuit.

Sol. Here, time of 1 cycle $T = 1/50$ s. So, we have to calculate the average energy as time >> T.

Energy consumed in time t

$$= V_{\text{rms}} I_{\text{rms}} \cos\phi t = \frac{I_M}{\sqrt{2}} \times \frac{V_M}{\sqrt{2}} \times \frac{R}{Z} t$$

$$V = \frac{V_{\text{M}}^2 R}{2Z^2} t \qquad \left(\because\ I_{\text{M}} = \frac{V_{\text{M}}}{Z}\right)$$

Now, $Z = \sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}$

$$= \sqrt{(50)^2 + \left(314 \times 20 \times 10^{-3} - \frac{1}{314 \times 100 \times 10^{-6}}\right)^2}$$

$$= \sqrt{3153.6} \approx 56\ \Omega$$

Energy consumed $= \dfrac{10^2 \times 50 \times 20 \times 60}{2 \times 3153.6} = 951$ J

When resistance is removed,

$$\cos\phi = \frac{R}{Z'} = 0 \quad \text{or,} \quad \phi = \frac{\pi}{2} \quad \text{or} \quad -\frac{\pi}{2}$$

$$Z' = \frac{1}{\omega C} - \omega L' = \frac{1}{314 \times 10^{-4}} - 314 \times 40 \times 10^{-3} = 19.3$$

[here $X_{\text{C}} > X_{\text{L}}$, hence $\phi = \pi/2$]

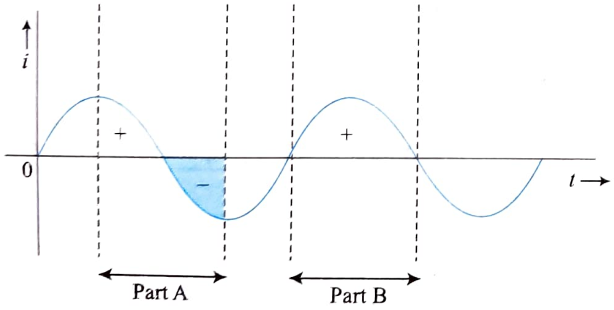
$$I = \frac{V_{\text{M}}}{Z'} \sin(\omega t + \phi) = \frac{10}{19.3} \sin\left(314t + \frac{\pi}{2}\right)$$

$$= 0.52 \cos 314t$$

EXAMPLE 6.9

Show graphically that the average of sinusoidally varying current in half cycle may or may not be zero.

Sol. Figure shows two parts, A and B, each half cycle. In part A we can see that the net area is zero.



$\therefore$ $\langle i \rangle$ in part A = 0.

In part B, area is positive hence in this part $\langle i \rangle \neq 0$.

EXAMPLE 6.10

Find the effective value of current
$i = 2\sin 100\pi t + 2\cos(100\pi t + 30°)$.

Sol. **Analytically:**

$$i = 2\sin 100\pi t + 2\cos(100\pi t + 30°)$$

$$= 2\sin 100\pi t + 2\cos 100\pi t \cos 30° - 2\sin 100\pi t \sin 30°$$

$$= \sin 100\pi t + \sqrt{3}\cos 100\pi t$$

$$= I_{\text{m}} \sin(100\pi t + \phi)$$

where $I_{\text{m}} = \sqrt{1 + (\sqrt{3})^2} = 2$ A

$$I_{\text{rms}} = \frac{I_{\text{m}}}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}\ \text{A}$$

Graphically:

The given equation can be written as:

$$i = 2\sin 100\pi t + 2\sin(100\pi t + 120°)$$

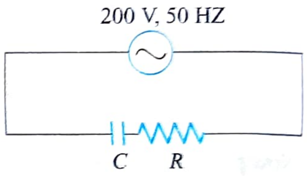
It is the sum of two phasors having phase difference 120°.

$$I_{\text{m}} = \sqrt{2^2 + 2^2 + 2 \times 2 \times 2\cos 120°} = 2\ \text{A}$$

$$I_{\text{rms}} = \frac{I_{\text{m}}}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}\ \text{A}$$

EXAMPLE 6.11

In an RC series circuit (see figure), the rms voltage of source is 200 V and its frequency is 50 Hz. If $R = 100\ \Omega$ and $C = \dfrac{100}{\pi}\ \mu\text{F}$, find



(a) impedance of the circuit,
(b) power factor angle,
(c) power factor,
(d) current,
(e) maximum current,
(f) voltage across R,
(g) voltage across C,
(h) maximum voltage across R,
(i) maximum voltage across C,
(j) $\langle P \rangle$,
(k) $\langle P_{\text{R}} \rangle$,
(l) $\langle P_{\text{C}} \rangle$.
(m) $i(t)$, $V_{\text{R}}(t)$, and $V_{\text{C}}(t)$

Sol. $X_{\text{C}} = \dfrac{10^6}{\dfrac{100}{\pi}(2\pi 50)} = 100\ \Omega$

(a) $Z = \sqrt{R^2 + X_{\text{C}}^2} = \sqrt{100^2 + (100)^2} = 100\sqrt{2}\ \Omega$

(b) $\tan\phi = \dfrac{X_{\text{C}}}{R} = 1 \quad \therefore \quad \phi = 45°$

(c) Power factor $= \cos\phi = \dfrac{1}{\sqrt{2}}$

(d) Current $I_{\text{rms}} = \dfrac{V_{\text{rms}}}{Z} = \dfrac{200}{100\sqrt{2}} = \sqrt{2}$ A

(e) Maximum current $= I_{\text{rms}} \sqrt{2} = 2$ A

(f) Voltage across $R = V_{\text{R, rms}} = I_{\text{rms}} R = \sqrt{2} \times 100$ V

(g) Voltage across $C = V_{\text{C, rms}} = I_{\text{rms}} X_{\text{C}} = \sqrt{2} \times 100$ V

(h) Max voltage across $R = \sqrt{2}\ V_{\text{R, rms}} = 200$ V

(i) Max voltage across $C = \sqrt{2}\ V_{\text{C, rms}} = 200$ V

(j) $\langle P \rangle = V_{\text{rms}} I_{\text{rms}} \cos\phi = 200 \times \sqrt{2} \times \dfrac{1}{\sqrt{2}} = 200$ W

(k) $\langle P_{\text{R}} \rangle = I^2_{\text{rms}} R = 200$ W

(l) $\langle P_C \rangle = 0$

(m) Applied voltage can be written as:

$$V_s(t) = V_0 \sin \omega t \qquad (\omega = 2\pi f = 2\pi\, 50 = 100\pi \text{ rad s}^{-1})$$

$$= 200\sqrt{2}\sin(100\pi t)$$

$$i(t) = I_0 \sin(100\pi t + \phi)$$

$$= \frac{V_0}{Z}\sin\left(100\pi t + \frac{\pi}{4}\right) = \frac{200\sqrt{2}}{100\sqrt{2}}\sin\left(100\pi t + \frac{\pi}{4}\right)$$

$$= 2\sin\left(100\pi t + \frac{\pi}{4}\right)$$

$$V_R(t) = V_{R0}\sin\left(100\pi t + \frac{\pi}{4}\right) = I_0 R\sin\left(100\pi t + \frac{\pi}{4}\right)$$

$$= 2\times 100\sin\left(100\pi t + \frac{\pi}{4}\right) = 200\sin\left(100\pi t + \frac{\pi}{4}\right)$$

$$V_C(t) = V_{C0}\sin\left(100\pi t + \frac{\pi}{4} - \frac{\pi}{2}\right)$$

$$= I_0 X_C \sin\left(100\pi t - \frac{\pi}{4}\right) = 200\sin\left(100\pi t - \frac{\pi}{4}\right)$$

EXAMPLE 6.12

An ac source of angular frequency ω is fed across a resistor R and a capacitor C in series. The current registered is I. If now the frequency of source is changed to $\omega/3$ (but maintaining the same voltage), the current in the circuit is found to be halved. Calculate the ratio of the reactance to resistance at the original frequency ω.

Sol. According to the given problem,

$$I = \frac{V}{Z} = \frac{V}{[R^2 + (1/C\omega)^2]^{1/2}} \qquad \text{...(i)}$$

and, $$\frac{I}{2} = \frac{V}{[R^2 + (3/C\omega)^2]^{1/2}} \qquad \text{...(ii)}$$

From Eqs (i) in (ii), $\dfrac{1}{C^2\omega^2} = \dfrac{3}{5}R^2$

So that, $$\frac{X}{R} = \frac{(1/C\omega)}{R} = \frac{\left(\frac{3}{5}R^2\right)^{1/2}}{R} = \sqrt{\frac{3}{5}}$$

EXAMPLE 6.13

A $9/100\pi$ inductor and a 12 Ω resistance are connected in series to a 225 V, 50 Hz ac source. Calculate the current in the circuit and the phase angle between the current and the source voltage.

Sol. Here $X_L = \omega L = 2\pi f L = 2\pi \times 50 \times \dfrac{9}{100\pi} = 9\ \Omega$

So, $$Z = \sqrt{R^2 + X_L^2} = \sqrt{12^2 + 9^2} = 15\ \Omega$$

So $$I = \frac{V}{Z} = \frac{225}{15} = 15\text{ A}$$

and $$\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{9}{12}\right) = \tan^{-1}(3/4) = 37^\circ$$

i.e., the current will lag the applied voltage by 37° in phase.

EXAMPLE 6.14

When an inductor coil is connected to an ideal battery of emf 10 V, a constant current 2.5 A flows. When the same inductor coil is connected to an ac source of 10 V and 50 Hz then the current is 2 A. Find out the inductance of the coil.

Sol. When the coil is connected to a dc source, the final current is decided by the resistance of the coil.

$$\therefore \quad r = \frac{10}{2.5} = 4\ \Omega$$

When the coil is connected to an ac source, the final current is decided by the impedance of the coil.

$$\therefore \quad Z = \frac{10}{2} = 5\ \Omega$$

But $$Z = \sqrt{(r)^2 + (X_L)^2} \Rightarrow X_L = 3\ \Omega$$

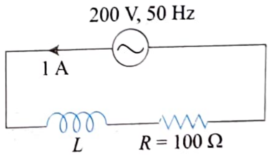
$$\therefore \quad \omega L = 2\pi f L = 3 \Rightarrow 2\pi\, 50\, L = 3 \Rightarrow L = 3/100\pi \text{ H}$$

EXAMPLE 6.15

A bulb is rated at 100 V, 100 W. It can be treated as a resistor. Find out the inductance of an inductor (called choke coil) that should be connected in series with the bulb to operate the bulb at its rated power with the help of an ac source of 200 V and 50 Hz.

Sol. From the rating of the bulb, the resistance of the bulb is

$$R = \frac{V^2}{P} = 100\ \Omega$$



For the bulb to be operated at its rated value, the rms current through it should be 1A.

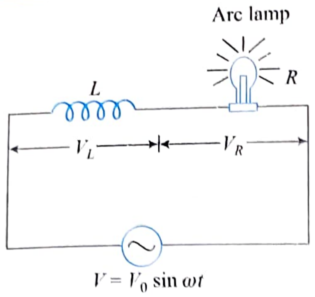
Also, $$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z}$$

$$\therefore \quad 1 = \frac{200}{\sqrt{100^2 + (2\pi 50 L)^2}} \Rightarrow L = \frac{\sqrt{3}}{\pi}\text{ H}$$

EXAMPLE 6.16

A choke coil is needed to operate an arc lamp at 160 V (rms) and 50 Hz. The arc lamp has an effective resistance of 5 Ω when running of 10 A (rms). Calculate the inductance of the choke coil. If the same arc lamp is to be operated on 160 V (dc), what additional resistance is required? Compare the power losses in both the cases.

Sol. As for the arc lamp $V_R = IR = 10 \times 5 = 50$ V, so when it is connected to 160 V ac source through a choke in series,



$$V^2 = V_R^2 + V_L^2 = \sqrt{160^2 - 50^2} = 152 \text{ V}$$

and as, $V_L = IX_L = I\omega L = 2\pi fLI$

So, $L = \dfrac{V_L}{2\pi fI} = \dfrac{152}{2 \times \pi \times 50 \times 10} = 4.84 \times 10^{-2}$ H

Now the lamp is to be operated at 160 V dc; instead of choke if additional resistance r is put in series with it,

$V = I(R + r)$, i.e., $160 = 10(5 + r)$

i.e., $r = 11\ \Omega$

In case of ac, as choke has no resistance, power loss in the choke will be zero while the bulb will consume,

$$P = I^2 R = 10^2 \times 5 = 500 \text{ W}$$

However, in case of dc as resistance r is to be used instead of choke, the power loss in resistance r will be.

$$PL = 10^2 \times 11 = 1100 \text{ W}$$

while the bulb will still consume 500 W, i.e., when the lamp is run on resistance r instead of choke more than double the power consumed by the lamp is wasted by the resistance r.

EXAMPLE 6.17

A coil of inductance 0.50 H and resistance 100 Ω is connected to a 240 V, 50 Hz ac supply. What are the maximum current in the coil and the time lag between voltage maximum and current maximum?

Sol. Impedance of the coil:

$$Z = \sqrt{R^2 + (2\pi fL)^2} = \sqrt{100^2 + 4\pi^2(50)^2(0.5)^2}$$

$$= 186.2\ \Omega$$

Maximum current: $I_0 = \dfrac{E_0}{Z} = \dfrac{240\sqrt{2}}{186.2} = 1.82$ A

$$\tan\phi = \frac{X_C - X_L}{R} = \frac{0 - 2\pi(50)(0.5)}{100} = -\frac{\pi}{2}$$

$$\Rightarrow \quad \phi = -\tan^{-1}\left(\frac{\pi}{2}\right) = -57.51° = -1.003 \text{ rad}$$

Let emf be maximum at t_1, then $\omega t_1 = \dfrac{\pi}{2}$

[we have $E = E_0 \sin \omega t$ and $I = I_0 \sin(\omega t + \phi)$]

Let current be maximum at t_2, then $\omega t_2 + \phi = \dfrac{\pi}{2}$

From above, we have $\omega(t_2 - t_1) = -\phi$

Time lag between voltage maximum and current maximum:

$$t_2 - t_1 = -\frac{\phi}{\omega} = \frac{1.003}{2\pi f} = 3.2 \times 10^{-3} \text{ s}$$

EXAMPLE 6.18

A 100 μF capacitor in series with a 40 Ω resistor is connected to a 110 V, 60 Hz supply. What is the maximum current in the circuit? What is the time lag between current maximum and voltage maximum?

Sol. Impedance of circuit is

$$Z = \sqrt{(40)^2 + \frac{1}{(120\pi \times 100 \times 10^{-6})^2}} = 48\ \Omega$$

Maximum current: $I_0 = \dfrac{E_0}{Z} = \dfrac{110\sqrt{2}}{48} = 3.24$ A

[we have $E = E_0 \sin \omega t$ and $I = I_0 \sin(\omega t + \phi)$]

$$\tan\phi = \frac{X_C}{R} = \frac{1}{120\pi(100 \times 10^{-6})40}$$

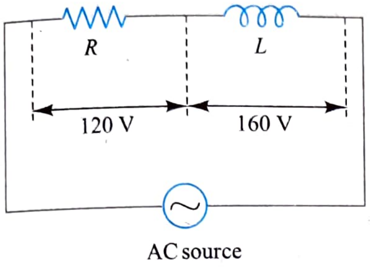
$$\Rightarrow \quad \phi = 33.5° = 0.186\pi$$

Required time lag: $t_0 = \dfrac{\phi}{\omega} = \dfrac{0.186\pi}{2\pi 60} = 1.55 \times 10^{-3}$ s

Exercises

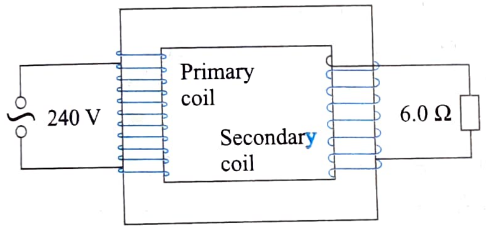
Single Correct Answer Type

1. The circuit given in figure has a resistanceless choke coil L and a resistance R. The voltages across R and L are also given in the figure. The virtual value of the applied voltage is



(1) 100 V (2) 200 V
(3) 300 V (4) 400 V

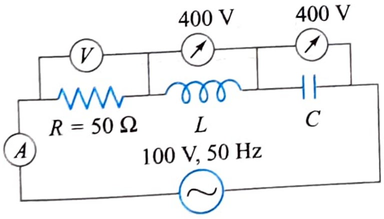
2. Figure shows an iron-cored transformer assumed to be 100% efficient. The ratio of the secondary turns to the primary turns is 1:20.



A 240 V ac supply is connected to the primary coil and a 6 Ω resistor is connected to the secondary coil. What is the current in the primary coil?

(1) 0.10 A (2) 0.14 A
(3) 2 A (4) 40 A

3. In the series LCR circuit (figure), the voltmeter and ammeter readings are:



(1) $V = 100$ V, $I = 2$ A (2) $V = 100$ V, $I = 5$ A
(3) $V = 1000$ V, $I = 2$ A (4) $V = 300$ V, $I = 1$ A

4. An LCR circuit contains resistance of 100 Ω and a supply of 200 V at 300 rad angular frequency. If only capacitance is taken out from the circuit and the rest of the circuit is joined, current lags behind the voltage by 60°. If, on the other hand, only inductor is taken out, the current leads by 60° with the applied voltage. The current flowing in the circuit is

(1) 1 A (2) 1.5 A
(3) 2 A (4) 2.5 A

5. The peak value of an alternating emf E given by $E = E_0 \cos\omega t$ is 10 V and frequency is 50 Hz. At time $t = (1/600)$ s, the instantaneous value of emf is

(1) 10 V (2) $5\sqrt{3}$ V
(3) 5 V (4) 1 V

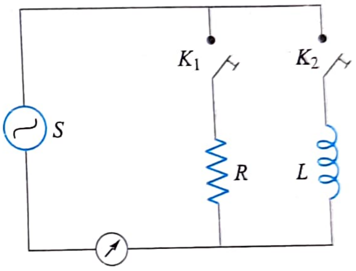
6. A coil has an inductance of 0.7 H and is joined in series with a resistance of 220 Ω. When an alternating emf of 220 V at 50 cps is applied to it, then the wattless component of the current in the circuit is (take $0.7\pi = 2.2$)

(1) 5 A (2) 0.5 A
(3) 0.7 A (4) 7 A

7. An inductive coil has resistance of 100 Ω. When an ac signal of frequency 1000 Hz is fed to the coil, the applied voltage leads the current by 45°. What is the inductance of the coil?

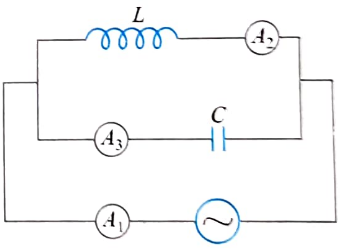
(1) 2 mH (2) 3.3 mH
(3) 16 mH (4) $\sqrt{5}$ mH

8. In the circuit shown in figure, R is a pure resistor, L is an inductor of negligible resistance (as compared to R), S is a 100 V, 50 Hz ac source of negligible resistance. With either key K_1 alone or K_2 alone closed, the current is I_0. If the source is changed to 100 V, 100 Hz the current with K_1 alone closed and with K_2 alone closed will be, respectively,



(1) $I_0, \dfrac{I_0}{2}$ (2) $I_0, 2I_0$
(3) $2I_0, I_0$ (4) $2I_0, \dfrac{I_0}{2}$

9. For the circuit shown in figure, the ammeter A_2 reads 1.6 A and ammeter A_3 reads 0.4 A. Then

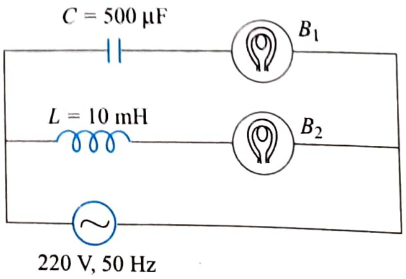


(1) $\omega = \dfrac{4}{\sqrt{LC}}$

(2) $f = \dfrac{2\pi}{\sqrt{LC}}$

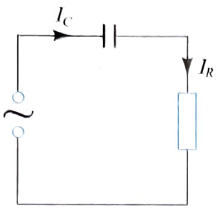
(3) the ammeter A_1 reads 1.2 A
(4) the ammeter A_1 reads 2A

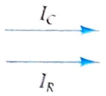
10. In the circuit shown in figure, if both the bulbs B_1 and B_2 are identical, then

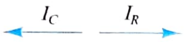


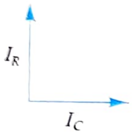
(1) their brightness will be the same
(2) B_2 will be brighter than B_1
(3) B_1 will be brighter than B_2
(4) only B_2 will glow because the capacitor has infinite impedance

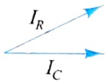
11. Figure shows a source of alternating voltage connected to a capacitor and a resistor. Which of the following phasor diagrams correctly describes the phase relationship between I_c, the current between the source and the capacitor, and I_R, the current in the resistor?



(1)


(2)


(3)


(4)


12. A sinusoidal alternating current of peak value I_0 passes through a heater of resistance R. What is the mean power output of the heater?

(1) $I_0^2 R$
(2) $\frac{I_0^2 R}{2}$
(3) $2I_0^2 R$
(4) $\sqrt{2}\, I_0^2 R$

13. A resistance of 20 Ω is connected to a source of an alternating potential $V = 220 \sin(100\,\pi t)$. The time taken by the current to change from the peak value to rms value, is

(1) 0.2 s
(2) 0.25 s
(3) 2.5×10^{-3} s
(4) 2.5×10^{-3} s

14. In LCR circuit current resonant frequency is 600 Hz and half power points are at 650 and 550 Hz. The quality factor is

(1) $\frac{1}{6}$
(2) $\frac{1}{3}$
(3) 6
(4) 3

15. An ac voltage is represented by

$$E = 220\sqrt{2}\cos(50\pi)t$$

How many times will the current become zero in 1 s?

(1) 50 times
(2) 100 times
(3) 30 times
(4) 25 times

16. A transmitter transmits at a wavelength of 300 m. A condenser of capacitance 2.4 μF is being used. The value of the inductance for the resonant circuit is approximately

(1) 10^{-4} H
(2) 10^{-6} H
(3) 10^{-8} H
(4) 10^{-10} H

17. A capacitor of capacitance 1 μF is charged to a potential of 1 V. It is connected in parallel to an inductor of inductance 10^{-3} H. The maximum current that will flow in the circuit has the value

(1) $\sqrt{1000}$ mA
(2) 1 mA
(3) 1 μA
(4) 1000 mA

18. Using an ac voltmeter, the potential difference in the electrical line in a house is read to be 234 V. If the line frequency is known to be 50 cycles per second, the equation for the line voltage is

(1) $V = 165 \sin(100\,\pi t)$
(2) $V = 331 \sin(100\,\pi t)$
(3) $V = 220 \sin(100\,\pi t)$
(4) $V = 440 \sin(100\,\pi t)$

19. An inductor and a resistor are connected in series with an ac source. In this circuit.

(1) the current and the PD across the resistance lead the PD across the inductance
(2) the current and the PD across the resistance lag behind the PD across the inductance by an angle $\pi/2$
(3) the current and the PD across the resistance lag behind the PD across the inductance by an angle π
(4) the PD across the resistance lags behind the PD across the inductance by an angle $\pi/2$ but the current in resistance leads the PD across the inductance by $\pi/2$

20. In a series LCR circuit, the voltage across the resistance, capacitance and inductance is 10 V each. If the capacitance is short circuited, the voltage across the inductance will be

(1) 10 V
(2) $10/\sqrt{2}$ V
(3) (10/3)V
(4) 20 V

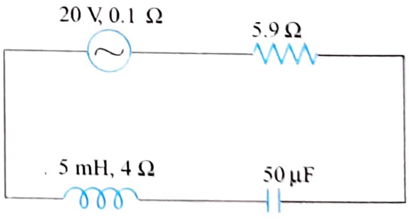
21. An ideal choke takes a current of 10 A when connected to an ac supply of 125 V and 50 Hz. A pure resistor under the same conditions takes a current of 12.5 A. If the two are connected to an ac supply of 100 V and 40 Hz, then the current in series combination of above resistor and inductor is

(1) $10/\sqrt{2}$ A
(2) 12.5 A
(3) 20 A
(4) 10 A

22. A direct current of 5 A is superimposed on an alternating current $I = 10 \sin \omega t$ flowing through a wire. The effective value of the resulting current will be

(1) (15/2) A
(2) $5\sqrt{3}$ A
(3) $5\sqrt{5}$ A
(4) 15 A

23. In the circuit of figure, the source frequency is $\omega = 2000$ rad s^{-1}. The current in the circuit will be

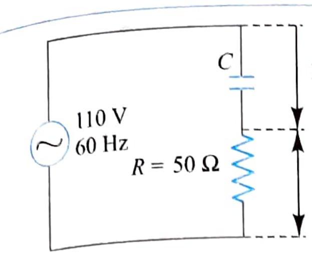


(1) 2 A
(2) 3.3 A
(3) $2/\sqrt{5}$ A
(4) $\sqrt{5}$ A

24. An rms voltage of 110 V is applied across a series circuit having a resistance 11 Ω and an impedance 22 Ω. The power consumed is

(1) 275 W
(2) 366 W
(3) 550 W
(4) 1100 W

25. In the circuit given in figure, $V_C = 50$ V and $R = 50$ Ω. The values of C and V_R are



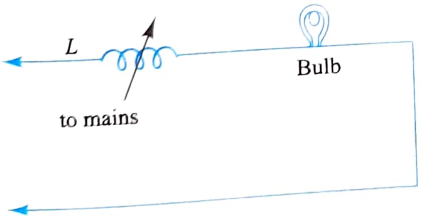
(1) 3.3 mF, 60 V (2) 104 μF, 98 V
(3) 52 μF, 98 V (4) 2 μF, 60 V

26. An 8 μF capacitor is connected across a 220 V, 50 Hz line. What is the peak value of charge through the capacitor?
(1) 2.5×10^{-3} C (2) 2.5×10^{-4} C
(3) 5×10^{-5} C (4) 7.5×10^{-2} C

27. A dc ammeter and a hot wire ammeter are connected to a circuit in series. When a direct current is passed through circuit, the dc ammeter shows 6 A. When ac current flows through circuit, the ac ammeter shows 8 A. What will be reading of each ammeter if dc and ac currents flow simultaneously through the circuit?
(1) dc = 6 A, ac = 10 A (2) dc = 3 A, ac = 5 A
(3) dc = 5 A, ac = 8 A (4) dc = 2 A, ac = 3 A

28. An ac is given by equation $I = I_1 \cos \omega t + I_2 \sin \omega t$. The rms value of current is given by
(1) $\dfrac{I_1 + I_2}{2}$ (2) $\dfrac{(I_1 + I_2)^2}{\sqrt{2}}$
(3) $\sqrt{\dfrac{I_1^2 + I_2^2}{2}}$ (4) $\dfrac{I_1^2 + I_2^2}{2}$

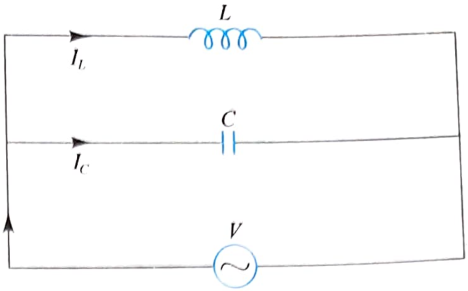
29. A typical light dimmer used to dim the stage lights in a theatre consists of a variable induction for L (where inductance is adjustable between zero and L_{max}) connected in series with a light bulb B as shown in figure. The mains electrical supply is 220 V at 50 Hz, the light bulb is rated at 220 V, 1100 W. What L_{max} is required if the rate of energy dissipated in the light bulb is to be varied by a factor of 5 from its upper limit of 1100 W?



(1) 0.69 H (2) 0.28 H
(3) 0.38 H (4) 0.56 H

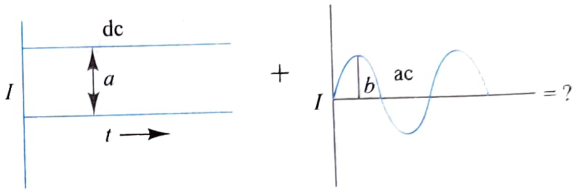
30. Two alternating voltage generators produce emfs of the same amplitude E_0 but with a phase difference of $\pi/3$. The resultant emf is
(1) $E_0 \sin[\omega t + (\pi/3)]$ (2) $E_0 \sin[\omega t + (\pi/6)]$
(3) $\sqrt{3}\, E_0 \sin[\omega t + (\pi/6)]$ (4) $\sqrt{3}\, E_0 \sin[\omega t + (\pi/2)]$

31. For the circuit shown in figure, current in inductance is 0.8 A while that in capacitance is 0.6 A. What is the current drawn from the source?



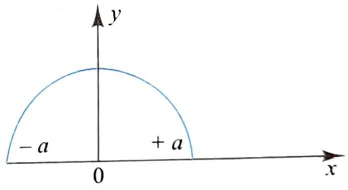
(1) 0.1 A (2) 0.3 A
(3) 0.6 A (4) 0.2 A

32. If a direct current of value a ampere is superimposed on an alternative current $I = b \sin \omega t$ flowing through a wire, what is the effective value of the resulting current in the circuit?



(1) $\left[a^2 - \dfrac{1}{2}b^2\right]^{1/2}$ (2) $[a^2 + b^2]^{1/2}$
(3) $\left[\dfrac{a^2}{2} + b^2\right]^{1/2}$ (4) $\left[a^2 + \dfrac{1}{2}b^2\right]^{1/2}$

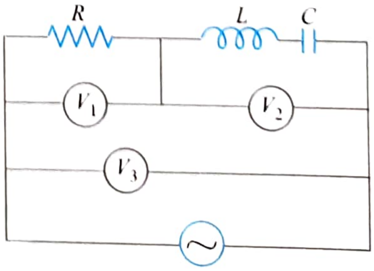
33. Determine the rms value of a semi-circular current wave which has a maximum value of a.



(1) $(1/\sqrt{2})\,a$ (2) $\sqrt{(3/2)}\,a$
(3) $\sqrt{(2/3)}\,a$ (4) $(1/\sqrt{3})\,a$

34. An alternating voltage $E = 50\sqrt{2}\ \sin(100\,t)$ V is connected to a 1 μF capacitor through an ac ammeter. What will be the reading of the ammeter?
(1) 10 mA (2) 5 mA
(3) $5\sqrt{2}$ mA (4) $10\sqrt{2}$ mA

35. Which voltmeter will give zero reading at resonance?



(1) V_1 (2) V_2
(3) V_3 (4) None

36. A 50 W, 100 V lamp is to be connected to an ac mains of 200 V, 50 Hz. What capacitor is essential to be put in series with the lamp?

(1) $\frac{25}{\sqrt{2}}$ μF　　(2) $\frac{50}{\pi\sqrt{3}}$ μF

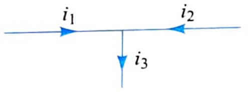
(3) $\frac{50}{\sqrt{2}}$ μF　　(4) $\frac{100}{\pi\sqrt{3}}$ μF

37. A capacitor of 10 μF and an inductor of 1 H are joined in series. An ac of 50 Hz is applied to this combination. What is the impedance of the combination?

(1) $\frac{5(\pi^2-5)}{\pi}\,\Omega$　　(2) $\frac{10(10-\pi^2)}{\pi}\,\Omega$

(3) $\frac{10(\pi^2-5)}{\pi}\,\Omega$　　(4) $\frac{5(10-\pi^2)}{\pi}\,\Omega$

38. If $i_1 = 3\sin\omega t$ and $i_2 = 4\cos\omega t$, then i_3 is



(1) $5\sin(\omega t + 53°)$　　(2) $5\sin(\omega t + 37°)$
(3) $5\sin(\omega t + 45°)$　　(4) $5\sin(\omega t + 53°)$

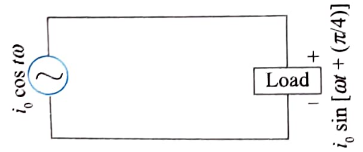
39. In an ac circuit, the potential differences across an inductance and resistance joined in series are, respectively, 16 V and 20 V. The total potential difference across the circuit is

(1) 20 V　　(2) 25.6 V
(3) 31.9 V　　(4) 53.5 V

40. Current in an ac circuit is given by $I = 3\sin\omega t + 4\cos\omega t$, then

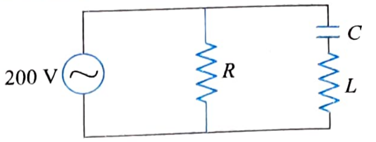
(1) rms value of current is 5 A.
(2) mean value of this current in any one half period will be $6/\pi$.
(3) if voltage applied is $V = V_m \sin\omega t$, then the circuit may be containing resistance and capacitance only.
(4) if voltage applied is $V = V_m \sin\omega t$, the circuit may contain resistance and inductance only.

41. A current source sends a current $I = i_0 \cos(\omega t)$. When connected across an unknown load, it gives a voltage output of $v = v_0 \sin[\omega t + (\pi/4)]$ across that load. Then the voltage across the current source may be brought in phase with the current through it by



(1) connecting an inductor in series with the load
(2) connecting a capacitor in series with the load
(3) connecting an inductor in parallel with the load
(4) connecting a capacitor in parallel with the load

42. In the circuit shown in figure, $X_C = 100\ \Omega$, $X_L = 200\ \Omega$ and $R = 100\ \Omega$. The effective current through the source is



(1) 2 A　　(2) $2\sqrt{2}$ A
(3) 0.5 A　　(4) $\sqrt{0.4}$ A

43. For an LCR series circuit with an ac source of angular frequency ω,

(1) circuit will be capacitive if $\omega > \frac{1}{\sqrt{LC}}$
(2) circuit will be inductive if $\omega = \frac{1}{\sqrt{LC}}$
(3) power factor of circuit will be unity if capacitive reactance equals inductive reactance
(4) current will be leading voltage if $\omega > \frac{1}{\sqrt{LC}}$

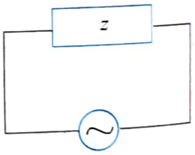
44. The value of current in two series LCR circuits at resonance is same. Then

(1) both circuits must be having same value of capacitance and inductance
(2) in both circuits ratio of L and C will be same
(3) for both the circuits X_L/X_C must be same at that frequency
(4) both circuits must have same impedance at all frequencies

45. In series LCR circuit voltage drop across resistance is 8 V, across inductor is 6 V and across capacitor is 12 V. Then

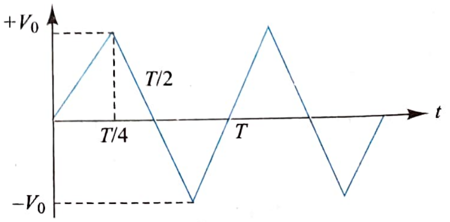
(1) voltage of the source will be leading current in the circuit
(2) voltage drop across each element will be less than the applied voltage
(3) power factor of circuit will be 4/3
(4) none of these

46. In a black box of unknown elements (L or R or any other combination), an ac voltage $E = E_0 \sin(\omega t + \phi)$ is applied and current in the circuit was found to be $I = I_0 \sin[\omega t + \phi + (\pi/4)]$. Then the unknown elements in the box may be



(1) only capacitor
(2) inductor and resistor both
(3) either capacitor, resistor, and inductor or only capacitor and resistor
(4) only resistor

47. The voltage time (V–t) graph for triangular wave having peak value V_0 is as shown in figure.



The rms value of V in time interval from $t = 0$ to $T/4$ is

(1) $\frac{V_0}{\sqrt{3}}$ (2) $\frac{V_0}{2}$

(3) $\frac{V_0}{\sqrt{2}}$ (4) None of these

48. In the above question, the average value of voltage (V) in one time period will be

(1) $\frac{V_0}{\sqrt{3}}$ (2) $\frac{V_0}{2}$

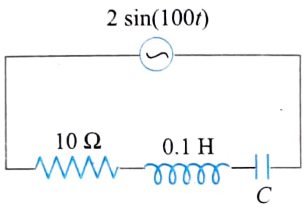
(3) $\frac{V_0}{\sqrt{2}}$ (4) 0

49. What reading would you expect of a square-wave current, switching rapidly between + 0.5 A and – 0.5 A, when passed through an ac ammeter?

(1) 0 (2) 0.5 A

(3) 0.25 A (4) 1.0 A

50. The power factor of the circuit in figure is $1/\sqrt{2}$. The capacitance of the circuit is equal to



(1) 400 μF (2) 300 μF

(3) 500 μF (4) 200 μF

51. In an ideal transformer, the voltage and the current in the primary coil are 200 V and 2 A, respectively. If the voltage in the secondary coil is 2000 V, then the value of current in the secondary coil will be

(1) 0.2 A (2) 2 A

(3) 10 A (4) 20 A

52. An AC source rated 100 V (rms) supplies a current of 10 A (rms) to a circuit. The average power delivered by the source:

(1) must be 1000 W

(2) may be greater than 1000 W

(3) may be less than 1000 W

(4) all of the above three are possible

53. The voltage of an AC source varies with time according to the relation: $E = 120 \sin 100\pi t \cos 100\pi t$ V. What is the peak voltage of the source?

(1) 60 V (2) 120 V

(3) 30 V (4) $\sqrt{2}$ V

54. A resistance of 20 Ω is connected to a source of an alternating potential $V = 220 \sin(100\pi t)$. The time taken by the current to change from the peak value to rms value is

(1) 0.2 s (2) 0.25 s

(3) 25 s (4) 2.5×10^{-3} s

55. A current is made up of two components 3 A dc component and ac component given by $I = 4 \sin \omega t$ A. The effective value of current is

(1) $\sqrt{17}$A (2) $\sqrt{7}$A

(3) $\sqrt{25}$A (4) $\sqrt{7 \times 25}$A

56. An alternating e.m.f. of 200 V and 50 cycles is connected to a circuit of resistance 3.142 Ω and inductance 0.01 H. The lag in time between the e.m.f. and the current is

(1) 1.5 ms (2) 2.5 ms

(3) 3.5 ms (4) None of these

57. In an AC circuit, a resistance of R ohm is connected in series with an inductance L. If phase angle between voltage and current by 45°, the value of inductive reactance will be

(1) $R/4$

(2) $R/2$

(3) R

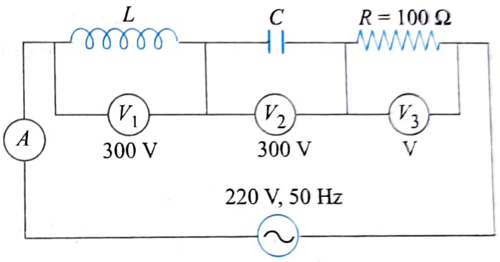
(4) cannot be found with the given data

58. A direct current of 2 A and an alternating current having a maximum value of 2 A flow through two identical resistances. The ratio of heat produced in the two resistances will be

(1) 1 :1 (2) 1 : 2

(3) 2 : 1 (4) 4 : 1

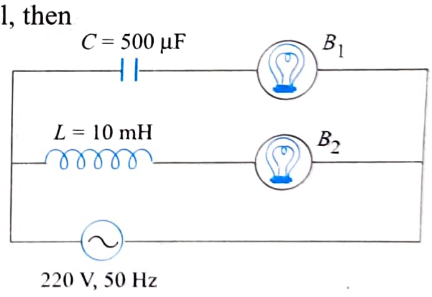
59. In the circuit shown in figure, what will be the reading of the voltmeter V_3 and ammeter A?



(1) 800 V, 2 A (2) 300 V, 2 A

(3) 220 V, 2.2 V (4) 100 V, 2 A.

60. In the circuit shown in figure, if both the bulbs B_1 and B_2 are identical, then



(1) their brightness will be the same

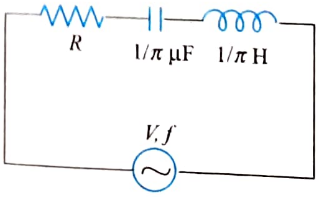
(2) B_2 will be brighter than B_1

(3) as frequency and that of B_2 will decrease

(4) only B_2 will glow because the capacitor has infinite impedance

Multiple Correct Answers Type

1. In an ac circuit shown below in figure, the supply voltage has a constant rms value V but variable frequency f. At resonance, the circuit



(1) has current I given by $I = \dfrac{V}{R}$

(2) has a resonance frequency 500 Hz

(3) has a voltage across the capacitor which is 180° out of phase with that across the inductor

(4) has a current given by $I = \dfrac{V}{\sqrt{R^2 + \left(\dfrac{1}{\pi} + \dfrac{1}{\pi}\right)^2}}$

2. Resonance occurs in a series LCR circuit when the frequency of the applied emf is 1000 Hz.

(1) when frequency = 900 Hz, then the current through the voltage source will be ahead of emf of the source

(2) the impedance of the circuit is minimum at $f = 1000$ Hz

(3) at only resonance the voltages across L and C differ in phase by 180°

(4) if the value of C is doubled, resonance occurs at $f = 2000$ Hz

3. Which of the following statements is true? Heat produced in a current carrying conductor depends upon

(1) the time for which the current flows in the conductor

(2) the resistance of the conductor

(3) the strength of the current

(4) the nature of current (ac or dc)

4. A choke coil of resistance 5 Ω and inductance 0.6 H is in series with a capacitance of 10 μF. If a voltage of 200 V is applied and the frequency is adjusted to resonance, the current and voltage across the inductance and capacitance are I_0, V_0, and V_1, respectively. We have

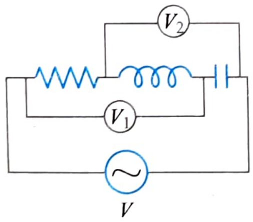
(1) $I_0 = 40$ A

(2) $V_0 = 9.8$ kV

(3) $V_1 = 9.8$ kV

(4) $V_1 = 19.6$ kV

5. In an RLC series circuit shown in figure, the readings of voltmeters V_1 and V_2 are 100 V and 120 V, respectively. The source voltage is 130 V. For this situation, mark out the correct statement(s).



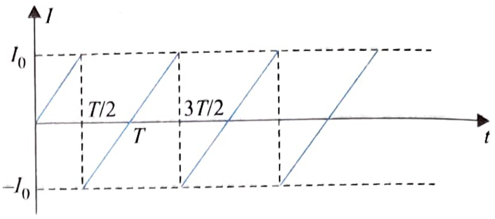
(1) Voltage across resistor, inductor and capacitor are 50 V, $50\sqrt{3}$ V, and $120 + 50\sqrt{3}$ V, respectively

(2) Voltage across resistor, inductor, and capacitor are 50 V, $50\sqrt{3}$ V, and $120 - 50\sqrt{3}$ V, respectively

(3) Power factor of the circuit is $\dfrac{5}{13}$

(4) The circuit is capacitive in nature

6. The current in a certain circuit varies with time as shown in figure All the straight lines are parallel to each other. Then for time interval $T/2$ to $3T/2$



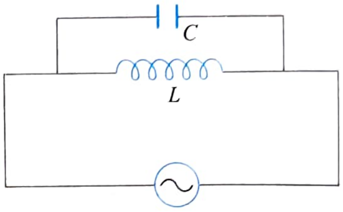
(1) average current is zero

(2) average current is $I_0/\sqrt{3}$

(3) rms current is $I_0/\sqrt{3}$

(4) rms current is $2I_0/\sqrt{3}$

7. For the circuit shown in figure, the emf of the generator is E. The current through the inductor is 1.6 A, while the current through the condenser is 0.4 A. Then



$E = E_0 \sin \omega t$

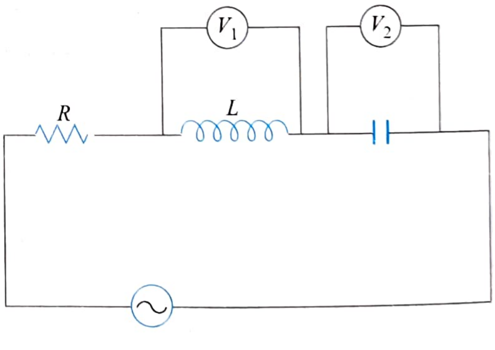
(1) current drawn from the generator is $I = 2$ A

(2) current drawn from the generator is $I = 1.2$ A

(3) $\omega = \dfrac{1}{2\sqrt{LC}}$

(4) $\omega = \dfrac{1}{4\sqrt{LC}}$

8. In figure shown $R = 100\ \Omega$, $L = \dfrac{2}{\pi}$ H and $C = \dfrac{8}{\pi}$ μF are connected in series with an ac source of 200 V and frequency f. V_1 and V_2 are two hot-wire voltmeters. If the readings of V_1 and V_2 are same, then



(1) $f = 125$ Hz

(2) $f = 250\ p$ Hz

(3) Current through R is 2 A

(4) $V_1 = V_2 = 1000$ V

Linked Comprehension Type

For Problems 1–3

If the voltage in an ac circuit is represented by the equation, $V = 220\sqrt{2}\sin(314t - \phi)$, calculate

1. rms value of the voltage

(1) 220 V (2) 314 V

(3) $220\sqrt{2}$ V (4) $200/\sqrt{2}$ V

2. average voltage

(1) 220 V (2) $622/\pi$ V

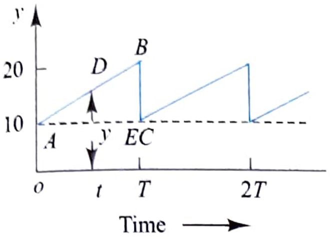
(3) $220/\sqrt{2}$ V (4) $220\sqrt{2}$ V

3. frequency of ac

(1) 50 Hz (2) $50\sqrt{2}$ Hz

(3) $50/\sqrt{2}$ Hz (4) 75 Hz

For Problems 4–5



4. The average value of the wave-form shown in figure is.

(1) $15\sqrt{2}$ (2) $10\sqrt{2}$

(3) 10 (4) 15

5. The rms value of the signal is

(1) $10\sqrt{\frac{7}{3}}$ (2) $\frac{10}{\sqrt{3}}$

(3) $10\sqrt{7}$ (4) $10\sqrt{3}$

For Problems 6–7

A current of 4 A flows in a coil when connected to a 12 V dc source. If the same coil is connected to a 12 V, 50 rad s^{-1} ac source, a current of 2.4 A flows in the circuit.

6. The inductance of the coil is

(1) 0.02 H (2) 0.04 H

(3) 0.08 H (4) 1.0 H

7. The power developed in the circuit of a 2500 μF capacitor that is connected in series with the coil is

(1) 17.28 W (2) 8.64 W

(3) 10 W (4) 15 W

For Problems 8–9

An inductor 20×10^{-3} H, a capacitor 100 μF, and a resistor 50 Ω are connected in series across a source of emf $V = 10 \sin 314\, t$.

8. Then the energy dissipated in the circuit in 20 min is

(1) 960 J (2) 900 J

(3) 250 J (4) 500 J

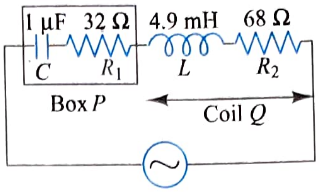
9. If resistance is removed from the circuit and the value of inductance is doubled, the variation of current with time in the new circuit is

(1) $0.52 \cos 314\, t$ (2) $0.52 \sin 314t$

(3) $0.52 \sin(314\, t + \pi/3)$ (4) None of these

For Problems 10–13

A box P and a coil Q are connected in series with an ac source of variable frequency. The emf of the source is constant at 10 V. Box P contains a capacitance of 1 μF in series with a resistance of 32 Ω. Coil Q has a self inductance of 4.9 mH and a resistance of 68 Ω in series. The frequency is adjusted so that maximum current flows in P and Q.



10. The impedance of P at this frequency is

(1) 77 Ω (2) 36 Ω

(3) 40 Ω (4) 125 Ω

11. The impedance of Q at this frequency is

(1) 200 Ω (2) $\sqrt{1350}$ Ω

(3) 55 Ω (4) $\sqrt{9524}$ Ω

12. The voltage across P is

(1) 12 V (2) 7.7 V

(3) 10 V (4) 24 V

13. The voltage across Q is

(1) 20 V (2) $\frac{\sqrt{1350}}{10}$ V

(3) 5.5 V (4) $\frac{\sqrt{9524}}{10}$ V

For Problems 14–16

A series LCR circuit containing a resistance of 120 Ω has angular resonance frequency 4×10^5 rad s^{-1}. At resonance the voltages across resistance and inductance are 60 V and 40 V, respectively.

14. The value of inductance L is

(1) 0.1 mH (2) 0.2 mH

(3) 0.35 mH (4) 0.4 mH

15. The value of capacitance C is

(1) $\frac{1}{32}$ μF (2) $\frac{1}{16}$ μF

(3) 32 μF (4) 16 μF

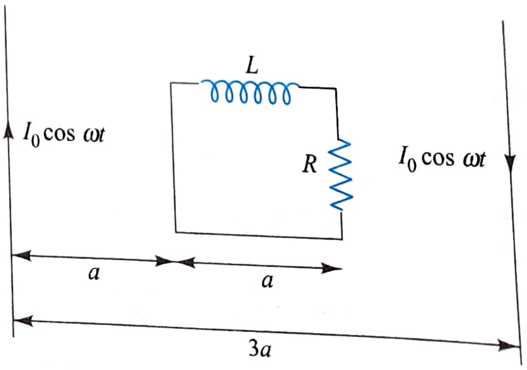
16. At what frequency, the current in the circuit lags the voltage by 45° ?

(1) 4×10^5 rad s^{-1} (2) 3×10^5 rad

(3) 8×10^5 rad s^{-1} (4) 2×10^5 rad s^{-1}

For Problems 17–19

In figure, a square loop consisting of an inductor of inductance L and resistor of resistance R is placed between two long parallel wires. The two long straight wires have time-varying current of magnitude $I = I_0 \cos \omega t$ but the directions of current in them are opposite.



17. Total magnetic flux in this loop is

(1) $\dfrac{\mu_0 Ia}{\pi}\ln 2$ (2) $\dfrac{2\mu_0 Ia}{\pi}\ln 2$

(3) $\dfrac{4\mu_0 Ia}{\pi}\ln 2$ (4) $\dfrac{\mu_0 Ia}{2\pi}\ln 2$

18. Magnitude of emf in this circuit only due to flux change associated with two long straight current carrying wires will be

(1) $\dfrac{\mu_0 a\ln 2\, I_0\omega}{\pi}\sin\omega t$ (2) $\dfrac{2\mu_0 a\ln 2\, I_0\omega}{\pi}\sin\omega t$

(3) $\dfrac{\mu_0 a\ln 2\, I_0\omega}{2\pi}\cos\omega t$ (4) $\dfrac{\mu_0 a\ln 2\, I_0\omega}{\pi}\cos\omega t$

19. The instantaneous current in the circuit will be

(1) $\dfrac{2\mu_0 a\ln 2\, I_0\omega}{\pi\sqrt{R^2+\omega^2 L^2}}\sin(\omega t-\phi)$

(2) $\dfrac{2\mu_0 a\ln 2\, I_0\omega}{\pi\sqrt{R^2+\omega^2 L^2}}\sin(\omega t+\phi)$

(3) $\dfrac{\mu_0 a\ln 2\, I_0\omega}{\pi\sqrt{R^2+\omega^2 L^2}}\sin\omega t$

(4) $\dfrac{\mu_0 a\ln 2\, I_0\omega}{\pi\sqrt{R^2+\omega^2 L^2}}\sin(\omega t-\phi]$

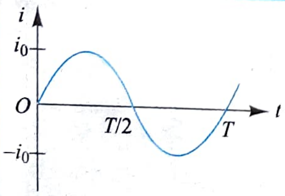
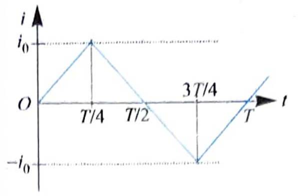
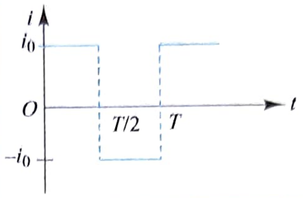
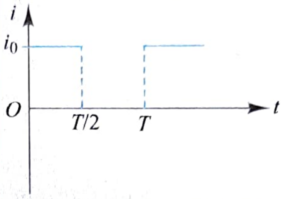
$\left(\text{where } \tan\phi=\dfrac{\omega L}{R}\right)$

Matrix Match Type

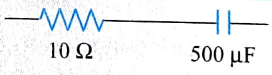
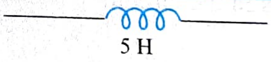
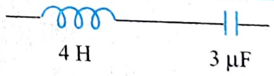
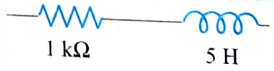
1. Match the following column.

	Column I		Column II
i.	Inductance of a coil	**a.**	Depends on resistivity
ii.	Capacitance	**b.**	Depends on shape
iii.	Impedance of coil	**c.**	Depends on medium inserted
iv.	Reactance of a capacitor	**d.**	Depends on external voltage source

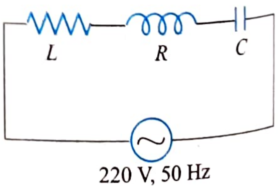
2. In column I, variation of current i with time t is given in the figure. In column II, root mean square current i_{rms} and average current are given. Match column I with corresponding quantities given in column II

	Column I		Column II
i.		**a.**	$i_{rms}=\dfrac{i_0}{\sqrt{3}}$
ii.		**b.**	Average current for positive half cycle is i_0
iii.		**c.**	Average current for positive half cycle is $\dfrac{i_0}{2}$
iv.		**d.**	Full cycle average current is zero

3. Four different circuit components are given in each situation of column I and all the components are connected across an ac source of same angular frequency $\omega = 200$ rad s^{-1}. The information of phase difference between the current and source voltage in each situation of column I is given in column II. Match the circuit components in column I with corresponding results in column II

	Column I		Column II
i.		**a.**	The magnitude of required phase difference is π/2.
ii.		**b.**	The magnitude of required phase difference is π/4.
iii.		**c.**	The current leads in phase to source voltage.
iv.		**d.**	The current lags in phase to source voltage.

4. In series R-L-C circuit, $R = 100\ \Omega$, $C = \dfrac{100}{\pi}\ \mu$F, and $L = \dfrac{100}{\pi}$ mH are connected to an ac source as shown in figure. The rms value of ac voltage is 220 V and its frequency is 50 Hz. In column I, some physical quantities are mentioned, while in column II, information about quantities are provided. Match the entries of column I with the entries of column II.



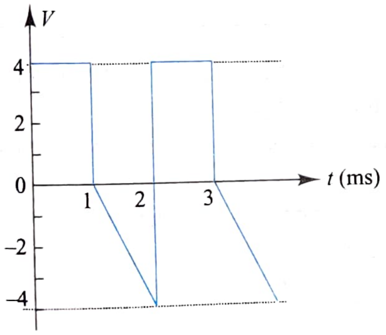
Column I	Column II
i. Average power dissipated in the resistor is	a. zero
ii. Average power dissipated in the inductor is	b. non-zero
iii. Average power dissipated in the capacitor is	c. 163 SI units
iv. The rms voltage across the capacitor is	d. 265.7 SI units

5. Consider all possibilities [L, R, C are non-zero]

Column I	Column II
i. In *L-R* series ac circuit	a. current lags inductor voltage by $\pi/2$
ii. In *R-C* series ac circuit	b. current lags voltage by an angle less than $\pi/2$
iii. In *L-C-R* series ac circuit (consider all possibilities)	c. current leads voltage by an angle less than $\pi/2$
iv. In purely resistive ac circuit	d. current and voltage are in phase

Numerical Value Type

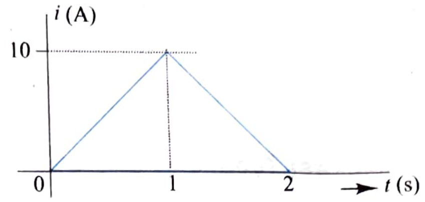
1. Variation of voltage with time is shown in figure.



(a) The rms voltage is found to be $N\sqrt{\frac{2}{3}}$ V. Find N.

(b) Also calculate the average value of voltage (in V).

2. Find the average value of current (in A) shown graphically in figure, from $t = 0$ to $t = 2$ s.

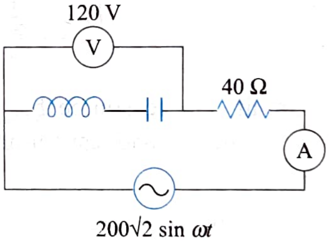


3. The average value of current $i = I_m \sin \omega t$ from $t = \frac{\pi}{2\omega}$ to $t = \frac{3\pi}{2\omega}$ is how many times of I_m?

4. An ac ammeter is used to measure current in a circuit. When a given direct current passes through the circuit, the ac ammeter reads 3 A. When another alternating current passes through the circuit, the ac ammeter reads 4 A. Then find the reading of this ammeter (in A), if dc and ac flow through the circuit simultaneously.

5. A coil of inductance $L = 5/8$ H and of resistance $R = 62.8\ \Omega$ is connected to the mains alternating voltage of frequency 50 Hz. What can be the capacitance of the capacitor (in μF) connected in series with the coil if the power dissipated has to remain unchanged? Take $\pi^2 = 10$.

6. In the given LCR series circuit find the reading (in A) of the hot wire ammeter. (here all hot wire meters are ideal)



Archives

JEE MAIN

Single Correct Answer Type

1. In a series *LCR* circuit, $R = 200\ \Omega$ and the voltage and the frequency of the main supply are 220 V and 50 Hz, respectively. On taking out the capacitance from the circuit, the current lags behind the voltage by 30°. On taking out the inductor from the circuit, the current leads the voltage by 30°. The power dissipated in the *LCR* circuit is
(1) 305 W
(2) 210 W
(3) Zero W
(4) 242 W **(AIEEE 2010)**

2. An arc lamp requires a direct current of 10 A at 80 V to function. If it is connected to a 220 V (rms), 50 Hz AC supply, the series inductor needed for it to work is close to
(1) 80 H
(2) 0.08 H
(3) 0.044 H
(4) 0.065 H **(JEE Main 2016)**

3. For an RLC circuit driven with voltage of amplitude v_m and frequency $\omega_0 = \frac{1}{\sqrt{LC}}$. The quality factor, Q is given by
(1) $\frac{CR}{\omega_0}$
(2) $\frac{\omega_0 L}{R}$
(3) $\frac{\omega_0 R}{L}$
(4) $\frac{R}{(\omega_0 C)}$

(JEE Main 2018)

4. In an ac circuit, the instantaneous emf and current are given by $\varepsilon = 100 \sin 30\,t$

$$i = 20\sin\left(30t - \frac{\pi}{4}\right)$$

In one cycle of alternate current, the average power consumed by the circuit and the wattless current are, respectively.

(1) 50, 0 (2) 50, 10

(3) $\frac{1000}{\sqrt{2}}, 10$ (4) $\frac{50}{\sqrt{2}}, 0$

(JEE Main 2018)

JEE ADVANCED

Single Correct Answer Type

1. An ac voltage source of variable angular frequency ω and fixed amplitude V_0 is connected in series with a capacitance C and an electric bulb of resistance R (inductance zero). When ω is increased
   (1) the bulb glows dimmer
   (2) the bulb glows brighter
   (3) total impedance of the circuit is unchanged
   (4) total impedance of the circuit increases

(IIT JEE, 2010)

Multiple Correct Answers Type

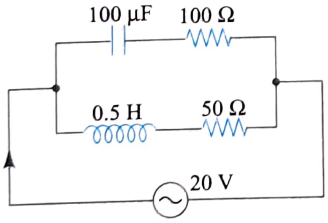
1. A series R-C circuit is connected to an ac voltage source. Consider two cases: (A) when C is without a dielectric medium and (B) when C is filled with a dielectric of constant 4. The current I_R through the resistor and voltage V_C across the capacitor are compared in the two cases. Which of the following is/are true?

   (1) $I_R^A > I_R^B$ (2) $I_R^A < I_R^B$

   (3) $V_C^A > V_C^B$ (4) $V_C^A < V_C^B$

(IIT-JEE 2011)

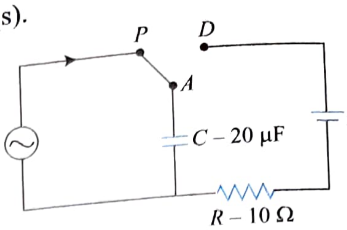
2. In the given circuit, the ac source has $\omega = 100$ rad s^{-1}. Considering the inductor and capacitor to be ideal, the correct choice(s) is/are



   (1) The current through the circuit, I is 0.3 A
   (2) The current through the circuit, I is $0.3\sqrt{2}$ A
   (3) The voltage across 100 Ω resistor is $10\sqrt{2}$ V
   (4) The voltage across 50 Ω resistor is 10 V

(IIT-JEE 2012)

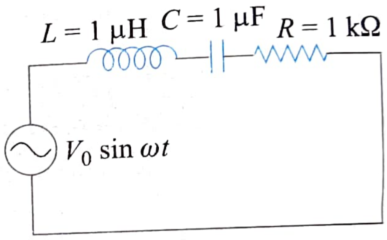
3. At time $t = 0$, terminal A in the circuit shown in the figure is connected to B by a key and an alternating current $I(t) = I_0 \cos(\omega t)$, with $I_0 = 1$ A and $\omega = 500$ rad/s starts flowing in it with the initial direction shown in the figure. At $t = 7\pi/6\omega$, the key is switched from B to D. Now onwards only A and D are connected. A total charge Q flows from the battery to charge the capacitor fully. If $C = 20$ μF, $R = 10$ Ω and the battery is ideal with emf of 50 V, identify the correct statement(s).



   (1) Magnitude of the maximum charge on the capacitor before $t = \frac{7\pi}{6\omega}$ is 1×10^{-3}C
   (2) The current in the left part of the circuit just before $t = \frac{7\pi}{6\omega}$ is clockwise.
   (3) Immediately after A is connected to D, the current in R is 10 A.
   (4) $Q = 2 \times 10^{-3}$C.

(JEE Advanced 2014)

4. In the circuit shown, $L = 1$ μH, $C = 1$ μF and $R = 1$ kΩ. They are connected in series with an a.c. source $V = V_0 \sin \omega t$ as shown. Which of the following options is/are correct?



   (1) At $\omega \sim 0$ the current flowing through the circuit becomes nearly zero
   (2) At $\omega >> 10^6$ rad s^{-1}, the circuit behaves like a capacitor
   (3) The frequency at which the current will be in phase with the voltage is independent of R
   (4) The current will be in phase with the voltage if $\omega = 10^4$ rads^{-1}

(JEE Advanced 2017)

5. The instantaneous voltages at three terminals marked X, Y and Z are given by

$$V_X = V_0 \sin \omega t$$

$$V_Y = V_0 \sin\left(\omega t + \frac{2\pi}{3}\right) \text{ and } V_Z = V_0 \sin\left(\omega t + \frac{4\pi}{3}\right)$$

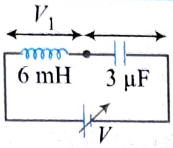
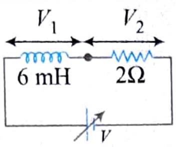
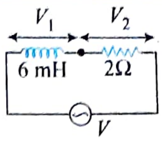
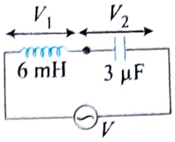
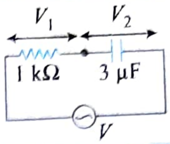
   An ideal voltmeter is configured to read rms value of the potential difference between its terminals. It is connected between points X and Y and then between Y and Z. The reading(s) of the voltmeter will be:

   (1) $V_{XY}^{rms} = V_0$
   (2) $V_{YZ}^{rms} = V_0\sqrt{\frac{1}{2}}$
   (3) Independent of the choice of the two terminals
   (4) $V_{XY}^{rms} = V_0\sqrt{\frac{3}{2}}$

(JEE Advanced 2017)

atrix Match Type

1. You are given many resistors, capacitors and inductors. These are connected to a variable dc voltage source (the first two circuits) or an ac voltage source of 50 Hz frequency (the next three circuits) in different ways as shown in Column II. When a current I (steady state for dc or rms for ac) flows through the circuit, the corresponding voltages V_1 and V_2 (indicated in circuits) are related as shown in Column I. Match the two.

	Column I		Column II
i.	$I \neq 0$, V_1 is proportional to I	a.	
ii.	$I \neq 0$, $V_2 > V_1$	b.	
iii.	$V_1 = 0$, $V_2 = V$	c.	
iv.	$I \neq 0$, V_2 is proportional to I	d.	
		e.	

(IIT-JEE 2010)

Numerical Value Type

1. A series R-C combination is connected to an ac voltage of angular frequency $\omega = 500$ rad s^{-1}. If the impedance of the R-C circuit is $R\sqrt{1.25}$, the time constant (in millisecond) of the circuit is

(IIT-JEE 2011)

Answers Key

EXERCISES

Single Correct Answer Type

1. (2) **2.** (1) **3.** (1) **4.** (3) **5.** (2)
6. (2) **7.** (3) **8.** (1) **9.** (3) **10.** (2)
11. (1) **12.** (2) **13.** (4) **14.** (3) **15.** (1)
16. (3) **17.** (1) **18.** (2) **19.** (2) **20.** (2)
21. (1) **22.** (2) **23.** (1) **24.** (1) **25.** (2)
26. (1) **27.** (1) **28.** (3) **29.** (2) **30.** (3)
31. (4) **32.** (4) **33.** (3) **34.** (2) **35.** (2)
36. (2) **37.** (2) **38.** (1) **39.** (2) **40.** (3)
41. (1) **42.** (2) **43.** (3) **44.** (3) **45.** (4)
46. (3) **47.** (1) **48.** (4) **49.** (2) **50.** (3)
51. (1) **52.** (3) **53.** (1) **54.** (4) **55.** (1)
56. (2) **57.** (3) **58.** (3) **59.** (3) **60.** (2)

Multiple Correct Answers Type

1. (1),(3) **2.** (1),(2) **3.** (1),(2),(3)
4. (1),(2),(3) **5.** (1),(3),(4) **6.** (1),(3)
7. (2),(3) **8.** (1),(3),(4)

Linked Comprehension Type

1. (1) **2.** (2) **3.** (1) **4.** (4) **5.** (1)
6. (3) **7.** (1) **8.** (1) **9.** (1) **10.** (1)
11. (4) **12.** (2) **13.** (4) **14.** (2) **15.** (1)
16. (3) **17.** (1) **18.** (1) **19.** (4)

Matrix Match Type

1. i. → b., c.; ii. → b., c.; iii. → a., b., c., d.; iv. → b., c., d.
2. i. → d.; ii. → a., d.; iii. → b., d.; iv. → b.
3. i. → b., c.; ii. → a., d.; iii. → a., c.; iv. → b., d.
4. i. → b., d.; ii. → a..; iii. → a.; iv. → b., c.
5. i. → a., b.; ii. → c.; iii. → a., b., c., d.; iv. → d.

Numerical Value Type

1. (a) (4), (b) 1 **2.** (5) **3.** (0) **4.** (5)
5. (8) **6.** (4)

ARCHIVES

JEE Main

Single Correct Answer Type

1. (4) **2.** (4) **3.** (2) **4.** (3)

JEE Advanced

Single Correct Answer Type

1. (2)

Multiple Correct Answers Type

1. (2),(3) **2.** (1),(3) **3.** (3),(4) **4.** (1),(3) **5.** (3),(4)

Matrix Match Type

1. i. → c., d., e.; ii. → b., c., d., e.; iii. → a., b.; iv. → b., c., d., e.

Numerical Value Type

1. (4)

7

Electromagnetic Waves

INTRODUCTION

In 1865, Maxwell discovered the displacement current. This brought together the phenomenon of electricity and magnetism into a coherent and unified theory. After this discovery, Maxwell predicted the transmission of energy by electromagnetic waves, which could travel with the speed of light. This led him to conclude that the light itself is an electromagnetic wave.

According to him, an accelerated charge produces a sinusoidal time-varying magnetic field, which in turn produces a sinusoidal time-varying electric field. The two fields so produced are mutually perpendicular time-varying electric and magnetic fields. They constitute electromagnetic waves which can propagate through empty space.

The velocity of electromagnetic waves in free space is given by

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

where $\mu_0 (= 1.257 \times 10^{-6}\,\mathrm{T\,m\,A^{-1}})$ and $\varepsilon_0 (= 8.854 \times 10^{-12}\,\mathrm{C^2\,N^{-1}\,m^{-2}})$ are, respectively, the absolute permeability and the absolute permittivity of free space. The velocity of electromagnetic waves in free space (vacuum) is equal to the velocity of light in vacuum, i.e., $3 \times 10^8\ \mathrm{m\ s^{-1}}$.

ILLUSTRATION 7.1

What physical quantity is the same for X-rays of wavelength 10^{-10} m, red light of wavelength 6800 Å, and radiowaves of wavelength 500 m?

Sol. Speed is the physical quantity which is the same in vacuum for given X-ray, red light and radiowave.

Value of speed of electromagnetic waves in vacuum is

$c = 3 \times 10^8\ \mathrm{m\ s^{-1}}$.

ILLUSTRATION 7.2

A plane electromagnetic wave travels in vacuum along z-direction. What can you say about the directions of its electric and magnetic field vectors? If the frequency of the wave is 30 MHz, what is its wavelength?

Sol. The electric field vector $\vec{E}$ and magnetic field vector $\vec{B}$ are in the xy plane.

Using $c = \lambda\nu$, we get $\nu = \dfrac{c}{\lambda}$ and $\lambda = \dfrac{3 \times 10^8}{30 \times 60} = 10$ m

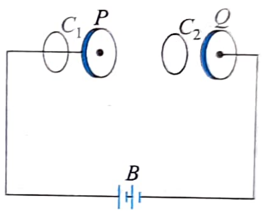
MAXWELL'S DISPLACEMENT CURRENT

According to Ampere's circuital law, the magnetic field B is related to steady current I as

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I \qquad \text{...(i)}$$

where I is the current travelling the surface bounded by closed path C.

In 1864, Maxwell showed that relation (i) is logically inconsistent. He accounted for this inconsistency as follows:



Consider a parallel plate capacitor having plates P and Q being charged with battery B.

During charging, a current I flows through the connecting wires which changes with time. This current will produce magnetic field around the wires which can be detected using a magnetic compass needle. Consider two loops c_1 and c_2 parallel to the plates P and Q of the capacitor. c_1 is enclosing only the connecting wire attached to the plate P of the capacitor and c_2 lies in the region between the two plates of capacitor. For the loop c_1, a current I is flowing through it, hence Ampere's circuital law for loop c_1 gives

$$\oint_{C_1} \vec{B} \cdot d\vec{l} = \mu_0 I \qquad \text{...(ii)}$$

Since the loop c_2 lies in the region between the plates of the capacitor, no current flows in this region. Hence Ampere's circuital law for loop c_2 gives

$$\oint_{C_2} \vec{B} \cdot d\vec{l} = 0 \qquad \text{...(iii)}$$

The relations (ii) and (iii) continue to be true even if two loops c_1 and c_2 are infinitesimally close to the plate P of the capacitor. On the other hand, as the loops c_1 and c_2 are infinitesimally close, it is expected that

$$\oint_{C_1} \vec{B} \cdot d\vec{l} = \oint_{C_2} \vec{B} \cdot d\vec{l} \qquad \text{...(iv)}$$

Thus, relation (iv) is in contradiction with relations (ii) and (iii). This led Maxwell to point out that Ampere's circuital law as given by (i) is logically inconsistent.

Idea of Displacement Current: Maxwell predicted that not only a current flowing in a conductor produces magnetic field but also a time-varying electric field (i.e., changing electric field) in a vacuum/free space (or in a dielectric) produces a magnetic field.

It means a changing electric field gives rise to a current which flows through a region so long as the electric field is changing there. Maxwell also predicted that this current produces the same magnetic field as a conduction current can produce. This current is known as 'displacement current'.

Thus, displacement current is that current which comes into play in the region in which the electric field and hence the electric flux is changing with time.

Maxwell defined this displacement current in space where electric field is changing with time as

$$I_D = \varepsilon_0 \frac{d\phi_E}{dt} \qquad \text{...(v)}$$

where ϕ_E is the electric flux.

Maxwell also found that conduction current (I) and displacement current (I_D) together have the property of continuity, although, individually, they may not be continuous.

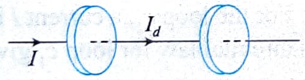
This idea led Maxwell to modify Ampere's circuital law in order to make the same logically consistent. He states Ampere circuital law to the form,

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 (I + I_D) = \mu_0 \left(I + \varepsilon_0 \frac{d\phi_E}{dt} \right)$$

It is now called as Ampere Maxwell law.

ILLUSTRATION 7.3

The following figure shows capacitor made of two circular plates. The capacitor is being charged by an external source which supply constant current equal to 0.15 A. Obtain displacement charged by constant current across plates. Given $\frac{dV}{dt} = 1.87 \times 10^9 \text{ V s}^{-1}$.



Sol. Displacement current $I_d = \frac{\varepsilon_0 \, d\phi_C}{dt}$

But $\phi_E = EA = \frac{d}{\varepsilon_0 A} A = \frac{q}{\varepsilon_0}$

$$\therefore \quad I_d = \varepsilon_0 \frac{d}{dt} \frac{q}{\varepsilon_0} = \frac{\varepsilon_0}{\varepsilon_0} \frac{dq}{dt} = \frac{dq}{dt} = I$$

Here $I = 0.15$ A $\therefore I_d = 0.15$ A

MAXWELL'S EQUATIONS

The four Maxwell's equations and Lorentz force law together constitute the foundations of classical electromagnetism. The Maxwell's equations are:

1st equation: $\oiint \vec{E} \cdot d\vec{S} = \frac{q}{\varepsilon_0}$

2nd equation: $\oiint \vec{B} \cdot d\vec{S} = 0$

3rd equation: $\oiint \vec{B} \cdot d\vec{l} = \frac{-d}{dt} \iint \vec{B} \cdot d\vec{S}$

4th equation: $\oiint \vec{B} \cdot d\vec{l} = \mu_0 \varepsilon_0 \frac{d}{dt} \cdot \iint \vec{B} \cdot d\vec{S} + \mu_0 \cdot I$

The Lorentz equation is

$$\vec{F} = q \left(\vec{E} \times \vec{v} \times \vec{B} \right)$$

$$E_z = Ez_0 \sin(\omega t - ky)$$

$$B_x = Bx_0 \sin(\omega t - ky), \text{ where } \frac{\omega}{k} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

Since $\omega = 2\pi f$, where f is the frequency and $k = 2\pi/\lambda$, where λ is the wavelength.

Therefore, $\frac{\omega}{k} = \frac{2\pi f}{2\pi/\lambda} = f\lambda$

But $f\lambda$ gives the velocity of the wave. Hence $f\lambda = c = \omega k$. Hence, the velocity of the waves is given by

$$c = \frac{\omega}{k} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

ILLUSTRATION 7.4

Suppose that the electric field amplitude of an electromagnetic wave is $E_0 = 120$ N/C and that its frequency is $v = 50.0$ MHz. **(a)** Determine B_0, ω, k, and λ. **(b)** Find expressions for $\vec{E}$ and $\vec{B}$.

Sol.

(a) (i) Using $\frac{E_{\text{max}}}{B_{\text{max}}} = c$, we get

$$B_{\text{max}} = \frac{E}{c} = \frac{120}{3 \times 10^8} = 40 \times 10^{-7} \text{ T}$$

i.e., $B_{\text{max}} = 400$ nT

(ii) $\omega = 2\pi v = 2 \times \pi \times 50 \times 10^6$

$= 3.14 \times 10^8$ rad s^{-1}

(iii) $k = \frac{2\pi}{\lambda} = \frac{2\pi}{6} = \frac{2 \times 3.14}{6} = 1.05 \text{ rad s}^{-1}$

(iv) $c = \lambda v$ or $\lambda = \frac{c}{v} = \frac{3 \times 10^8}{50 \times 10^6} = 6$ m

(b) $\vec{E} = E_{\text{max}} \sin(kx - \omega t) \hat{j}$

$= 120 \sin(1.05x - 3.14 \times 10^8 \times t) \hat{j}$

$\vec{B} = B_{\text{max}} \sin(kx - \omega t) \hat{k}$

$= 400 \sin(1.05x - 3.14 \times 10^8 \times t) \hat{k}$

ILLUSTRATION 7.5

Suppose that the electric field part of a electromagnetic wave in vacuum is

$\vec{E} = (3.1 \text{ N/C}) \cos (1.8 \text{ rad/m})y + (5.4 \times 10^6 \text{ rad/s})t\}$

(a) What is the direction of propagation?

(b) What is the wavelength λ?

(c) What is the frequency v?

(d) What is the amplitude of the magnetic field part of the wave?

(e) Write an expression for the magnetic field part of the wave.

Sol. Here $\vec{E} = 3.1\cos\{1.8y + (5.4 \times 10^6)t\}\hat{i}$

Comparing it with standard equation

$$\vec{E} = E_0 \cos(kx + \omega t)d$$

(a) $-\hat{j}$ direction

(b) Using $k = \frac{2\pi}{\lambda}$, we get $\lambda = \frac{2\pi}{k} = \frac{2\pi}{1.8} = 3.5$ m

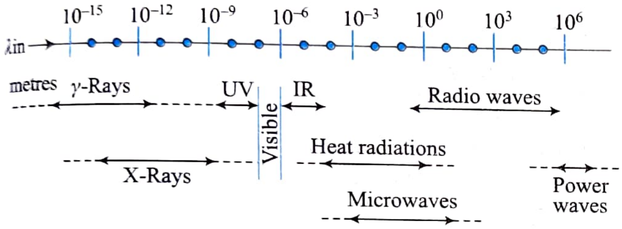
(c) Frequency, $\nu = \frac{\omega}{2\pi} = \frac{5.4 \times 10^6}{2\pi} = 0.86$ MHz

(d) $c = \frac{E}{B}$ or $B = \frac{E}{c} = \frac{3.1}{3 \times 10^8} = 10$ nT

(e) $\vec{B} = 10\cos(1.8y + 5.4 \times 10^6)\hat{k}$

ELECTROMAGNETIC WAVE SPECTRUM

Broadly, the various regions of the electromagnetic wave spectrum may be assigned the wavelength ranges as follows:



ENERGY DENSITY

The electric and magnetic fields in a plane electromagnetic wave are given by

$$E = E_0 \sin\omega(t - x/c)$$

and $B = B_0 \sin\omega(t - x/c)$

In any small volume dV, the energy of the electric field is

$$U_E = \frac{1}{2}\varepsilon_0 E^2\, dV$$

and the energy of the magnetic field is $U_B = \frac{1}{2\mu_0} B^2\, dV$

Thus, the total energy is $U = \frac{1}{2}\varepsilon_0 E^2\, dV + \frac{1}{2\mu_0} B^2\, dV$

The energy density is $u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu Z_0} B^2$

$$u = \frac{1}{2}\varepsilon_0 E_0^2 \sin^2 \omega(t - x/c) + \frac{1}{2\mu_0} B_0^2 \sin^2 \omega(t - x/c)$$

If we take the average over a long time, the $\sin^2$ terms have an average value of 1/2

Thus, $u_{av} = \frac{1}{4}\varepsilon_0 E_0^2 + \frac{1}{4\mu_0} B_0^2$

Now, $E_0 = cB_0$ and $\mu_0\varepsilon_0 = \frac{1}{c^2}$ so that, $\mu_0 = \frac{1}{\varepsilon_0 c^2}$ and

$$B_0 = \frac{E_0}{c} \quad \frac{1}{4\mu_0} B_0^2 = \frac{\varepsilon_0 c^2}{4}\left(\frac{E_0}{c}\right)^2 = \frac{1}{4}\varepsilon_0 E_0^2$$

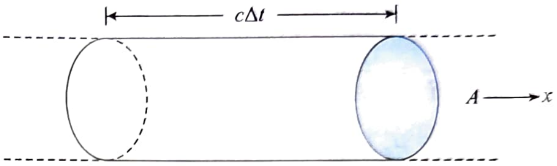
Thus, the electric energy density is equal to the magnetic density in average,

or, $u_{av} = \frac{1}{4}\varepsilon_0 E_0^2 + \frac{1}{4}\varepsilon_0 E_0^2 = \frac{1}{2}\varepsilon_0 E_0^2$

Also $u_{av} = \frac{1}{4\mu_0} B_0^2 + \frac{1}{4\mu_0} B_0^2 = \frac{1}{2\mu_0} B_0^2$

INTENSITY

The energy crossing per unit area per unit time perpendicular to the direction of propagation is called the intensity of a wave.



Consider a cylindrical volume with area of cross section A and length $c\,\Delta t$ along the x-axis. The energy contained in this cylinder crosses the area A in time Δt as the wave propagates at speed c. The energy contained is

$$U = u_{av}\,(c\,\Delta t)A$$

The intensity is $I = \frac{U}{A\Delta t} = u_{av}c$ in terms of maximum electric field, $I = \frac{1}{2}\varepsilon_0 E_0^2 c$

MOMENTUM

The electromagnetic wave also carries linear momentum with it. The linear momentum carried by the portion of wave having energy U is given by $p = U/c$.

Thus, if the wave incident on a material surface is completely absorbed, it delivers energy U and momentum $p = U/c$ to the surface. If the wave is totally reflected, the momentum delivered is $2U/c$ because the momentum of the wave changes from p to $-p$. It follows that electromagnetic waves incident on a surface exert a force on the surface.

PROPERTIES OF ELECTROMAGNETIC WAVES

(a) In electromagnetic waves the electric field vector $\vec{E}$, magnetic field vector $\vec{B}$ and propagation vector $\vec{k}$ are mutually perpendicular such that $\vec{E}$, $\vec{B}$, and $\vec{k}$ form a right angle system. Hence electromagnetic waves are transverse in nature.

(b) Electromagnetic waves travel with speed of light. In vacuum their speed is

$$c = \frac{1}{\sqrt{\mu_0\,\varepsilon_0}}$$

In isotropic medium, their speed is

$$v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu . \mu_0 \varepsilon . \varepsilon_0}}$$

$$= \frac{1}{\sqrt{\mu_r \varepsilon_r}} \cdot \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = \frac{c}{n}$$

where $n = \sqrt{\mu_r \, \varepsilon_r}$ is the refractive index of the medium,
μ_r = relative permeability of medium, and
ε_r = relative permittivity of medium or electric dielectric constant.

(c) The Poynting vector $\vec{S} = \vec{E} \times \vec{H}$ represents the direction of energy flow per unit area per second along the direction of wave propagation. Its unit is $\mathrm{Wm^{-2}}$.

(d) The medium offers hindrance to the propagation of wave. Such a hindrance is called Wave Impedance (Z) and its value in a medium is given by

$$Z = \frac{E}{H} = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_r}{\varepsilon_r}} \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

$$(\therefore \; \mu = \mu_r \, \mu_0 \text{ and } \varepsilon : \varepsilon_0 \, \varepsilon_r \;)$$

where μ_r and ε_r are relative permeability and relative permittivity of the medium.
For vacuum or free space

$$Z = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 376.6 \; \Omega$$

Also in free space

$$\frac{E}{B} = \frac{E}{\mu_0 H} = \frac{1}{\mu_0} \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

$$\frac{E}{B} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = c$$

$$E = Bc$$

(e) In free space
The electrostatic energy density is equal to magnetostatic energy density.
Average electric energy density = Average magnetic energy density

$$= \frac{1}{4} \varepsilon_0 \, E_0^2 = \frac{1}{4} \mu_0 \, H_0^2 = \frac{B_0^2}{4\mu_0}$$

$$\text{Total average energy density} = \frac{1}{2} \varepsilon_0 \, E_0^2 \text{ (in free space)}$$

(f) In a medium

$$\text{Electric energy density} = \frac{1}{2} \varepsilon \, E^2$$

$$\text{Magnetic energy density} = \frac{B^2}{2\mu}$$

$$\text{Total energy density} = \frac{1}{2} \varepsilon E^2 + \frac{B^2}{2\mu}$$

(g) The electric and magnetic fields satisfy the following wave equations, which can be obtained from Maxwell's third and fourth equations.

$$\frac{\partial^2 E}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$

$$\frac{\partial^2 B}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B}{\partial t^2}$$

(h) Electromagnetic waves carry momentum and hence can exert pressure (P) on surfaces, which is known as radiation pressure. For an electromagnetic wave with Poynting vector $\vec{S}$, incident upon a perfectly absorbing surface.

$$P = \frac{S}{c}$$

and if incident upon a perfectly reflecting surface, then

$$P = \frac{2S}{c}$$

(i) The electric and magnetic fields of a sinusoidal plane electromagnetic wave propagating in the positive x-direction can also be written as

$$E = E_0 \sin \, (kx - \omega t)$$

$$B = B_0 \sin \, (kx - \omega t)$$

where ω is the angular frequency of the wave and k is the angular wave number or propagation constant which are given by

$$\omega = 2\pi f \text{ and } k = \frac{2\pi}{\lambda}$$

(j) The intensity of a sinusoidal plane electromagnetic wave is defined as the average value of Polynting vector taken over one cycle.

ATMOSPHERE

The atmosphere of earth extends upon about 300 km. The composition of atmosphere is as given below.

TROPOSPHERE

Troposphere extends up to a height of about 12 km. This layer consists of water droplets, vapour, dust particles etc. Density of air decreases as we move from bottom to top of this layer. This layer is responsible for all the weather phenomena affecting our environment.

STRATOSPHERE

This layer extends from about 10 km to about 50 km. In the upper part of the atmosphere, we have a layer of ozone. The density of air at the top of the stratosphere is about 10^{-3} times the density at the surface of earth.

MESOSPHERE

The mesosphere extends from about 50 km to about 80 km.

IONOSPHERE

The atmosphere above the mesosphere is called ionosphere. This layer is composed partly of electrons and positive ions. The rest of the atmosphere is composed mostly of neutral molecules.

The atmosphere is transparent to visible radiation and we can see the sun and the stars through it clearly. However, most infrared radiation is not able to pass through, as it is absorbed by the atmosphere. Low lying clouds in the atmosphere also prevent infrared radiation from passing through. The ozone layer blocks the passage of ultraviolet radiation from the sun.

ILLUSTRATION 7.6

About 5% of the power of a 100 W light bulb is converted to visible radiation. What is the average intensity of visible radiation

(a) At a distance of 1 m from the bulb?

(b) At a distance of 10 m?

Assume that the radiation is emitted isotropicailly and neglect reflection.

Sol.

(a) Visible power = 5 W

$\therefore$ Average intensity of radiation at 1 m

$$= \frac{\text{Power}}{4\pi r^2} = \frac{5}{4\times\pi\times1} = 0.4\text{ W m}^{-2}$$

(b) Average intensity of radiation at 10 m

$$= \frac{\text{Power}}{4\pi r^2} = \frac{5}{4\times\pi\times10^2} = 0.004\text{ W m}^{-2}$$

ILLUSTRATION 7.7

Given below are some famous numbers associated with electromagnetic radiation in different contexts in physics. State the part of the em spectrum of which each belongs.

(a) 21 cm (wavelength emitted by atomic hydrogen in interstellar space)

(b) 1057 MHz (frequency of radiation arising from two close energy levels in hydrogen, known as Lamb shift)

(c) 2.7 K (temperature associated with the isotropic radiation filling all space-thought to be a relic of the 'big-bang' origin of the universe)

(d) 5890 Å–58963 Å (double lines of sodium)

(e) 14.4 keV (energy of a particular transition n ^{57}Fe nucleus associated with a famous high resolution spectroscopic method of sodium)

Sol.

(a) Wavelength is of the order of 10^{-2} m, i.e., short radio wave.

(b) Frequency is of the order of 10^{-2} Hz, i.e,. short radio wave.

(c) $\lambda_m T = 0.29$ or

$$\lambda_m = \frac{0.29}{T} = \frac{0.29}{2.7} = 0.09\text{ cm} = 0.0009\text{ m}$$

Wavelength is of the order of 10^{-4} m, i.e., microwave.

(d) Wavelengths are of the order of 10^{-7} m, i.e., visible radiations.

(e) Energy $= \dfrac{hc}{\lambda e}$ or $\lambda = \dfrac{hc}{\text{Energy}\times e}$

$$\lambda = \frac{6.63\times10^{-34}\times3\times10^{8}}{14.4\times10^{3}\times1.6\times10^{-19}}$$

i.e., $\lambda = 0.86\times10^{-10}$

$= 8.6\times10^{-11}$ m

i.e., X-rays.

Solved Examples

EXAMPLE 7.1

In an apparatus, the electric field was found to oscillate with an amplitude of 18 V/m. The magnitude of the oscillating magnetic field will be

(1) 4×10^{-6}T (2) 6×10^{-8}T
(3) 9×10^{-9}T (4) 11×10^{-11}T

Sol. (2) $c = \dfrac{E}{B} \Rightarrow B = \dfrac{E}{c} = \dfrac{18}{3\times10^8} = 6\times10^{-8}\text{T}$.

EXAMPLE 7.2

According to Maxwell's hypothesis, a changing electric field gives rise to

(1) an e.m.f. (2) electric current
(3) magnetic field (4) pressure radiant

Sol. (3) According to the Maxwell's EM theory, the EM waves propagation contains electric and magnetic field vibration in mutually perpendicular direction. Thus the changing of electric field give rise to magnetic field.

EXAMPLE 7.3

In an electromagnetic wave, the electric and magnetising fields are 100 Vm^{-1} and 0.265 Am^{-1}. The maximum energy flow is

(1) 26.5 W/m^2 (2) 36.5 W/m^2
(3) 46.7 W/m^2 (4) 765 W/m^2

Sol. (1) Here $E_0 = 100$ V/m, $B_0 = 0.265$ A/m.

$\therefore$ Maximum rate of energy flow $S = E_0 \times B_0$

$$= 100\times.265 = 26.5\frac{\text{W}}{\text{m}^2}$$

EXAMPLE 7.4

The 21 cm radio wave emitted by hydrogen in interstellar space is due to the interaction called the hyperfine interaction is atomic hydrogen. the energy of the emitted wave is nearly

(1) 10^{-17} joule (2) 1 joule
(3) 7×10^{-3} joule (4) 10^{-24} joule

Sol. (4) $E = \dfrac{hc}{\lambda} = \dfrac{6.6\times10^{-34}\times3\times10^{8}}{21\times10^{-2}} = 0.94\times10^{-24} \approx 10^{-24}$ J

EXAMPLE 7.5

The oscillating electric and magnetic vectors of an electromagnetic wave are oriented along

(1) The same direction but differ in phase by 90°
(2) The same direction and are in phase
(3) Mutually perpendicular directions and are in phase
(4) Mutually perpendicular directions and differ in phase by 90°

Sol. (3) $\vec{E}$ and $\vec{B}$ are mutually perpendicular to each other and are in phase i.e. they become zero and minimum at the same place and at the same time.

EXAMPLE 7.6

In which one of the following regions of the electromagnetic spectrum will the vibrational motion of molecules give rise to absorption

(1) ultraviolet (2) microwaves
(3) infrared (4) radio waves

Sol. **(2)** Molecular spectra due to vibrational motion lie in the microwave region of EM-spectrum. Due to Kirchhoff's law in spectroscopy the same will be absorbed.

EXAMPLE 7.7

An electromagnetic wave travels along z-axis. Which of the following pairs of space and time varying fields would generate such a wave

(1) E_x, B_y (2) E_y, B_x (3) E_z, B_x (4) E_y, B_z

Sol. **(1)** E_x and B_y would generate a plane EM wave travelling in z-direction. $\vec{E}$, $\vec{B}$ and $\vec{k}$ form a right handed system $\vec{k}$ is along z-axis. As $\hat{i}\times\hat{j}=\hat{k}$

$\Rightarrow E_x\hat{i}\times B_y\hat{j}=C\hat{k}$ i.e. E is along x-axis and B is along y-axis.

EXAMPLE 7.8

A signal emitted by an antenna from a certain point can be received at another point of the surface in the form of

(1) sky wave (2) ground wave
(3) sea wave (4) both (1) and (2)

Sol. **(4)** Ground wave and sky wave both are amplitude modulated wave and the amplitude modulated signal is transmitted by a transmitting antenna and received by the receiving antenna at a distance place.

EXAMPLE 7.9

The electromagnetic waves do not transport

(1) energy (2) charge (3) momentum (4) information

Sol. **(2)** EM waves transport energy, momentum and information but not charge. EM waves are uncharged

EXAMPLE 7.10

A plane electromagnetic wave is incident on a material surface. If the wave delivers momentum p and energy E, then

(1) $p=0, E=0$ (2) $p\neq0, E\neq0$
(3) $p\neq0, E=0$ (4) $p=0, E\neq0$

Sol. **(2)** EM waves carry momentum and hence can exert pressure on surfaces. They also transfer energy to the surface so $p\neq0$ and $E\neq0$.

EXAMPLE 7.11

An electromagnetic wave, going through vacuum is described by $E=E_0\sin(kx-\omega t)$. Which of the following is independent of wavelength

(1) k (2) ω (3) k/ω (4) $k\omega$

Sol. **(3)** The angular wave number $k=\frac{2\pi}{\lambda}$; where λ is the wave length. The angular frequency is $\omega=2\pi v$.

The ratio $\frac{k}{\omega}=\frac{2\pi/\lambda}{2\pi v}=\frac{1}{v\pi}=\frac{1}{c}=$ constant

EXAMPLE 7.12

An electromagnetic wave going through vacuum is described by $E=E_0\sin(kx-\omega t)$; $B=B_0\sin(kx-\omega t)$. Which of the following equation is true

(1) $E_0k=B_0\omega$ (2) $E_0\omega=B_0k$
(3) $E_0B_0=\omega k$ (4) None of these

Sol. **(1)** $\frac{E_0}{B_0}=C$. also $k=\frac{2\pi}{\lambda}$ and $\omega=2\pi v$

These relation gives $E_0K=B_0\omega$

EXAMPLE 7.13

A radio receiver antenna that is 2 m long is oriented along the direction of the electromagnetic wave and receives a signal of intensity 5×10^{-16} W/m². The maximum instantaneous potential difference across the two ends of the antenna is

(1) 1.23 μV (2) 1.23 mV (3) 1.23 V (4) 12.3 mV

Sol. **(1)** $I=\frac{1}{2}\varepsilon_0 CE_0^2$

$$\Rightarrow \quad E_0=\sqrt{\frac{2I}{\varepsilon_0 C}}=\sqrt{\frac{2\times5\times10^{-16}}{8.85}}=0.61\times10^{-6}\,\frac{\text{V}}{\text{m}}$$

Also $E_0=\frac{V_0}{d}$ $\Rightarrow V_0=E_0d=0.61\times10^{-6}\times2=1.23\,\mu\text{V}$

EXAMPLE 7.14

Electromagnetic waves travel in a medium which has relative permeability 1.3 and relative permittivity 2.14. Then the speed of the electromagnetic wave in the medium will be

(1) 13.6×10^{6} m/s (2) 1.8×10^{2} m/s
(3) 3.6×10^{8} m/s (4) 1.8×10^{8} m/s

Sol. **(4)** $v=\frac{c}{\sqrt{\mu_r\varepsilon_r}}=\frac{3\times10^8}{\sqrt{1.3\times2.14}}=1.8\times10^8$ m/s

EXAMPLE 7.15

If c is the speed of electromagnetic waves in vacuum, its speed in a medium of dielectric constant K and relative permeability μ_r is

(1) $v=\frac{1}{\sqrt{\mu_rK}}$ (2) $v=c\sqrt{\mu_rK}$
(3) $v=\frac{c}{\sqrt{\mu_rK}}$ (4) $v=\frac{K}{\sqrt{\mu_rC}}$

Sol. **(3)** Speed of light of vacuum $c=\frac{1}{\sqrt{\mu_0\varepsilon_0}}$ and in another medium $v=\frac{1}{\sqrt{\mu\varepsilon}}$

$$\therefore \quad \frac{c}{v}=\sqrt{\frac{\mu\varepsilon}{\mu_0\varepsilon_0}}=\sqrt{\mu_rK}\Rightarrow v=\frac{c}{\sqrt{\mu_rK}}$$

Exercises

Single Correct Answer Type

1. The intensity of sunlight (in W/m^2) at the solar surface will be
(1) 5.6×10^6 (2) 5.6×10^7
(3) 4.2×10^6 (4) 4.2×10^7

2. Which of the following has zero average value in a plane electromagnetic wave?
(1) Electric field (2) Magnetic potential
(3) Electric energy (4) Magnetic energy

3. The wave impedance of free space is
(1) 0 Ω (2) 376.6 Ω
(3) 1883 Ω (4) 3776 Ω

4. The frequency 1057 MHz of radiation arising from two close energy levels in hydrogen belongs to
(1) radio waves (2) infrared waves
(3) micro waves (4) γ-rays

5. The wave associated with 2.7 K belongs to
(1) radio waves (2) micro waves
(3) ultraviolet ways (4) infrared waves

6. The percentage power of X-ray increases with the increases in its
(1) velocity (2) intensity
(3) frequency (4) wavelength

7. The shortest wavelength of X-rays remitted from an X-ray tube depends upon
(1) nature of the gas in the tube
(2) voltage applied to the tube
(3) current in the tube
(4) nature of target of the tube

8. X-rays are not used for radar purposes, because they are not
(1) reflected by target
(2) partly absorbed by target
(3) electromagnetic waves
(4) completely absorbed by target

9. The energy of X-rays photon is 3.3×10^{-16} J. Its frequency is
(1) 2×10^{19} Hz (2) 5×10^{18} Hz
(3) 5×10^{17} Hz (4) 5×10^{16} Hz

10. In X-ray tube, the accelerating potential applied at the anode is V volt. The minimum wavelength of the emitted X-ray will be
(1) eV/h (2) h/eV
(3) eV/ch (4) hc/eV

11. The area of television telecast is made twice, the height of antenna will be changed as
(1) halved (2) doubled
(3) quardupled (4) kept unchanged

12. If a source is transmitting electromagnetic wave of frequency 8.2×10^6 Hz, then the wavelength of the electromagnetic waves transmitted from the source will be
(1) 36.6 m (2) 40.5 m
(3) 42.3 m (4) 50.9 m

13. A plane electromagnetic wave

$$E_z = 100\cos(6 \times 10^8 t + 4x)\ \text{V/m}$$

propagates in a medium of dielectric constant
(1) 1.5 (2) 2.0
(3) 2.4 (4) 4.0

14. If μ_0 be the permeability and K_0 the dielectric constant of a medium, its refractive index is given by
(1) $\dfrac{1}{\sqrt{\mu_0 K_0}}$ (2) $\dfrac{1}{\mu_0 K_0}$
(3) $\sqrt{\mu_0 K_0}$ (4) $\mu_0 K_0$

15. If ε_0 and μ_0 represent the permittivity and permeability of vacuum and ε and μ represent the permittivity and permeability of medium, the refractive index of the medium is given by
(1) $\sqrt{\dfrac{\varepsilon_0\mu_0}{\varepsilon\,\mu}}$ (2) $\sqrt{\dfrac{\varepsilon\mu}{\varepsilon_0\mu_0}}$
(3) $\sqrt{\dfrac{\varepsilon}{\mu_0\,\varepsilon_0}}$ (4) $\sqrt{\dfrac{\mu_0\,\varepsilon_0}{\varepsilon}}$

16. In a plane electromagnetic wave, the electric field oscillates sinusoidally at a frequency of 2.5×10^{10} Hz and amplitude 480 V/m. The amplitude of the oscillating magnetic field will be
(1) 1.52×10^{-8} Wb/m^2 (2) 1.52×10^{-7} Wb/m^2
(3) 1.6×10^{-6} Wb/m^2 (4) 1.6×10^{-7} Wb/m^2

17. The magnetic field between the plates of radius 12 cm separated by a distance of 4 mm of a parallel plate capacitor of capacitance 100 pF along the axis of plates having conduction current of 0.15 A is
(1) zero (2) 1.5 T
(3) 15 T (4) 0.15 T

18. A flood light is covered with a fitter that transmits red light. The electric field of the emerging beam is represented by a sinusoidal plane wave

$$E_x = 36\sin(1.20 \times 10^7 z - 3.6 \times 10^{15} t)\ \text{V/m}$$

The average intensity of beam in W/m^2 will be
(1) 6.88 (2) 3.44
(3) 1.72 (4) 0.86

19. The average energy-density of electromagnetic wave given by $E = (50\ \text{N/C})\sin(\omega t - kx)$ will be nearly
(1) 10^{-8} J/m^3 (2) 10^{-7} J/m^3
(3) 10^{-6} J/m^3 (4) 10^{-5} J/m^3

20. A larger parallel plate capacitor, whose plates have an area of 1 m^2 are separated from each other by 1 mm. If the plates has dielectric constant 10, then the displacement current at this instant is
(1) 25 μA (2) 11 μA
(3) 2.2 μA (4) 1.1 μA

21. A parallel plate capacitor with plate area A and separation between the plates d, is charged by a constant current i. Consider a plane surface of area $A/2$ parallel to the plates and drawn simultaneously between the plates. The displacement current through this area is
(1) i (2) $i/2$
(3) $i/4$ (4) $i/8$

22. The sum delivers 10^4 W/m² of electromagnetic flux to the earth's surface. The total power that is incident on a roof of dimensions (10×10) m² will be
(1) 10^4 W (2) 10^5 W
(3) 10^6 W (4) 10^7 W

23. The average value of electric energy density in an electromagnetic wave is (E_0 is peak value)
(1) $\frac{1}{2}\varepsilon_0 E_0^2$ (2) $\frac{E_0^2}{2\varepsilon_0}$
(3) $\varepsilon_0 E_0^2$ (4) $\frac{1}{4}\varepsilon_0 E_0^2$

24. The sun delivers 10^3 W/m² of electromagnetic flux to the earth's surface. The total power that is incident on a roof of dimensions 8 m × 20 m will be
(1) 2.56×10^4 W (2) 6.4×10^5 W
(3) 4.0×10^5 W (4) 1.6×10^5 W

25. In Q. 24, the radiation force on the roof will be
(1) 8.53×10^{-5} N (2) 2.3×10^{-3} N
(3) 1.33×10^{-3} N (4) 5.33×10^{-4} N

26. A plane electromagnetic wave of wave intensity 6 W/m² strikes a small mirror of area 39 cm², held perpendicular to the approaching wave. The momentum transferred in kg ms⁻¹ by the wave to the mirror each second will be
(1) 1.2×10^{-10} (2) 2.4×10^{-9}
(3) 3.6×10^{-8} (4) 4.8×10^{-7}

27. The wave of wavelength 5900 Å emitted by any atom or molecule must have some finite total length which is known as coherence length. For sodium light, this length is 2.4 cm. The number of oscillations in this length will be
(1) 4.068×10^8 (2) 4.068×10^4
(3) 4.068×10^6 (4) 4.068×10^5

28. A radiowave has a maximum electric field intensity of 10^{-4} V m^{-1} on arrival at a receiving antenna. The maximum magnetic flux density of such a wave is
(1) zero (2) 3×10^4 T
(3) 5.8×10^{-9} T (4) 3.3×10^{-13} T

29. The transmitting antenna of a radio-station is mounted vertically. At a point 10 km due north of the transmitter the peak electric field is 10^{-3} V/m. The amplitude of the radiated magnetic field is
(1) 3.33×10^{-10} T (2) 3.33×10^{-12} T
(3) 10^{-3} T (4) 3×10^5 T

30. A circular ring of radius r is placed in a homogeneous magnetic field perpendicular to the plane of the ring. The field B changes with time according to the equation $B = Kt$, where K is a constant and t is the time. The electric field in the ring is
(1) $\frac{Kr}{4}$ (2) $\frac{Kr}{3}$
(3) $\frac{Kr}{2}$ (4) $\frac{K}{2r}$

31. A plane em wave of wave intensity of 10 W/m² strikes a small mirror of area 20 cm², held perpendicular to the approaching wave. The radiation force on the mirror will be
(1) 6.6×10^{-11} N (2) 1.33×10^{-11} N
(3) 1.33×10^{-10} N (4) 6.6×10^{-10} N

32. In a region of free space the electric field at some instant of time is $\vec{E} = (80\hat{i} + 32\hat{j} - 64\hat{k})$ V/m and the magnetic field is $\vec{B} = (0.2\hat{i} + 0.08\hat{j} - 0.29\hat{k})\mu$T. The pointing vector for these fields is
(1) $-11.52\hat{i} + 28.8\hat{j}$ (2) $-28.8\hat{i} + 11.52\hat{i}$
(3) $28.8\hat{i} - 11.52\hat{j}$ (4) $11.52\hat{i} - 28.8\hat{j}$

33. A plane electromagnetic wave propagating in the x-direction has wavelength of 60 mm. The electric field is in the y-direction and its maximum magnitude is 33 V/m^{-1}. The equation for the electric field as function of x and t is
(1) $11 \sin \pi(t - x/c)$ (2) $33 \sin \pi \times 10^{11}(t - x/c)$
(3) $33 \sin \pi (t - x/c)$ (4) $11 \sin \pi \times 10^{11}(t - x/c)$

34. In an electromagnetic wave, the electric field oscillated sinusoidally with amplitude 45 Vm^{-1}, the rms value of oscillating magnetic field will be
(1) 1.6×10^{-8} T (2) 16×10^{-9} T
(3) 144×10^{-8} T (4) 11.3×10^{-8} T

35. The magnetic field in the plane electromagnetic wave is given by

$$B_z = 2 \times 10^{-7} \sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t) \text{ tesla.}$$

The expression for electric field will be .
(1) $E_z = 30\sqrt{2}\sin(0.5\times10^3 x + 1.5\times10^{11} t)$ V/m
(2) $E_z = 60 \sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t)$ V/m
(3) $E_y = 30\sqrt{2}\sin(0.5\times10^{11} x + 0.5\times10^3 t)$ V/m
(4) $E_y = 60 \sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t)$ V/m

Archives

JEE MAIN

Single Correct Answer Type

1. An electromagnetic wave in vacuum has the electric and magnetic field $\vec{E}$ and $\vec{B}$, which are always perpendicular to each other. The direction of polarization is given by $\vec{X}$ and that of wave propagation by $\vec{k}$. Then

(1) $\vec{X} \parallel \vec{B}$ and $\vec{k} \parallel \vec{B} \times \vec{E}$ (2) $\vec{X} \parallel \vec{E}$ and $\vec{k} \parallel \vec{E} \times \vec{B}$

(3) $\vec{X} \parallel \vec{B}$ and $\vec{k} \parallel \vec{E} \times \vec{B}$ (4) $\vec{X} \parallel \vec{E}$ and $\vec{k} \parallel \vec{B} \times \vec{E}$

(AIEEE 2012)

2. The magnetic field in a travelling electromagnetic wave has a peak value of 20 nT. The peak value of electric field strength is

(1) 6 V/m (2) 9 V/m

(3) 12 V/m (4) 3 V/m **(JEE Main 2013)**

3. Match List-I (Electromagnetic wave type) with List-II (its association/application) and select the correct option from the choices given below the lists:

List I	List II
(A) Infrared waves	(i) To treat muscular strain
(B) Radio waves	(ii) For broadcasting
(C) X-rays	(iii) To detect fracture of bones
(D) Ultraviolet rays	(iv) Absorbed by the ozone layer of the atmosphere

	(A)	(B)	(C)	(D)
(1)	(iii)	(ii)	(i)	(iv)
(2)	(i)	(ii)	(iii)	(iv)
(3)	(iv)	(iii)	(ii)	(i)
(4)	(i)	(ii)	(iv)	(iii)

(JEE Main 2014)

4. During the propagation of electromagnetic waves in a medium:

(1) Electric energy density is equal to the magnetic energy density.

(2) Both electric and magnetic energy densities are zero.

(3) Electric energy density is double of the magnetic energy density.

(4) Electric energy density is half of the magnetic energy density. **(JEE Main 2014)**

5. A red LED emits light at 0.1 watt uniformly around it. The amplitude of the electric field of the light at a distance of 1 m from the diode is

(1) 1.73 V/m (2) 2.45 V/m

(3) 5.48 V/m (4) 7.75 V/m

(JEE Main 2015)

6. An EM wave from air enters a medium. The electric fields are

$$\vec{E}_1 = E_{01}\hat{x}\cos\left[2\pi\nu\left(\frac{z}{c}-t\right)\right] \text{ in air}$$

$$\vec{E}_2 = E_{02}\,\hat{x}\cos\left[k(2z-ct)\right] \text{ in medium,}$$

where the wave number k and frequency ν refer to their values in air. The medium is non-magnetic, if ε_{r1} and ε_{r2} refer to relative permittivities of air and medium respectively, which of the following options is correct ?

(1) $\frac{\varepsilon_{r1}}{\varepsilon_{r2}} = \frac{1}{2}$ (2) $\frac{\varepsilon_{r1}}{\varepsilon_{r2}} = 4$

(3) $\frac{\varepsilon_{r1}}{\varepsilon_{r2}} = 2$ (4) $\frac{\varepsilon_{r1}}{\varepsilon_{r2}} = \frac{1}{4}$

(JEE Main 2018)

Answers Key

EXERCISES

Single Correct Answer Type

1. (2) 2. (1) 3. (3) 4. (1) 5. (2)
6. (3) 7. (2) 8. (1) 9. (3) 10. (4)
11. (2) 12. (1) 13. (2) 14. (3) 15. (2)
16. (3) 17. (1) 18. (3) 19. (1) 20. (3)
21. (2) 22. (3) 23. (4) 24. (4) 25. (4)
26. (1) 27. (2) 28. (4) 29. (2) 30. (3)
31. (3) 32. (4) 33. (2) 34. (4) 35. (4)

ARCHIVES

JEE Main

Single Correct Answer Type

1. (2) 2. (1) 3. (2) 4. (1) 5. (2)
6. (4)